This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

GODFREY LOWELL CABOT SCIENCE LIBRARY
of the Harvard College Library

This book is
FRAGILE
and circulates only with permission.
Please handle with care
and consult a staff member
before photocopying.

Thanks for your help in preserving
Harvard's library collections.

33.19

[illegible] X.720

Price U.X72

[illegible]

S. N. Swaim.

[illegible]

DYNAMICS,

CONSTRUCTION OF MACHINERY,

EQUILIBRIUM OF STRUCTURES,

AND THE

STRENGTH OF MATERIALS.

BY

George

G. FINDEN WARR.

ILLUSTRATED WITH TWO HUNDRED AND SEVENTY-SEVEN CUTS.

BEING

A CONTINUATION OF THE TREATISES ON MECHANICS, IN THE
LIBRARY OF USEFUL KNOWLEDGE.

LONDON:—ROBERT BALDWIN,
47, PATERNOSTER ROW.

1851.

Eng 258.51

1872, Oct. 10
Gift of
Joseph S. Swain,
of Cambridge
(H. U. 1873.)

LONDON:
GEORGE WOODFALL AND SON,
ANGEL COURT, SKINNER STREET.

PREFACE.

One object of the writer of the present Treatises is, to supply that which was wanting in the former part of the "Library of Useful Knowledge" on mechanical science. In three treatises Dr. Lardner has taken up the subjects—of the mechanical agents, or first movers;—the elements of machinery (statics);—friction, and the rigidity of cordage. In the present are considered the principles of moving forces or dynamics, the equilibrium of artificial structures, and the strength of materials used in the arts: the first is theoretical, and the latter three treatises are occupied with the most important matters of practical mechanics.

Dynamics has been too frequently omitted in elementary works as a distinct branch of mechanical science, or at most stated dogmatically and without demonstration of its principles, from the circumstance of its being treated by mathematical reasoning of a high order, and extended into other sciences, as physical astronomy, where dynamical problems attain their greatest intricacy; the endeavour here has been to use only so much of the simplest mathematics as appeared to be unavoidable, and more frequently to exclude it altogether, proving the truth of principles and laws by common logic, or reference to experimental evidence; and to confine the subject to its practical limits as a part of mechanical knowledge. In all cases the writer has endeavoured to illustrate the application of abstract principle by reference to objects in nature and proceedings in art.

The treatise on the Construction of Machinery has been so divided as to lay before the reader:—in the first place, a number of elements or simple parts used in various kinds of machinery to carry on and change the direction of motion, to produce straight from curvilineal, and curvilineal from straight, slow from fast, and irregular from regular movements. They are, therefore, the fundamental portions of every machine, and the explanation of their functions will enable the student to analyze the working parts of any machinery:—Secondly, to illustrate the building of our most modern machinery by examples of those used for the most general work, such as turning, planing, hammering, punching, cutting, wheel-cutting, and printing: these have been chosen because their operations are so evident as to require little description of the work done, others, perhaps more remarkable to the mechanical eye, such as spinning and weaving machines, developing their beauty by a long explanation of the articles made:—Thirdly, to explain the instruments by which the working power of machines is determin-

1 11/11

able:—and lastly, to supply the latest information relative to that grand consideration in the formation and use of any machine—the amount of the friction of substances when at rest or in motion. The apparatus of M. Morin, and his manner of using it to determine the coefficient of friction, have been given with some detail, as much instruction may be gained from the elegance of the instruments and the mode of experiment.

The EQUILIBRIUM of STRUCTURES is devoted to buildings in wood and stone. The former embraces structures made up of framework, and, commencing with the stability of the simple frame often used at the well's mouth to draw up a bucket, proceeds to roofs, domes, and centres for bridge-buildings. In the latter part, those structures built up of small pieces, as stone and brick, are considered, from the equilibrium of two irregular stones, one resting on the other, to walls and arches. The latter part of the treatise is mostly practical, in which the writer has endeavoured to explain, in few words, the proceedings of engineers in erecting bridges, in stone, wood, and iron, including the suspension principle, and to give some of the results of their experience. Illustrative descriptions of a number of bridges are added, which generally prove most instructive; those of the Britannia and Conway tubular bridges, to which comparatively considerable space has been devoted, deserve much attention, as showing what may be effected by persevering experiment on well-founded principles.

With the last treatise, on the STRENGTH of MATERIALS, some trouble has been taken to collect and condense the great amount of scattered information supplied by numerous experimenters. Our knowledge of this subject has greatly increased within the last fifteen years, and deserves a collection and careful collation: this is wanting; but it is hoped that the reader who desires not to be ignorant of the principal things known, and the principles deduced from modern experiments, will find a practical statement of them in the present treatise.

The mention of our age and nation are sufficient to answer for the remark, that the subjects contained in the present volume deserve some attention from all in the community: it is enough for those who set their hands to any division of labour to study each point in its practical minuteness; but of the leading principles which guide in the construction and cause the right action and endurance of machinery and other structures, none should be ignorant. For this general information the treatises are designed, and the writer has endeavoured, throughout, to state the results of human invention in a manner that should require nothing but the simple exercise of human reason and judgment to understand.

G. FINDEN WARR.

CONTENTS.

DYNAMICS.

CHAPTER I.

PAGE
Nature of the Subject.—Definitions 1

CHAPTER II.

Uniform Motion.—Inertia.—Composition of Forces.—Relation of Bodies in Motion 2

CHAPTER III.

Momentum and Collision 7

CHAPTER IV.

Accelerating Forces.—GRAVITY 12

CHAPTER V.

Motion on the Inclined Plane.—Motion on a Curve.—Principles of the Pendulum and its Application.—Compensating Pendulums . . . 14

CHAPTER VI.

Motion of Projectiles.—Resistance of the Air to Bodies moving in it . . 23

CHAPTER VII.

On ROTATION:—Rotation about a Fixed Axis.—The Centres of Oscillation, Percussion, and Gyration.—Illustrations of Rotation.—Centrifugal Force.—Principal Axes of Rotation 26

CHAPTER VIII.

Mathematical Illustrations:—The Parallelogram of Forces.—Uniformly Accelerated Motion.—Rotation about a Fixed Axis 40
Historical Notice of Dynamics 43

CONSTRUCTION OF MACHINERY.

CHAPTER I.

PAGE

ELEMENTS OF MACHINES.—Wheelwork: Pitch-Circle; Forms of Teeth; Friction Wheels.—Shafts and Spindles.—Simple and Differential Movements.—Gearing Apparatus 45

CHAPTER II.

ILLUSTRATIONS OF MACHINERY.—Turning-Lathe.—Planing Machine.—Steam Forge Hammer.—Punching and Plate-Cutting Machines.—Drilling Machines.—Wheel-Cutting Machine.—Printing Machines 71

Times Printing Machine 111

CHAPTER III.

DYNAMOMETERS: Regnier's; Prony's Friction; Colladon's Lever 94

CHAPTER IV.

FRICTION.—Morin's Apparatus and Experiments.—Friction of Axles.—Limiting Angle of Resistance.—Tables of Friction of Plane Surfaces and Axles . 99

EQUILIBRIUM OF STRUCTURES.

CHAPTER I.

Action of Forces.—Relation of contending Forces with respect to a Body on which they act.—Moment of a Force.—Stability of the Centre of Gravity . 117

CHAPTER II.

FRAMEWORK 120

CHAPTER III.

ROOFS.—Rafter and Tie-Beam; Thrust of the Parts; Truss with King-Post; Truss with Two Posts; Mansarde Roof; Roofs of the Teatro d'Argentino, St. Paul's (Rome), Mr. Nicholson's, the Birmingham Theatre, M. Larnier's, the Riding-House (Moscow).—Curved Roofs.—Norman Roofs.—Iron Roofs . 122

CHAPTER IV.

DOMES AND CENTRES.—Philibert Delorme's Dome; Domes of the Edinburgh Register Office and the Pantheon.—Strains on Centres: Centres of Neuilly, Blackfriars, and London Bridges.—Removal of Centres 134

CHAPTER V.

General Equilibrium of Structures in Stone, &c.—WALLS of Buildings and Revetments 139

CHAPTER VI.

PAGE

The ARCH.—Flat Arch; Equilibrium of different Forms; Coulomb's Investigation of the failing of an Arch; Line of Pressure in an Arch; Points of Rupture 144

CHAPTER VII.

BRIDGES.—Materials.—*Stone Bridges:* Forms of Arches; Foundations; Piers and Abutments; Laying the Arch-Courses; Spandrels; Key-Stone.—Bridges—London, Chester, Turin, Santissima Trinita, Waterloo, Y Pryd . 151

CHAPTER VIII.

BRIDGES (continued).—*Timber Bridges:* Character of Wood; Simple Beam and Trussed Girder; Lattice Bridges; Large Timber Trusses;—Bridges over the Clyde, Kandel, Cismone, Rhine (Schaffhausen);—Curved Timber Arches; Price's; Bridges near Kandy, at Bamberg, at Philadelphia, over the Ottawa, Tweed, and Delaware 168

CHAPTER IX.

BRIDGES (continued).—*Iron Bridges:* Nature of Iron Girders; Lattice and Busse's Forms; Fairbairn's Tubular Girder; Tubular Bridges over the Conway River and Menai Strait; Iron Arches at Colebrook Dale, Buildwas, Bishopwearmouth, Southwark, &c. 177

CHAPTER X.

SUSPENSION BRIDGES.—Historical Notice.—Catenary Curve; its Form and Tensions; Variation of Chain-Section.—Oscillations produced in Chain Bridges.—Of the Chains: Figure of least Tension; Effect of Temperature; Length of Links; Strength of Bolts; Wire Cables.—Of the Piers, and Chain Fastenings.—The Freiburg, Menai, Hungerford Market, Hammersmith, Niagara, Pesth, Berwick, and Dredge's Bridges 207

Tables of the Dimensions of the Principal Bridges in the World . . . 230

THE STRENGTH OF MATERIALS.

CHAPTER I.

General Notice.—Cohesion.—Elasticity, and its Modulus 233

CHAPTER II.

TENSION:—Its Laws; Extension of Bodies.—Strongest Form of Section.—Tensile Strength of Timber: Musschenbroek and Barlow; Causes of different Results.—Tensile Strength of Cast Iron: Early Estimates; Mr. Hodgkinson's Experiments on Hot and Cold Blast.—Minard and Desormes' Experiments.—Wrought Iron: Telford's, Brunel's, and G. Rennie's Experiments; Mr. Fairbairn's Trials of Iron Plate.—Wires and Mixed Metals.—Practical Reference to forming Joints 235

CHAPTER III.

PAGE

COMPRESSION; Physical Laws of Rupture by Crushing.—Compression-Power of Wood; Iron, Cast and Wrought; and Stone.—Strength of Pillars of Iron, Wood, and Stone 240

CHAPTER IV.

TRANSVERSE STRAIN.—The Neutral Axis.—Length, Depth, and Breadth Relations in a Beam.—Rules for calculating the Strength of Beams.—Equated Beams.—Deflection: its Laws; Defective Elasticity.—Effect of Impact and Motion on Beams.—Applications of the Principle of a Neutral Axis.—Transverse Strength of Wood, Iron, and Stone.—*Detrusion.*—*General Comparison of Strength* 250

CHAPTER V.

TORSION.—Nature of the Strain.—General Laws of Torsion; Rules for calculating it.—Experiments on Torsion; Mr. Bevan's Investigation, and Moduli.—Comparative Torsion 268

APPENDIX.

TABLE I.

Solid uniform Cylinders of Cast Iron (with rounded ends) 273

TABLE II.

Solid uniform Cylinders of Cast Iron (with flat ends) 274

TABLE III.

Short Solid Pillars of Cast Iron (flat at the ends) 275

TABLE IV.

Hollow Cylindrical Pillars, flat at the ends, of Low Moor Iron, No. 3. Length of each pillar, 7 feet 6¾ inches, except otherwise specified 275

TABLE V.

Hollow Cylindrical Pillars, rounded at the ends, of Low Moor Iron, No. 3, cast in dry sand; length of cylinder 7 feet 6¾ inches 277

TABLE VI.

Hollow uniform Cylindrical Pillars 279

TABLE VII.

Results of Experiments by Messrs. Fairbairn and Hodgkinson on Bars of Cast Iron from the principal Iron Works of the United Kingdom . . . 280

MECHANICS.

FOURTH TREATISE.

DYNAMICS.

CHAPTER I.—*Nature of the Subject—Definitions.*

(1.) IN the study of the phenomena of solid bodies with reference to the action of forces upon them, which form the proper objects of the science of Mechanics, it is both philosophical and convenient to divide the subject into two parts—one under which is classed that condition of matter termed rest, or where forces are opposed by equal forces; the other comprising every condition of motion, or where forces are not opposed by equal forces. These branches are called *statics*, or equilibrium; and *dynamics*.

(2.) In the former treatises, published under the direction of the Society for the Diffusion of Useful Knowledge, the science of equilibrium or statics was partially considered: in the present treatise, therefore, we shall direct our attention to that division of the science called dynamics.

(3.) The word *dynamics* means the science of moving forces generally; it is understood, however, to be applied to the conditions of *solid* bodies in motion, the effects of forces, and relation between bodies in motion. We have thus presented to us a wide, though most useful, field of inquiry.

(4.) The principal element which distinguishes dynamics is *time*—a principle which enters into every problem; time is measured by motion; without motion we should have no idea of time. The apparent motion of the sun and stars is the foundation of our information respecting time. In simplifying any problem for the purpose of calculation, the time taken into account is generally one second.

(5.) In the science of statics every case becomes simple on account of the non-interference of such circumstances as the weight of materials, and the particular constitution of matter; but when we attempt to calculate the effects of forces acting on mobile bodies, all particulars must be considered. The qualities of substances, as density, mass, and weight, are in frequent use in dynamical investigations.

(6.) The density of a solid body may be defined to be the relative closeness of its particles or atoms; the more compact a body is, the greater is its density. For instance, platinum, the heaviest of the metals, is above 22 times as dense as pumice-stone, which will float upon water. Water at 60° (Fahrenheit's scale) is taken as the unit of measure of the specific gravities or densities of solid bodies. In the treatise on *Hydrostatics* the reader will find a table of specific gravities.

(7.) The word *mass* means quantity of matter; so that to find the mass of a body it is necessary to take the product of the density and the size or volume. If we have a volume of lead, and one of iron, each containing eight cubic inches, their masses are not equal; for the density of lead is about 7, and of iron about 11 times that of water; therefore their masses will be found respectively $7 \times 8 = 56$, and $11 \times 8 = 88$, or the mass of the iron will be to the mass of lead as 88 to 56. Confusion frequently arises in the mind from considering the mass and weight as identical, which they are not, as may be seen from the following consideration. A cubic inch of lead weighs on the surface of the earth $6\frac{1}{2}$ oz.; on the sun it would weigh $11\frac{4}{10}$ lbs.; so that the weight of a body varies according to its position with reference to another body which attracts it, but the mass remains the same in both cases.

(8.) The term *force* is applied to signify a producer of motion, or a resistant to motion. The idea of force is generally associated in the mind with activity; we know nothing of the nature of forces in themselves, but by the effects produced on any body. When motion commences, we say a force caused it; or if a motion be suddenly destroyed or retarded, we also say a force effected it. There is, however, a measure of force in the velocity of motion which it creates.

(9.) *Velocity* is associated with force in consequence of an apparent relation between them. If we suppose a body to be composed of a number of particles, any force applied to it will produce in each particle a certain rate of motion or velocity; if there be twice the number of particles the force will be distributed amongst twice the number, so that half the velocity will result. Forces are therefore known by the velocities they can produce in any masses. Practically considered they are supplied by air, steam, water, gravity, springs, and animal exertion.

Chapter II.—*Uniform motion.—Forces acting obliquely.—Relation of bodies in motion.*

(10.) A body is said to move uniformly when its velocity continues the same during the time of motion; so if a ball could be made to run along the ground at the rate of ten feet in one second, and twenty feet were described in two seconds, we should say it moved uniformly. This is simple enough if we could obtain motion thus equable, but that is not possible naturally: we meet with no uniform motion in nature; the qualities of matter are such that when any motion takes place it is either accelerated or retarded, from the stone thrown carelessly into the air to the great planetary bodies which move through space; the same circumstances may be observed, that where matter causes motion in matter by its attractive power, that motion will not be uniform.

It might be considered, from these remarks, that it is of little value to consider uniform motion; but it will be seen that we found our calculations of variable motions upon the supposition of uniform motions. Again, we have uniform motion, practically speaking, in machinery.

(11.) We observed, in art. (9.), that the velocity of any motion was to be measured by the time and space, or distance. Suppose a ball was allowed to run with a uniform motion for 100 feet, and that it occupied 5 seconds in its passage; to find its rate *per second* we must of course divide the whole distance run by the time of running, that is, $\frac{100}{5} = 20$; therefore the velocity per second of the ball's motion was 20 feet. Showing the velocity, we can find the time which any body will take to run any required distance, or the distance it will run in any required time. In the first case, if the distance be given, as 100 feet, and the velocity 20 feet per second, then, $\frac{100}{20} = 5''$, the same as stated above; in the second case, using the same velocity, the distance run by a body in 5 seconds will be $5 \times 20 = 100$ feet.

(12.) From this simple illustration we deduce the following results:—

1. The velocity of motion of a body is equal to the quotient of the space divided by the time.
2. The time of motion is equal to the space divided by the velocity.
3. The space described is equal to the product of the velocity and the time*.

(13.) Although we took the motion of a ball on a level surface for illustration, and considered it uniform, we know by experience that, with whatever force a body may be made to move on any surface, the velocity gradually decreases; if the ball be made to move 100 feet in 5 seconds, it moved more than 20 feet in the first second, and less than 20 feet in the last second. The ball, in its state of motion, had to contend against the opposing power of friction and the resistance of the air, which are continually retarding its motion; whereas the force which impels the ball was merely a momentary impulse. According to our conclusions stated above, the velocity was supposed uniform, which would be the case if the retarding forces could be put aside; this assertion cannot be shown to be true by any form of experiment, but may be deduced from the nature of the thing, and the argument which experiment supplies. If we attempt to roll a ball along a rough road, it quickly comes to a state of rest; if it be tried on a smooth and

---

* These rules may be stated more conveniently, by putting v for velocity, t for time, and s for space. Then,

$$v = \frac{s}{t}, \qquad t = \frac{s}{v}, \qquad s = vt.$$

level pavement, it will run much farther; if it be projected on a sheet of ice, it will appear for some time to move with the velocity at first given, and run to a great distance. It may be argued from this, that if we could prevent the little friction between the ball and the ice, and the resistance of the air, the ball would never stop.

(14.) The principle we have here deduced forms part of the first of the celebrated laws of motion, as stated by Sir Isaac Newton in his *Principia**, which is, that all bodies will continue in their state of rest or motion; that is, if they be in motion they have no tendency or power to stop, and if at rest no power to move; the latter assertion is easily acknowledged, as we find that every body resists our efforts to move it, but the former cannot, as we have seen, be so plainly shown to the physical senses; indeed many of the old philosophers supposed that rest was the natural state of matter, which it always sought when in motion, and accordingly termed this passiveness of matter *inertia*, or idleness.

(15.) The effect of this law is of great consequence in mechanical contrivances for producing motion; for it is easily seen that when the least possible expenditure of power is required to be made we must reduce the retarding causes, of which friction is the most important at moderate velocities; and also that when we have made the machine work as freely as possible, we require but the power to balance the retarding forces when the whole is in motion; when that is effected, the machine will continue to work uniformly if the work to be done offers a uniform resistance. Such considerations prevent the undue expenditure of power and useless wear of the parts of a machine, which are points of great importance in practical mechanics.

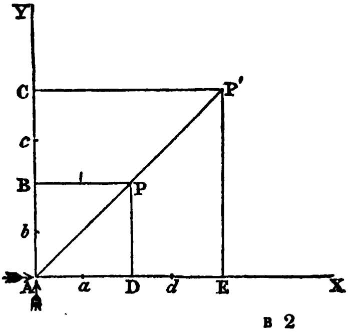
(16.) It may also be observed, that whatever amount of force is exerted, above that which is required merely to produce motion, will increase the velocity of the mobile parts of the machine; in the case of a railway train, for instance, a certain amount of power is exerted by the locomotive engine, which produces motion in the train; once moving, the engine is required only to overcome the friction of the moving train and the resistance of the air, when the train runs on a level; but as the engine has more power than is required for those purposes, it continues to exert a pulling force on the train, which increases its velocity till it reaches a certain rate called its *maximum*, that is when all the forces become equal.

(17.) We have hitherto been considering the effect produced on a body by one force; if two forces act at the same time, the result will vary according to the relative direction and intensity of the two forces. When a body in motion is impressed with a fresh force acting in the same direction, the effect will be an increase of velocity in the original direction. If the force act oppositely there will be a decrease in the velocity of the moving body proportional to the force. The third condition proposes to consider the effects of forces acting obliquely, or perpendicularly to each other.

(18.) In a former treatise on Mechanics it was shown that when two or more *pressures* act on a body it will take up some position of rest or equilibrium, which may be drawn geometrically, exhibiting both the direction and amount of the pressures. The same may be done in the present instance; the amount of the forces impressed and the direction of the body's motion being expressed by a simple construction. We suppose at present that the forces are acting in one plane, and the body free to move in that plane.

(19.) Let it be required to find the position of a body at any time, and the distance run when impressed by two equal forces acting at right angles to each other.

Fig. 1.



---

* 1. That every body perseveres in its state of rest or uniform motion in any direction, unless urged to change that state by forces impressed.

2. That change of motion is proportional to the moving power impressed, and is in the straight line in which that force is impressed.

3. Action is always contrary and equal to re-action, or the actions of two bodies upon each other are equal, and in contrary directions.

Draw the two lines A X, A Y to represent the direction of the two forces at right angles to each other, and measure an equal length, as A B, A D, equal to the distance the body would have run by the effect of each force separately, then draw B P parallel to A D, and D P parallel to A B, and the point P will show the relative position of the body impelled by both forces conjointly, having occupied the same time as it would to move from A to B or D by the solitary action of each force*.

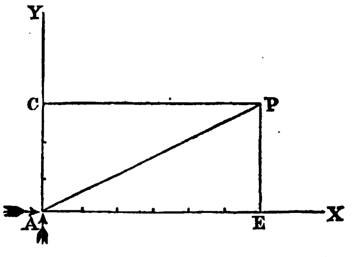
(20.) As an illustration of this construction, suppose a ball at A to rest on a smooth surface and be struck by two equal forces, each of which would produce a velocity of 2 feet per second; what velocity will the two forces acting perpendicular to each other produce, and how far will the ball have travelled in 2 seconds? By the principles before mentioned, the space described in 2 seconds by a body moving at the rate of 2 feet per second is 4 feet; then taking any unit of measure we please, as A *a*, we may measure it four times on each line forming the two lengths A E, A C; the body therefore will describe the line A P′ in the two seconds. To find the real amount required, it must be remembered that to calculate the hypothenuse of a right-angled triangle we take the square root of the sum of the squares of the other sides†, so that in our figure $AP' = \sqrt{AE^2 + EP'^2}$; we have the length of $AE = 4$ feet, and E P′ is also equal to 4 feet, being equal to A C, therefore

$$AP' = \sqrt{16 + 16} = \sqrt{32} = 5\tfrac{1}{2}$$

nearly. From this we find that the resulting velocity will be about 2¾ feet, and the distance run in two seconds will be above 5½ feet.

(21.) When the forces are unequal, we have merely to measure on that side in the direction of which the force acts, a proportional number of units. In *fig.* 2, the forces are represented unequal; that in the direction of A Y producing a velocity of 3 per second, while that in the direction A X is 6, and the body will describe the diagonal A P, and may be found as before.

Fig. 2.



(22.) Where the two forces do not act at right angles, they either partially destroy or assist each other. The latter is the consequence when their directions make an acute angle, as in *fig.* 3, by which it will be seen that the increase D E, in the space described in the direction A X, is owing to the obliquity of the direction of the force in A Y. The calculation of the space and velocity in this case is rather more difficult, as the distance D E must be measured. Suppose the velocities and times as before, namely 2 feet per second, and 2 seconds the time of motion; then,

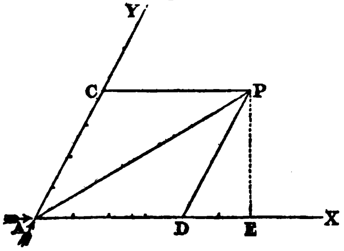
$$AP^2 = AD^2 + DP^2 + 2\,AD \times DE^*$$

in the figure $DE = 2$ of the spaces or units, therefore we have

$$AP^2 = 16 + 16 + 16 = 48,$$

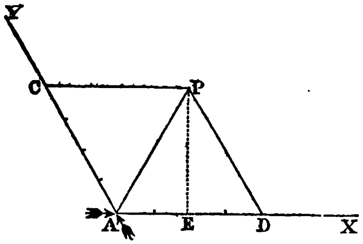
and $AP = \sqrt{48} = 7$ nearly, so that the ball will have a velocity of 3½ feet per second, and move in two seconds through 7 feet.

Fig. 3.



(23.) The other form of oblique ac-

Fig. 4.



---

* See Chap. 7, *Mathematical Illustrations*.

† *Geometry*, Book I. Prop. 36, or *Euclid*, i. 47.

* *Geometry*, I. Prop. 37. *Euclid*, ii. Prop. 12, 13.

tion is shown in *fig.* 4, whence it appears that the distance run is diminished, or that some part of the two forces has been destroyed. Measuring the units on each line as before, we find that the ball has progressed in the direction A X but one-half the distance due to the force in that direction alone, for, drawing the perpendicular P E, it bisects the line A D. Then to calculate the real space described and velocity of motion as before, again using the same amounts,

$$AP^2 = AD^2 + DP^2 - 2AD \times ED.$$

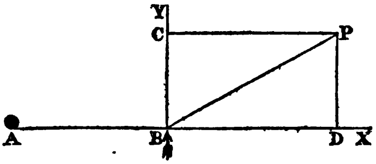
D E being equal to 2 units,

$$AP^2 = 16 + 16 - 16 = 16,$$

and $AP = \sqrt{16} = 4$; therefore the ball will move with a velocity of 2 feet, and the distance run in 2 seconds will be 4 feet.

(24.) When any body in motion is acted on by a force at right angles to its line of motion, a similar effect to that already described is produced. If the ball A, *fig.* 5, be moving toward X and when it reaches the point B, a force acts upon it in the direction B Y, the ball will move towards P, and describe B P in the same time in which it would have described B D or B C by the action of either force alone.

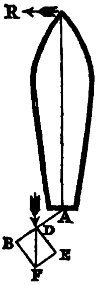
Fig. 5.



(25.) What we have stated in the preceding articles follow from the second law of motion. In the first treatise the principle was referred to statically, that is, showing that two or more pressures might be equilibrated by a pressure in the direction of the resultant of all the pressures; in the present case we suppose no equilibrium to be established, but use the same means to find the resulting motion.

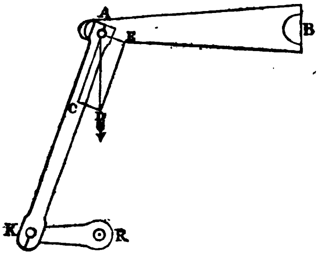
(26.) The effect of the action of two forces is very strikingly illustrated in the turning of a vessel by its rudder,—a little instrument, but holding an astonishing influence over the motions of the vessel, which presents but little direct resisting surface to the current, while the rudder, when turned so as to be perpendicular to the stream, presents a complete obstruction to the current. Let A B, *fig.* 6, represent a section of the rudder, the current running in the direction of the arrow D; the effect of the force upon the oblique surface of the rudder may be found by drawing the line D E perpendicular to B A, to exhibit the resistance of the rudder, F E parallel to B D, and joining B F; then the force represented by D F is resolved into two, D B and D E; the former exerts no influence on the rudder, but the latter tends to push it back; this being prevented by the helmsman, the head of the boat will move as indicated by the arrow R, until the rudder is again in the direction of the stream.

Fig. 6.



(27.) Again, the arrangement of the crank-rod of a steam-engine affords an instance of loss of power, which may be shown by this principle. Let A B, *fig.* 7, represent part of the beam of a steam-engine, and A K the crank-rod; then the beam, in descending, pushes in the direction of the arrow D; this, however, is not the line in which the resistance of the crank-rod is directed, which is K A; therefore if we draw the line A C along the crank-rod, A D for the direction of the motion of the beam,

Fig. 7.



and form the parallelogram, A E C D, we find that the crank-rod receives the force A C, the remaining force A E serving merely to wear away the axis at A, and derange the parts of the machine. We may here observe that the less the crank K R, and consequently the less the angle C A D, the less will A E be; the crank, however, must not be shortened beyond a certain amount, otherwise the loss there would be as great as the loss from the first-mentioned cause.

(28.) We have hitherto examined the effect of forces on a single body, where two or more bodies are in motion, and we require to establish a relation between them, or to find their relative position at any time, the motion of each must be first determined, and then their relation may be found. For instance, suppose two men to be running in the same direction and at the same speed, one having the start of the other by 20 feet, they will evidently be 20 feet distant from each other at any part of their journey, and 20 feet also when they stop; but if the speed of the first be greater or less than that of the other, he will either increase his relative distance, or the second will come up with him. On account of its simplicity we will again make use of a rolling ball to establish a rule.

(29.) According to a former rule (art. 13), a ball moving at the rate of 3 feet per second, will have run in 5 seconds, $5 \times 3 = 15$ feet; suppose another to start at the same time in the same direction, with a velocity of 2 feet, it will have travelled in 5 seconds, $5 \times 2 = 10$ feet; therefore the distance between them is the difference of these distances, or $15 - 10 = 5$ feet; if either had any start of the other, it is only necessary to add the amount to its distance; thence we deduce as a rule for bodies moving in the same direction:—

The distance between two bodies at any time, when moving uniformly, is equal to the difference of the distances moved by each[*].

(30.) Suppose an army to set out for military operations 100 miles distant, travelling at the rate of 20 miles a day; and a day after a brigade commences the same march, but at the rate of 25 miles a day; how far will they be from each other at the conclusion of the fourth day's march, reckoning from the time the brigade started? In this case the army has 20 miles start of the brigade, and is moving at a slower speed. Then, $20 \times 4 + 20 - 25 \times 4 = 0$. They will consequently meet after four days' march; and also they will have reached the place of action.

(31.) Again, suppose it were required to make a siding[†] on a line of railway at such a distance from the terminus that a luggage or slow train, moving 16 miles an hour, and starting regularly three hours before a fast train which moves 36 miles an hour, may arrive in time to run off and allow the fast train to pass without stopping. The luggage train has a start of $16 \times 3 = 48$ miles, but in this instance no time is given, because the time of their meeting is required to find the distance run. Taking the form given by the last example, we have, $16 \times \text{time} + 48 - 36 \times \text{time} = 0$, that is, the distance between them is nothing, or they meet, which is according to the conditions of the problem. Transpose $36 \times$ time to the other side, and divide by the time; then,

$$16 + \frac{48}{\text{time}} = 36, \text{ or } \frac{48}{\text{time}} = 36 - 16 = 20, \text{ or } 48 = 20 \times \text{time},$$

and lastly $\frac{48}{20} = \text{time}, = 2\frac{2}{5}$. We have thus easily found the time; then we have for the distance of each train from the terminus,

$$16 \times 2\tfrac{2}{5} + 48 = 86\tfrac{2}{5}, \text{ and } 36 \times 2\tfrac{2}{5} = 86\tfrac{2}{5};$$

therefore the siding must be made about $86\frac{2}{5}$ miles from the terminus.

---

[*] If t = time, v = lesser velocity, V = greater velocity, s and S = space described by the balls moving with the lesser and greater velocity respectively, and D = start of either ball, then $S - s = (V - v)\, t \mp D$[*]; so that to find the time when the two will meet, if that be possible, $t = \frac{D}{V - v}$.

[†] A "siding" is an extra pair of rails laid beside the regular line, on which luggage and parliamentary trains may run, to enable regular passenger trains to pass by them.

---

[*] The negative sign being used when the ball moving with the lesser velocity has the start, and the positive in the other case.

Chapter III.—*Momentum and Collision.*

(32.) In the application of forces to produce any required effect, an instrument or machine is generally made use of to modify and direct the force in a proper manner, so that the applied force acts either as a pressure or as an impulsive power. These two forms are generally quite distinct from each other, their effects being of such a kind as to prevent any just comparison between them. We shall, in this chapter, consider the nature of this impulsive force or impact.

In our previous examinations, forces were treated of independently of the body or bodies to which they may be applied. By experience we know that to raise a hundred weight we must bring into action more force than is required to raise half that amount; and if the two bodies be thrown from the hand, they will strike any obstruction with very different amounts of force: a worsted ball may be thrown against a window without breaking it, while a brass ball of a similar size would immediately produce a fracture. Again, when a light hammer is used to perform any work, we make it descend with greater speed than if it were a heavy hammer. We learn, therefore, from daily observation, that in the intervention of matter between forces and the objects to which they are applied, there are circumstances which ought to be considered.

Of equal masses the velocities are proportional to the forces.

With equal forces the velocities are inversely as the masses moved.

With equal velocities the forces are proportional to the masses.

(33.) The quantities which enter into the calculation of the effects produced by moving bodies are the mass of body and its velocity of motion. These, multiplied together, constitute the momentum of the body, or its quantity of motion.

This force can therefore be found and compared in any number of bodies when these two quantities are known. We say compared, because it is not referable to any standard, as we speak of a pound or ounce weight. Thus, suppose a cannon-ball of 15 lbs. weight, and moving with a velocity of 1500 feet per second, to strike against a wall, we might say its momentum, according to the above rule, is $1500 \times 15 = 22{,}500$, not pounds weight, for weight is a pressure, with which it cannot be compared, but greater than a ball moving at the same rate, and of 1 lb. weight. Again, if there be two balls of 21 lbs. each, giving them the same velocity, they will produce an equal amount of effect; but if one be driven with a velocity of 1000, and the other a velocity of 2000 feet per second, the momenta are, for the first, $1000 \times 25 = 25{,}000$; and for the second, $2000 \times 25 = 50{,}000$, which is double the first amount.

So also we may find the velocity required of any moving body whose weight we know, to produce an effect equal to any given amount; for as the momentum = mass × velocity, so the velocity required $= \dfrac{\text{momentum}}{\text{mass.}}$ If a ball of 50 lbs. weight have a velocity of 1000 feet per second, its momentum is 50,000; then a ball of 20 lbs. weight, to have an equal amount of momentum, must have a velocity equal to $\dfrac{50{,}000}{20} =$ 2500 feet per second.

(34.) The force of momentum is a result which follows from the principle of inertia: if we expend so much force to put any body in motion, by the nature of matter it will require as much force to destroy that motion. Although this is self evident, its general effect was very imperfectly considered in the early states of society. They were content with the most rude applications of the power, as is seen in their military engines.

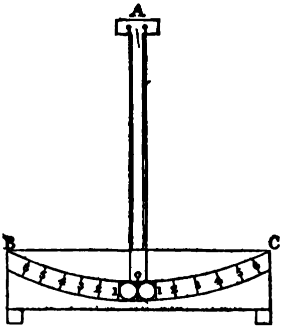
The peculiarity of the force of momentum is that its effects upon solid bodies are local, or confined to the spot where the force is applied, especially when a body moves with a great velocity. If a child press on a pane of glass with his finger, if not sufficient to withstand the force it will fracture in several directions; yet a small shot, moving with considerable velocity, will penetrate the glass and form a small hole, leaving the other part of the pane undisturbed. It appears, in such a case, that in consequence of the instantaneous action of the force, sufficient time does not elapse to enable the particles of glass which are struck by the shot to communicate motion to the particles around; they are instantly torn away, and the action of the force is no longer exerted on the pane. In the pressure of the finger there is time enough for the effects of the force to reach every particle of glass in the pane; and as the pressure becomes greater those parts give way which, from several causes, have the least cohesion. No pressure would therefore produce the same effect as the small shot moving with a high velocity.

From this distinctive character of the effects of momentum, it is always used where a sudden effort is required and great resistance to be overcome. For the purpose of driving a nail into a piece of hard wood a pressure would be incompetent; but a few quick blows from a small hammer are generally sufficient. In the operations of a siege this force is most useful: it may be required to destroy a wall or other obstruction which is of so great a mass that no pressure would be serviceable; but by cannonading, the stones forming the wall are loosened, a groove is gradually cut out; and this being repeated at another part, the wall between is weakened: the whole fire is then directed against the weakened portion till it falls.

(35.) There is an important consideration arising from these statements, which is that every machine should be made of a weight or mass proportional to the weight of the work for which it is intended. This forms a conspicuous element in the construction of any machine; and proper adjustment in this respect contributes mainly to give regularity and apparent ease in the performance of the several parts. The fly-wheel of a steam engine affords a striking instance of the value of mass in a machine; for its momentum when in motion being so greatly superior to any sudden resistance which it is likely to receive from the machinery driven, the engine will suffer no perceptible derangement in its motion.

(36.) When two or more bodies come into collision with one another, and are moving or free to move, the subject becomes more extensive; for a most useful and well-known property of matter must be considered, namely the *elasticity* of solid bodies; that is the power which any substance has, when compressed or bent, of returning to its original form. This singular property is not possessed equally by all substances: lead has but little elasticity, while glass is almost perfectly elastic, and the most elastic substance with which we are acquainted; so that all matter is but partially elastic.

Fig. 8.



(37.) The different elastic power of substances is very plainly observed by the following apparatus. From the hooks at A are hung, by fine threads, two balls of the substance intended to be tried. They must be of equal weight, and suspended so that, when they touch, their centres may be at the same distance from the point of suspension. Immediately behind them is the scale B C, on which is described the arc of a circle, divided into equal parts, commencing from the lowest point, which is 0, or zero, and numbering upwards on each side. If the balls be raised to equal distances on each side of the point 0, and allowed to fall, they will meet again at the point 0, when their elastic quality allows of an instantaneous compression, when the balls separate and fly backwards from each other, ascending the arc through which they descended, but not exactly to the same height; the difference between the length of the arc descended and the length ascended, will exhibit the elasticity of the substance as compared with perfect elasticity. If it had been perfect, the balls would have ascended to the point whence they commenced descending. Suppose they were two balls of box wood, it would be found that, if lifted to the division marked 6, and let fall, they would, after striking, return up the arc to about the division marked 5. Perfect elastic power would have returned them again to the sixth division; but being partial only, these numbers express the ratio of the elasticity of box wood to perfect elasticity, or as 5 to 6, or $\frac{5}{6}$. This apparatus was used by Newton, and is described in his great work, the *Principia*. He thus found that, with balls of worsted, the ratio was 5 to 9; with ivory, 8 to 9; and glass, 15 to 16. The elastic power was therefore $\frac{5}{9}$, $\frac{8}{9}$, $\frac{15}{16}$*.

(38.) If any substance were quite inelastic, equal masses of it, moving in opposite directions with equal velocities and coming into collision, would immediately stop; for the momenta would be equal, and consequently neutralize each other. The investigation of the effects of moving bodies on other moving or moveable bodies is much simplified by considering them as non-elastic. We shall therefore state a few general laws of collision, when it will be easy to apply the correction for partial elasticity to the results obtained.

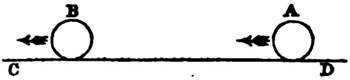
1. If two solid bodies be moving in the same direction, the common velocity, after collision, is equal to the sum of the products of the masses and their velocities, divided by the sum of the masses.

2. If two bodies be moving in opposite directions, the common velocity is equal to the difference of the products of the masses and velocities divided by the sum of the masses

3. If one body be at rest and the other in motion, the common velocity is equal to the product of the mass and velocity divided by the sum of the masses.

(39.) The most simple manner of illustrating these laws is by supposing the masses to be balls; because it is requisite to exhibit the proper effects of collision that the two bodies strike each other so that the line joining their centres of gravity may pass through the point of concussion; if it do not, the effect of the impact will not merely be an increase or decrease of velocity, as by the above rules, but complex motions ensue, as we shall have occasion to notice when speaking of rotation. In spheres the centre of gravity is situated in the centre; consequently, when two balls of the same size strike each other on a plane, their centres of gravity and point of contact will be in the same straight line.

Fig. 9.

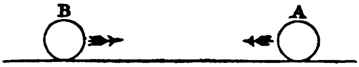


Suppose the ball A, of 2 lbs. weight, (*fig.* 9,) to be moving along C D at the rate of 4 feet per second, and to strike another ball B, also 2 lbs. weight, moving in the same direction, as indicated by the arrows, with a velocity of 3 feet per second, the rate of motion of both, after collision, will be

$$\frac{A \times \text{its velocity} + B \times \text{its velocity}}{A + B};$$

that is $\frac{2 \times 5 + 2 \times 3}{2 + 2} = 4$ feet per second. If the balls be unequal in size, so that A = 2 lbs., while B = 6 lbs., then $\frac{2 \times 5 + 6 \times 3}{2 + 6} = 3\frac{1}{2}$ feet per second for their resulting velocity.

Fig. 10.

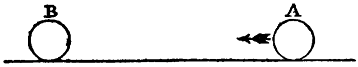


In the second case the bodies move in opposite directions. Let A = 3 lbs., having a velocity of 5 feet per second; and B = 3 lbs., with a velocity of 3 feet per second; move in the directions denoted by the arrows; after impact their velocity will be

$$\frac{A \times \text{its vel.} - B \times \text{its vel.}}{A + B}; \text{ or}$$

$\frac{3 \times 5 - 3 \times 3}{3 + 3} = 1$. That is, B will have its original motion destroyed, and move with a velocity of 1 foot per second in the direction of A's motion. If A's motion were less than that of B, the resulting quantity would be *minus;* that is, it would move in B's direction.

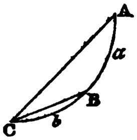
Fig. 11.



In the third case one body is at rest; then if A be a ball of 5 lbs. weight, moving at the rate of 4 feet per second,

---

* The above is merely a rough statement of the method of determining the elasticity; for the velocities acquired in falling down arcs of circles are as the chords of the arcs; so that if we let a ball fall from A, and after collision with the ball at C, it returns as far as B, the ratio would not be the lengths of the arcs, C *b* B, and C *b* B *a* A, but the chords of the arcs, C B, C A, which express the ratio of the velocities. In experiments, the chords of the arcs must be measured. The chords of small arcs, however, differ but little from the arcs themselves in length; and the smaller the arc the smaller the difference. Some excellent experiments have been made on the elasticities of different substances by Mr. Eaton Hodgkinson, to which we shall have occasion to refer.

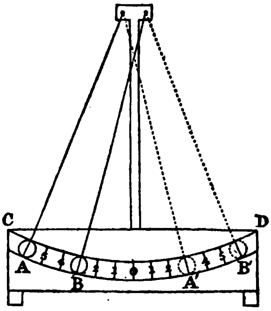


and B another ball of 5 lbs. weight, at rest, both balls, after impact, will move on with a velocity of $\frac{5 \times 4}{5 + 5} = 2$ feet per second. Thus the body in motion transmits half its velocity to the body at rest, both being of equal mass.

(40.) In the collision of partially elastic substances we shall find that if two balls, under the above-mentioned conditions, meet, they will not move on together, there being a recoil of the impinging ball proportional to its superior velocity to that of the second ball, and the elastic power of the substance. In each case two equal elastic balls meeting will exchange their states.

Thus, if they move in similar or opposite directions, the ball having the greater velocity will transfer it to the other and receive the lesser velocity. Where one strikes another at rest, the whole motion of the first is given to the ball at rest; the striking ball, receiving no motion, remains at rest.

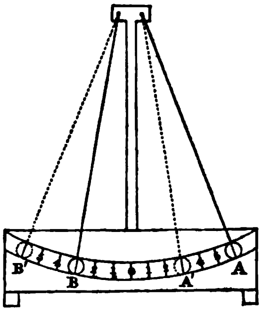
Fig. 12.



The simple apparatus shown in *fig.* 8 will well illustrate these circumstances. If A B be two equal balls of wood or ivory, and they be raised on one side of the perpendicular to different heights, as in *fig.* 12, A will strike B when they reach the lowest point, or 0, and recoil a little, giving greater velocity to B; so that when they arrive at their highest point on the other side, their positions will be A′ and B′; whence it is plain they have exchanged velocities.

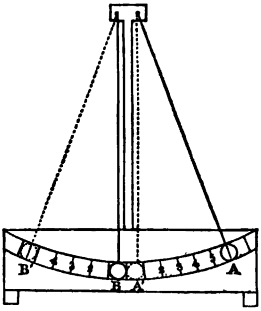
Again, we have said that when the two partially elastic balls strike each other with equal and opposite velocities, they rebound to about the point whence they set out; they really exchange velocities. This may be shown by raising the ball B, as in *fig.* 13, to the division

Fig. 13.



marked 3, and A to the division 6; when allowed to fall they will meet at 0, and be driven back, A to the point A′, or 3, and B to B′, or 6, nearly; so that they evidently exchange velocities.

Fig. 14.



Also, if B be at rest, as in *fig.* 14, and A raised and let fall, on striking B it will stop; while B, taking A's motion, will describe the arc B B′, nearly equal to that down which A fell.

(41.) Perfect elasticity would cause the following alteration in the rule for bodies meeting each other. If the ball A, *fig.* 9, struck against B, moving in the same direction, they would not have a *common* velocity; but the velocity of A would be

$$\frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} + 2\,\text{B} \times \text{its vel.}}{\text{A} + \text{B}};$$

and the velocity of B =

$$\frac{\text{B} \times \text{its vel.} - \text{A} \times \text{B's vel.} + 2\,\text{A} \times \text{its vel.}}{\text{A} + \text{B}};$$

so that the difference between this and the former rule consists in the addition of twice the weight of A and B, multiplied by their respective velocities. Therefore the ratio of the elasticity of any substance must enter as a multiplier into the calculation. Suppose the impact of two ivory balls to be under consideration, the form of the above would be—

$$\text{Velocity of A after impact} = \frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times \frac{5}{9} + (1 + \frac{5}{9})\,\text{B} \times \text{its vel.}}{\text{A} + \text{B}}$$

$$\text{Velocity of B after impact} = \frac{\text{B} \times \text{its vel.} - \text{A} \times \text{B's vel.} \times \frac{5}{9} + (1 + \frac{5}{9})\,\text{A} \times \text{its vel.}}{\text{A} + \text{B}}$$

Thus, suppose two balls of ivory, A = 3 lbs., its velocity = 5 feet per second, and B = 3 lbs., its velocity = 3 feet per second, to come into collision, the velocity of A would afterwards be

$$\frac{3 \times 5 - 3 \times 5 \times \frac{5}{9} + (1 + \frac{5}{9})\, 3 \times 3}{3 + 3} = \frac{15 - \frac{75}{9} + (9 + 5)}{3 + 3} = \frac{20\frac{2}{3}}{6} = 3\tfrac{8}{18}.$$

And the velocity of B =

$$\frac{3 \times 3 - 3 \times 3 \times \frac{5}{9} + (1 + \frac{5}{9})\, 3 \times 5}{3 + 3} = \frac{9 - 5 + (15 + 8\frac{1}{3})}{3 + 3} = \frac{27\frac{1}{3}}{6} = 4\tfrac{10}{18}.$$

The rule may be applied to the two other cases of impact. When A and B move in opposite directions—

$$\text{Velocity of A} = \frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times \frac{5}{9} - (1 + \frac{5}{9})\,\text{B} \times \text{its vel.}}{\text{A} + \text{B}}$$

$$\text{Velocity of B} = \frac{\text{A} \times \text{B's vel.} \times \frac{5}{9} - \text{B} \times \text{its vel.} + (1 + \frac{5}{9})\,\text{B} \times \text{its vel.}}{\text{A} + \text{B}}$$

When A moves, and B is at rest,—

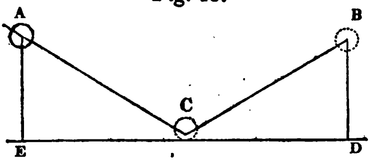
$$\text{Velocity of A} = \frac{\text{A} \times \text{its vel.} - \text{B} \times \text{A's vel.} \times \frac{5}{9}}{\text{A} + \text{B}}$$

$$\text{Velocity of B} = \frac{\text{A} \times \text{its vel.}\,(1 + \frac{5}{9})}{\text{A} + \text{B}}.$$

These rules apply to all cases where parts of machines come into collision; and the effects produced may then be easily determined.

(42.) The effect of partial elasticity is observed when a stone is thrown upon the ground, or a ball impelled against the side of a billiard table. If a round stone be let fall on hard pavement, it rebounds to a certain height; if it were quite elastic it would reach the point whence it fell. If it be thrown obliquely, as in *fig.* 15, it will be reflected to an angle B C D less than that at which it struck the fixed object, which cannot take any of its motion; the ratio of the lines A E, B D consequently express the ratio of its elasticity. Although the fixed object does not move, its particles are put into a state of vibration by the impact of the moving body; and if another ball rest against the opposite side, it would be impelled with the velocity of the striking ball, no matter how thick or long the interposing body. This may be strikingly shown by fixing a long rod of wood so that it will not move along when struck at either end, and suspending a ball so as to touch one end of the rod. If the other end of the rod be gently struck, the ball will instantly fly off, while the rod has not moved at all from its position.

Fig. 15.



(43.) In estimating the effects of momentum there has been much dispute whether the weight should be multiplied by the velocity or the square of the velocity to express its magnitude, which has arisen from the manner in which the subject is considered. The first method follows from what we have said of uniform forces: a uniform force acting on a body for two seconds will produce twice the velocity which would be acquired in one second; and also, if double the force acts for one second, the same effect is produced as in the first case; and as the moving body is supposed the same in both cases, the momentum or quantity of motion will be the same. However, in many instances, the observed effects of moving bodies on striking solid substances appear to require the square of the velocity to express the true value. In such cases the calculation is made with the weight of the velocity, multiplied by the height which it must have fallen through to acquire the velocity, and the heights are proportional to the square of the velocities.[*] Thus, if we have a velocity of 2 feet per second in a hammer by allowing it to fall 4 feet, to obtain double that velocity we must raise it 16 feet; for the square root of 16 is 4, which is the velocity required. When, therefore, we multiply by the height, we multiply by the square of the velocity acquired in falling through that height. The quantity thus obtained is called *the vis viva*, *force vive*, living force, or energy of the body in motion.

"We can value the effect of a power in two ways; for example, if we take the case of a man, we consider either what load he can support, or what work he can do in a given time. In the first case the forces are compared with a *dead force*, that is with a force which can equilibrate them—the term arising from the circumstance that they are destroyed as soon as they are brought into action. We have shown, then, that the forces are to one another as the products of the masses and the velocities; in the second case we compare them to a *living force*, that is to one which would elevate a weight to the same height in the same time; and it is evident that, in this way of estimating the effect of a force, the effect is composed of the weight and height[†]. . . . . We wish to compare together the elements which form the result of the work when the effect of machines or movers in motion is required, that is the weight, height, and time. The effect is then measured by the quantity $\frac{\text{weight} \times \text{height}}{\text{time}}$."[‡]

CHAPTER IV.—*Accelerating forces.—Gravity.*

(44.) UNDER the general term accelerating forces are included all those which continue their action upon a body for any length of time, for a uniform motion is the result of the momentary action of a force, but when it acts continuously it will increase or accelerate the motion of a moving body; a force acting under such conditions is called an accelerating force.

(45.) We shall confine ourselves, in this chapter, to the consideration of the accelerating natural force of gravity, the higher branches of the subject belonging properly to the science of astronomy, although it is the action of the same kind of force; by the same law we may find the path of a planet revolving round the sun, or the course of a stone thrown in the air. The most simple observation which we can make of the action of the force of gravity is the effect of a body let fall from the hand; experience teaches us that the longer it continues to fall, the more forcibly will it strike any obstacle, and by the principles of the preceding chapter, as the weight of the falling body is the same, its velocity must increase to account for the increased momentum. The ancient philosophers were well aware of these facts, but they did not consider the true cause of the motion, and were led into many vague and singular ideas about the cause of these apparently irregular movements.

Galileo was the first philosopher who reasoned properly on the subject, and by experiment verified his theory of the nature of the force of gravity. He allowed a ball to run freely on an inclined plane, and measured the time it occupied in descending, by the weight of water which dropped regularly from a prepared vessel during the ball's descent. In this way it was found that when the ball ran down the whole plane in 4 seconds, it performed but a quarter of the length in 2 seconds, thus showing that the spaces described are as the squares of the times; for suppose the whole length to be divided into 16 equal parts, then in 2 seconds the ball will run down 4 of those parts, or $2 \times 2 = 4$; and to describe the whole length it will require 4 seconds, and 4×4, or the square of $4 = 16$, the whole length of the inclined plane.

(46.) The force of gravity, or attraction of the earth, was thus proved to be an accelerating force; in uniform motion the spaces are as the times (art. 11), but in accelerated motion this law does not hold good. In order that a body may move uniformly, it must, as is remarked above, receive but an impulse, whereas the action of gravity is continual, so that a body in falling must increase its speed. A question here arises as to what velocity the moving body has at any time, and how it may be found. The method which is adopted to discover this problem, affords a simple instance of the principle which enables the philosopher to reduce questions where the circumstances are variable and irregular for the purpose of calculation. Let us suppose that the body whose velocity we want to find has fallen from A to B, *fig.* 16, and that it then describes the minute space, B *a*, in a very small space of time, its velocity during that period will be $\frac{B\,a}{\text{time}}$ or the minute space divided by the time it takes to describe it; we therefore consider the motion in this little space uniform, which is strictly wrong, because the velocity continually increases; but if B *a* be a very small space—a thousandth of a foot for instance—the error will be very small, and if B *a* be still less the error will also be smaller, we may thus suppose it finally to be nothing, that is when *a* and B coincide; we then find the velocity which the falling body has at the point B.

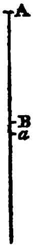
---

[*] See Mechanics, Treatise I., end of Chap. 8.

[†] Let m = mass, f = force, ds = small space moved in the small portion of time dt; then $mfds$, or $mvdv$, is equal to the effect produced in the time dt, and $\int mvdv = \frac{1}{2}mv^2$, the effect during the time t.

[‡] Francœur, "Traité Elémentaire de Méchanique," p. 308, 5th edit.

Fig. 16.



(47.) By this method we may determine the velocity of motion in all cases. Experiments have shown that a body in the latitude of London falls about 16* feet in the first second of time from the beginning of motion, and according to what its velocity at the end of the first second of time is 16 + 16 or 32 feet, at the end of the second second it will be twice the last amount or 64 feet, and similarly for all succeeding times of descent. [*See* Treatise I. p. 11.] The following are useful rules for finding the time, space, or velocity of bodies moving by the force of gravity.

1. *Space.*—The space described in a given time is equal to the square of that time multiplied by the space fallen through in one second; or, if the velocity and time be given, it is equal to one-half the product of the velocity and the time; or, if the velocity only be given, it is equal to the square of the velocity divided by twice the velocity acquires in falling for one second.

2. *Velocity.*—The velocity is equal to the product of the velocity acquired by falling for one second and the time of falling; or, it is equal to twice the space divided by the time; or, it is equal to the square root of the product of twice the space and the velocity acquired by falling for one second.

3. *Time.*—The time of falling is equal to velocity divided by twice the space fallen in one second; or, it is equal to twice the space divided by the velocity; or, it is equal to the square root of twice the space divided by the velocity acquired by falling for one second.

(48.) The above rules are continually used in calculating motions caused by the force of gravity, and will often be referred to as we proceed in our subject. The following instances illustrate their application.

How far will a body descend in 5 seconds? In this question we have the time stated and the space to be found, which according to the first rule is equal to the square of the time multiplied by the space described in one second, or 16 feet, therefore the depth a body will fall in 5 seconds is 5 × 5 × 16 = 400 feet. The question would be very similar if we were on the top of a high tower, or at the edge of a precipice, and wished to know the height; it would be necessary to let fall a stone or other heavy body and notice the time of its descent. Suppose a stone to be dropped into a well, and after 3½ seconds it is heard to splash in the water, we should find the depth to the surface of the water, to be 12¼ × 16 = 196 feet.

The rapid increase in the velocity of any falling body by the action of gravity, accounts for the surprising effects produced by large hail, which fall from a great height. Hailstones have frequently been seen of a considerable weight; in the Orkney Islands, in the year 1818, a hailstorm occurred, when the hailstones were of four ounces weight and upwards; one of them descending from a height of 2000 feet would acquire a velocity of $\sqrt{2} \times 2000 \times 32 = 359$ feet per second, and strike any body with as much force as a ball of iron weighing about $11\frac{1}{4}$ lbs. falling through the space of one foot.

---

* This number is not the exact amount. By experiments with the pendulum it has been found that a body in the latitude of London falls 193.14 inches, or 16.095 feet in the first second. [See *Mathematical Illustrations.*]

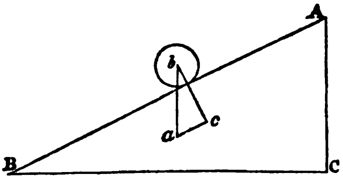
CHAPTER V.—*Motion on the Inclined Plane.—Motion on a Curve.—Principles of the Pendulum and its Applications.—Compensating Pendulums.*

(49.) THE constant force which is afforded by gravity may be made subservient to useful purposes, not only by its direct action on any body capable of moving freely, but by mechanical arrangements its effects may be varied to a certain extent, so as to produce any required result. One of the most extensive applications which is made of the force of gravity is the production of horizontal motion by means of the inclined plane; on an incline the railway train readily descends without the assistance of the locomotive engine; at Newcastle an inclined plane is used to transport the excavated coals from the mines to the vessels. In this case the application is ingenious; for as the train of loaded waggons descend, by their weight they draw up a train of empty waggons, whose contents have been transferred to the colliers; these waggons are filled, and in descending draw up the first-mentioned train, which has been disburdened of its load during the interval. Also the properties of motion in a curved line, which may be considered as composed of a number of planes varying in their inclination, produced by the same force, furnishes us with that most important instrument the pendulum, whose applications are at once useful and astonishing.

(50.) In a former treatise on mechanics, the equilibrium of inclined planes was considered; in that case the forces were required to produce rest, but in the present view of the subject the opposing forces in practice, which are friction and the resistance of the air, are supposed unequal to the force of gravity acting on the body, which will therefore descend along the plane. Our purpose is therefore to observe the laws which govern the motion of a heavy body placed upon an inclined plane.

(51.) If we place a ball on an inclined plane, we find it descends in a time and acquires a velocity varying with the length and height of the plane. Suppose A B, *fig*. 17, to represent the inclined plane, with the ball *b* on its surface, we can find, by geometrical construction, what amount of force urges it down the plane; draw the line *b a* of any length, so that it may represent the force of gravity acting in a vertical direction, similarly draw *b c* at right angles or perpendicular to the plane A B, which will represent the resistance of the plane, and *a c* parallel to A B; then *a c* will show both the relative amount and direction of the force producing its motion towards B. The triangle *a b c* is similar to the triangle A B C, and their sides therefore are in the same proportion, consequently *b a* is to *a c* as A B to A C, and we may use the line A C to express the accelerating force acting down the plane instead of the line *a c*; the line A C is the height of the inclined plane, and A B its length, whence we may say that the force on the inclined plane is to the whole force of gravity in the proportion of the height of the plane to its length. For instance, suppose the length A B to be 20 feet, and the height A C 10 feet, the accelerating force on the plane would be as 10 : 20, or $\frac{10}{20}$ths, or $\frac{1}{2}$; that is, the force producing the descent of the ball *b* upon it is $\frac{1}{2}$ that of gravity, and it would move with a velocity according to this amount of force. Also, if it were required to draw a body up the plane, a force greater than the proportion of A C to A B must be used.

Fig. 17.



(52.) From this determination of the force acting along an inclined plane, we may find the time of descent, and the velocity acquired during the descent, of any body placed upon the plane. With the assistance of the rules of the free descent of bodies by the force of gravity, we deduce the following rules for the time and velocity.

1. The *time* of descending down any inclined plane is equal to the square root of twice the length of the plane, divided by the ratio of the height to the length of the plane.

2. The *velocity* acquired in descending any inclined plane is equal to the ratio of the height of the plane to its length multiplied by the force of gravity and the time of descending. Or, it is equal to the square root of twice the length multiplied by the ratio of the length to the height of the plane and the force of gravity.

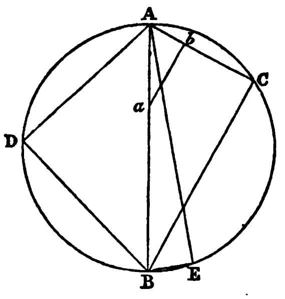
Suppose we required to know the time a body would take to descend an inclined plane of the same proportions as before, namely the height being 10 feet, and the length 20 feet. By the first rule the time is—

$$\sqrt{\frac{40}{\frac{10 \times 32}{20}}} = \sqrt{\frac{40}{16}} = \sqrt{2\tfrac{1}{2}} = 1\tfrac{1}{2}'' \text{ nearly.}$$

The velocity acquired may be obtained by the first rule for the velocity, whence it is $\frac{10}{20} \times 32 \times 1\frac{1}{2} = 24$ feet per second.

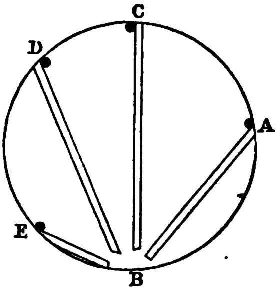
(53.) We have supposed the body descending to *roll*, because the friction of sliding under ordinary circumstances would be very considerable. Otherwise the body should slide; for when rolling, the motion is not so rapid as when sliding. A rolling body descends in consequence of its centre of gravity tumbling over and over; and two motions are given to it, one of progression down the plane, and another of rotation about its centre. (*See* chap. vii.)

Fig. 18.



(55.) The space through which any body will move freely in the same time that it descends an inclined plane may be found by a simple geometrical relation. As the ratio of the height to the length is proportional to the force acting along the plane and the force of gravity, the lines become measures of those forces. In any circle, as A C B D, *fig*. 18, draw a diameter, A B, and the lines A C, C B; then the force acting along A C is to the force of gravity as A C to A B; and if a body move from A, it will arrive at *b* in the same time that it would have fallen freely to *a*, or to C in the time it would have reached B. The same may be said of any other line drawn from the point A to the circumference, as A D or A E. In the latter case the difference of the spaces described, A E, A B, will be very small, in consequence of the near approach of the line A E to the perpendicular. A body would also descend along E B in the same time that it would fall from A to B. In the figure the lines A C, A E, A D, and E B, are the chords of the arcs, A C, A C E, A D, and E B, and A B is the diameter of the circle; therefore the time of descent of a body down any chord of an arc is equal to the time it would require to descend through the diameter of the circle; and also that the times of descent down all the chords are equal. This may be verified experimentally by attaching smooth planes of wood to a circle, as in *fig*. 19, forming chords, and meeting near the lowest point B of the circumference. If balls be placed on these planes, as at A C D E, and allowed to descend simultaneously, they will reach the bottom at the same time.

Fig. 19.



(56.) In applying the principle of the inclined plane to useful purposes, it is necessary to take into consideration the velocity required in a descending body, or the force which is applied to drag any body up an inclined plane; for unnecessary wear and tear of the parts of a machine may take place unless there is a proper proportion between the effect required and the forces brought into action. Friction, also, and the resistance of the air at great velocities have to be compensated.

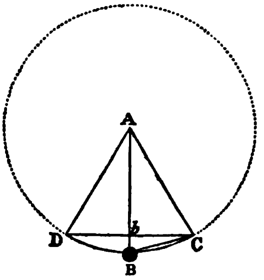
Suppose it were required to find the number of horses necessary to pull a cart, whose weight, with its contents, is 2000 lbs., up an inclined plane 500 feet long and 50 feet in height, at the rate of two and a half miles per hour. According to some authorities, a horse can draw 2000 lbs., or with a force of 200 lbs., at the rate of two and a half miles per hour. In consequence of the inclination of the road, the force of gravity pulls backward, acting as an addition to the load, and is equal to the force urging the cart down the incline; therefore $\frac{50}{500} = \frac{1}{10}$; that is $\frac{1}{10}$ of the load, or 200 lbs., would be the pressure acting on the load; and as a horse pulls with that force, two horses would be required to pull the cart up the incline.

To find the power of a steam engine necessary to draw a railway train up an incline, a similar calculation may be made. On a railway a force of about 9 lbs. will move a ton weight; and if the heaviest train be 100 tons, it will require, on a level, a force of 900 lbs. to move it. If the incline be 1000 feet long, and 10 feet high, or, as it is termed, a gradient of 1 in 100, the accelerating downward pressure will be $\frac{1}{100}$ that of gravity; consequently the load will be increased by the $\frac{1}{100}$ of 100 tons, or 1 ton. Thus the whole load to be drawn up will be 101 tons, and the force required 909 lbs. The engine must therefore exert a free force of above 909 lbs., or 4½ horses power.

(57.) What we have said of motion on an inclined plane may be applied also to motion along a curved line. If, however, the curve be circular, as is generally the case, it is found that the times of descent of any body, and also the accelerating force at every point of the descent are not equal, as is the case with the chords of the arcs of circles.

(58.) The principal feature of interest in the examination of ordinary curvilineal motion, is its application to the purpose of measuring time by means of the pendulum. In this beautiful instrument the force constraining the bob or ball to move in a circular arc is the pendulum rod, which, for the purpose of investigation, is supposed to possess no weight. We shall therefore, at first, consider the nature of the motion of a simple pendulum.

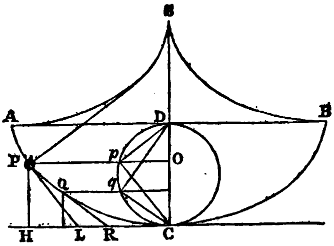
Fig. 20.



If we suspend a ball by a very fine thread, and allow it to vibrate, its motion, whether through a large or small arc, appears to be performed in an equal time; this, however, is found not to be the case. It may be easily shown that the times of descent of a pendulum, under such conditions, is not equal. Suppose the simple pendulum A B to oscillate in the arc C D, the time of performing an oscillation is equal to the square root of the length of the pendulum, divided by the force of gravity, and multiplied by the ratio of the circumference of a circle to its diameter. Suppose the length of the pendulum were 39.14 inches, and the force of gravity as before, 32 feet (art. 47.); the ratio of the circumference to the diameter of a circle is 3.1416 nearly to 1; then to find the time, in seconds, of oscillation of this pendulum, we have, by the above rule, $3.1416\sqrt{\frac{39.14}{32}} = 1''$ nearly, which would be true if the accelerating force were equal in all arcs which the pendulum might describe. It is, however, as the sine of the angle B A C, or the line *b* C; and the time of describing any arc increases as *b* B increases, which it evidently does as the arc of vibration increases; and to find the real time of oscillation, we must multiply our former sum by this quantity. In arcs of a few degrees, *b* B, or as it is called, the versed sine of the angle B A C, is a very minute quantity; if the pendulum vibrates in an arc of 5° on each side of the vertical or lowest point B, the versed sine *b* B of 5° is little more than .0038 of the radius A B, or the length of the pendulum; supposing the arc to decrease, it will also decrease, and finally may be considered as nothing.

(59.) This, however, is but an approximation towards the truth; and mathematicians have been led to investigate the form of curve required, in order that the times of descent in all arcs may be equal. The celebrated Huyghens solved this problem, and found the required curve to be a *cycloid*. This curve may be drawn by placing a point or pencil on the circumference of a circle, so that it may delineate a curve as the circle travels along a plane. In the following figure the point C, on the circle C D F moving along the straight line A B, will describe the curve B C Q A, which is called a cycloid; thus a point on the circumference of a carriage-wheel, when running along the road, describes the cycloid curve. Here the accelerating force, acting on a pendulum at P, is to the force of gravity as P H to P L; because the ball P is evidently moving in the direction P L at the instant it reaches P, and is therefore treatable according to the laws of bodies descending inclined planes; or, as the triangle C D *p* is similar to the triangle C *p* O, which is evidently similar to the triangle P L H, we may say that the force acting along the curve at P is to the force of gravity acting freely, as C *p* is to C D. This may be observed in any position of the pendulum, as at Q, where the ratio is as C *q* to C D. We observe that the lines P L, Q R, which represent the directions in which the ball is moving when at the points P, Q, are parallel with the chords drawn in the generating circle D *p* C, whence the laws of descent down the arcs of circles (art. 55) apply also to the cycloid. The two halves A S, S B, of a cycloid are placed as seen in the figure in order that the pendulum bob may describe a cycloidal curve.

Fig. 21.



(60.) Although a pendulum moving in such a curve is isochronous or *equal-timed* in its vibrations, yet it is used merely to illustrate the fact; in practice it would be most inconvenient to make a pendulum vibrate in a cycloidal arc, and the common pendulum is therefore used as the only practical method of measuring time. Its error, as we have seen, is reduced as we lessen the arc of vibration; and it may also be observed that there is no material difference between the curve of the circle and that of the cycloid for a little distance on each side of the point C; we shall therefore consider the times of oscillation in small arcs as isochronous.

(61.) The length of a pendulum which shall oscillate in a second of time may be easily found from the relation which has been shown to exist between the times of descent down the arcs of circles and their diameters. In art. 58 it was said that the time of an oscillation of a simple pendulum was equal to the square root of the length of the pendulum divided by the force of gravity, and multiplied by 3.1416. From this the length of the pendulum required to beat seconds or any other given time may be calculated; for if we square the whole sum, we have the square of the time $= (3.1416)^2 \times \frac{\text{length of pendulum}}{386 \text{ inches}}$; and the length of a pendulum will therefore be equal to $\frac{386 \text{ inches} \times (\text{time})^2}{(3.1416)^2}$. If the time be given, as 1 second, its square is also 1, which will not multiply; therefore the length of a pendulum which shall beat seconds is $= \frac{386 \text{ inches}}{9.87} = 39.12$ inches, which is very near the true amount. A more accurate determination has shown that the length required for a pendulum to beat seconds in the latitude of London, is 39.138 inches.

(62.) The length of a pendulum necessary to beat half-seconds, or any other time, may be found in a similar manner; for it will be seen in the above calculation, that the only quantity which changes its value is the length of the pendulum, the figures 3.1416 and 386 remaining constant; consequently the time of oscillation varies as the square root of the length of the pendulum. The following are useful rules:—

1. The *length* of a pendulum requisite to vibrate in any particular time is found by multiplying the square of the time by the force of gravity, and dividing the product by the square of 3.1416 (or 9.87).

2. The *time* of oscillation of any pendulum may be found by dividing its length by the force of gravity, taking the square root of the quotient, and multiplying by 3.1416.

3. If a seconds pendulum be found to *gain* or *lose* any number of seconds per day, the correction in length required may be found by multiplying twice the length of a seconds pendulum by the observed number of seconds which the clock has gained or lost during the day, and dividing the whole by the number of seconds in a day (86400″).

Their application may be readily shown by a few examples.

If we wish to find the length of a pendulum which shall oscillate in half-seconds, we can use the first rule, whence it is $\frac{.5 \times .5 \times 386}{9.87} = 9.7$ inches, nearly; a simple pendulum of this length would therefore vibrate twice in every second of time. A pendulum vibrating once in the course of several seconds would have to be of a most unwieldy length; thus to vibrate once in a minute, the length must be

$$\frac{60 \times 60 \times 386}{9.89} = 14{,}079 \text{ inches, or}$$

1,173¼ feet, nearly.

The following lengths of pendulums are given from accurate calculations:—

	Seconds.	Inches.
To vibrate in	¼	2.44616
”	½	9.78465
”	1	39.13860
”	2	156.65544

The second rule can be used when we have a pendulum of any length, and wish to know the time of its oscillation as compared with a seconds or other pendulum. For instance, suppose we have a simple pendulum 10 feet (120 inches) long, the time it will occupy in making an oscillation is—

$$3.1416 \times \sqrt{\frac{120}{39.14}} = 5\tfrac{1}{2}'', \text{ nearly.}$$

By the third rule may be found the alteration necessary in the length of any pendulum when the clock is found to go too fast or too slow; the number of oscillations over or under the proper amount made during the day must be noticed, when we have the elements for calculation. Suppose a clock loses 20 seconds in a day, the pendulum is evidently a little too long; then

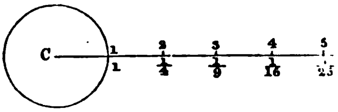
$$\frac{2 \times 39.138 \times 20}{86400} = .0018 \text{ of an inch,}$$

which is the amount by which the pendulum must be shortened. In ordinary pendulums the position of the bob is regulated by a nut and screw, the size of the threads of which we should know; suppose there are 100 threads in an inch of the screw, the width of each thread will therefore be .01 of an inch. Then $\frac{.0018}{.01} = \frac{1}{6}$ of a turn of the nut, will correct the error.

(63.) The pendulum is an instrument of singular delicacy; the most minute changes in its conditions is sufficient to affect its time of vibration: a pendulum having a length of 39.1386 inches will not vibrate seconds except on the latitude of London; at Leith a vibration would be performed in 0″.99997, and at Jamaica in 1″.0303. Again, if the same pendulum be carried down the mine at Dolcoath, in Cornwall, which is 1050 feet below the level of the sea, a vibration would occupy but 0″.99995; while on the summit of Mont Blanc, which is 15,780 feet above the level of the sea, it would take 1″.000753 to complete an oscillation. Thus we find that there are circumstances which it is necessary to consider in our calculations of the lengths of pendulums at different places on the surface of the earth, and at different heights above the level of the ocean.

(64.) We will first notice the cause of the variation in time observed at different elevations. It was remarked in chap. IV., that the force of gravity produced a velocity of 32 feet in a body, in the first second of falling, 64 in the second, and that 32 multiplied by the time of falling would give the final velocity. This is not strictly true; for the law of the force of attraction or gravitation is, that it varies inversely as the square of the distance from the centre of attraction. This may be shown by means of the following diagram. If any

Fig. 22.



body be placed on the circumference of the circle, as at 1, *fig.* 22, and it be attracted with a force of 100 lbs., it would be attracted with a force of ¼ of 100 lbs., or 25 lbs., at the point 2, or twice the former distance from the centre; and at the point 5 with a force of 4 lbs. In considering the fall of bodies on the surface of the earth, there is no occasion for introducing this into our calculations, as the space through which any body falls is generally so very small compared with the radius of the earth. We do not ordinarily require to find the velocity of a body falling from a height of one mile; yet, even at this considerable elevation, the variation in the accelerating force would be but small, as we may easily determine from the proportion above mentioned. The radius of the earth, or half its mean diameter, is 3965 miles, or 20,898,240 feet; and, for a distance of one mile above its surface, we must add 5,280 feet, which is equal to an English mile, making the sum of 20,903,520 feet. The squares of these sums being taken, we have the following inverse proportion:— $20{,}903{,}520^2 : 20{,}898{,}240^2 :: 32.19 :$ the force of gravity at the distance of one mile above the level of the sea. If this be calculated, it will be seen that the accelerating force is 32.12 feet, nearly.

Although this difference is not worthy of consideration when we are engaged in investigating the fall of bodies freely or on inclined planes, we are obliged to take it into account in the construction of pendulums. It is not that the error occasioned by neglecting it would be material in one or two vibrations; no effect would be observed in so small a time; but there is a little error in each oscillation, which, after a considerable time, becomes very perceptible. If, for example, a seconds pendulum took 1″.001, or a second and one-thousandth, to make a complete vibration, after the lapse of a day, it would be found that the clock to which it was attached would lose about 86″, or nearly a minute and a half; a very noticeable quantity, although the error in one second could not be detected.

(65.) This circumstance is sufficient to show the necessity of attending to the height above the level of the sea at which any pendulum is used, in order that its length may be adjusted. This can be easily effected by the following simple rule:—Multiply the height of the pendulum above the level of the sea by the number of seconds in a day (86,400″), and divide the product by the radius of the earth (3965 miles), which will give the number of seconds lost in a day. Knowing the number of seconds, we can apply the rule given in art. (62) to find the amount of alteration required in the length of the pendulum. This correction amounts to a considerable quantity at some places on the globe which are situated at great elevations. The beautiful city of Quito is situate on the Pinchincha mountain, amongst the immense range of the Andes, in South America, at the great elevation of nearly 9500 feet, or $1\frac{8}{10}$ mile above the level of the sea: a pendulum beating seconds in that latitude, at the level of the sea, would, if taken up to this city, lose in one day $\frac{86400 \times 1.8}{3965} = 39$ seconds.

(66.) The other cause of variation is in consequence of the pendulum vibrating in different times on different latitudes. In the instances mentioned it was shown that the pendulum vibrates faster the nearer it is to the poles of the earth; for at Jamaica, London, and Leith, it makes a vibration in the respective times, 1″.030, 1″.000, 0″.999; and the latitudes of these places are 18° 0′ 0″, 51° 31′ 8″, and 55° 58′ 37″, all north of the equator, which leads to the conclusion that the rapidity of vibration increases from the equator to the poles.

(67.) The cause of this increasing rate of vibration is the same as that which affects the motion of the pendulum when it is removed to a considerable height above the surface of the earth, namely, the varying force of attraction to the centre of the earth; hence we may readily infer that, in approaching the poles of the earth, we also approach a little nearer to its centre. From this remarkable circumstance, therefore, an idea might be formed of the true figure of the globe we inhabit, and we might find the flattening, as it is sometimes termed, which occurs at the poles. The pendulum has been applied with great success to this singular and interesting problem; scientific men have visited different parts of the earth, and measured the intensity of gravity by the rate of a pendulum's motion, and afforded materials for calculating the figure of the earth with considerable accuracy. From these observations rules have been established for finding the length of a pendulum to beat seconds on different latitudes; it may be found from the following proportion:—The square of the number of oscillations made by a pendulum in a day is to the square of the number of oscillations made in a day at the place in question as the length of an adjusted seconds pendulum is to the length of the pendulum which shall vibrate in one second at the place required. If the adjusted seconds pendulum be that of the latitude of London, the rule is—the length required at any other latitude =

$$39.138 \times \frac{(\text{No. of vibrations made there})^2}{(\text{No. of vibrations at London})^2}.$$

An illustration of this rule may be taken from the observations made some time ago for the purpose of finding the force of gravity at different places on the earth's surface. It was observed that, at London, a pendulum made 86061 oscillations in 24 hours; the same instrument made 86079 vibrations in the same time at Leith, in Scotland; then to determine from these data the length of a pendulum which shall vibrate in a second at Leith, we have the following calculation:—$39.139 \times \frac{(86079)^2}{(86061)^2}$; by this it may be found that the length required is about 39.155 inches; so that an increase in length of .016, or $\frac{1}{62}$ of an inch, must be given to the pendulum which is adjusted on the latitude of London. This is a very small quantity, though it may be seen that, if this correction were not made, the clock regulated by such a pendulum would gain rather more than 17⅓″ in the course of a day.

(68.) We have been speaking hitherto of the simple pendulum, which is supposed to be a heavy bob or ball attached to a weightless string or wire; this, however, is not the instrument for practice. All pendulums have a rod or wire of considerable weight, which destroys the necessary characteristic of the simple pendulum. The pendulum, also, when in motion, is subject to the resistance of the air, which affects the regularity of its movements in a small degree, although this is not comparable with the powerful effects of temperature. All substances are subject to an alteration in dimensions under a change of temperature; when the change is not very great, the alteration cannot be perceived by the eye, or even measured by the common methods of admeasurement; but the pendulum soon makes apparent any variation in the length of the rod by a change of a few degrees of temperature. It becomes, then, a matter of importance to investigate the effects of these different disturbing causes, in order that they may be compensated.

(69.) The weight of the pendulum rod influences the time of vibration, in consequence of a property of bodies moving about a fixed axis to have a certain point when in motion called the centre of oscillation—that is, a point in the length of the vibrating rod, where, if its whole mass were collected and suspended, as in the case of the simple pendulum, an oscillation would be performed in the same time as it will in the real condition of the rod; therefore the distance between the centre of oscillation and the point of suspension is the proper measure of the length of a pendulum. The calculation of this important particular will be shown in chap. vii. With respect to the resistance of the air, it may be neglected as inappreciable in ordinary instruments, the bob being flattened so that it cuts the air, and presents very little surface for its action. It is in the most delicate instruments only that this resistance is at all perceptible.

(70.) The effect of a change of temperature on the pendulum being of so serious a nature the subject has received a great deal of investigation. Many plans have been proposed and used for the purpose of compensating this cause of error. In the ordinary clock pendulum we have the following construction:—The pendulum rod A, *fig.* 23, is a steel or iron wire, of about 40 inches in length; at its upper end it is supported by a steel spring B, which allows it to vibrate freely; and at the lower end is placed a bob C, through which the rod passes. This bob is adjusted and kept in its place by means of a nut D, working on a screw which is cut on the rod. When the height of the bob C is adjusted so that the centre of oscillation of the whole is

Fig. 23.

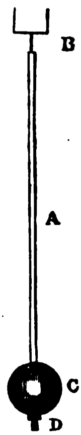
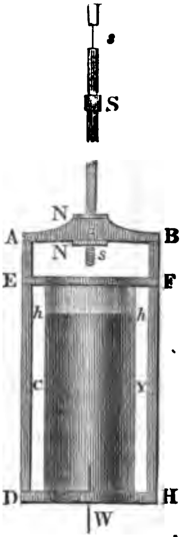


Fig. 24.



39.138 inches from the point of suspension, the pendulum will vibrate once every second. Such a pendulum, however, is subject to vary in its rate from the cause we are now considering; for the rod A will become longer or shorter according as the temperature of the air in which it is placed increases or decreases. It has been found, from accurate experiments, that a steel rod will expand .0000063596, or $\frac{1}{157242}$, of its length, for every degree of increasing temperature; thus a rod of the length of a seconds pendulum will expand .000248 of an inch nearly, for an increase of one degree of temperature; and as the temperature of summer is about 30° above that of winter, the pendulum rod will be .00744 of an inch longer in summer than in winter. This increase of length would cause the clock to lose 8″ daily in summer, if the pendulum were adjusted in the winter. In constructing a pendulum which shall not be liable to this irregularity, as we cannot by any means prevent the linear expansion of the rod, we must create a counter expansion, so that the centre of oscillation may remain constantly at the proper distance from the point of suspension. Of the various methods adopted to obtain this corrective expansion, the following illustrations will exhibit some of the most simple and elegant.

Mr. Graham, a chronometer maker, was the first to construct a compensating pendulum. In pursuing his inquiries, he came to the conclusion that, owing to the small expansion of metal rods, the compensation could not be effected by them; he then tried mercury, and brought out that beautiful instrument which is known as the mercurial pendulum. It was the first ever constructed, and remains the most perfect compensating pendulum at present in use. The following figure exhibits a form of it, the whole length of the rod not being shown in the figure. The rod R is of steel, suspended by means of a spring *s*, and carries a small sliding weight S, to effect a final adjustment of the centre of oscillation when all the other parts are arranged. Its length is about 34 inches, its lower part being made a screw to carry the stirrup A B D H, which is fixed in its proper position by the two nuts N N. The height of this stirrup between the top and bottom is about 8 inches; the base D H being made to receive the glass cylinder C Y, which is covered by the cap E F, holding it so as to prevent lateral motion. In this cylinder a quantity of mercury is placed, whose surface is indicated at *h h*, sufficient, by its expansion, which must of course be upwards, to compensate for the downward expansion of the rod and stirrup. A fine pointer W is fixed underneath the stirrup, to show the arc of vibration upon a scale placed behind it when mounted.

Soon after the mercurial pendulum was made, Mr. Harrison tried to effect a compensation with metal bars, and succeeded in forming the instrument which has been called, from its appearance, the gridiron pendulum. This is represented in *fig.* 25, the rod R being of steel, fixed at its lower extremity to a cross-piece, upon which rests two brass rods. These support another cross-bar *a b*, which holds up two rods of steel, as shown by the dark lines, sustaining the cross-bar *c d*, which forms the support of the bars of brass *c a*, *d b;* the uppermost cross-bar is fixed upon these, carrying the steel bars A C, B D; to the bar C D, joining these rods, the bob is attached. The upper cross-bars are not

***Fig.* 25.**

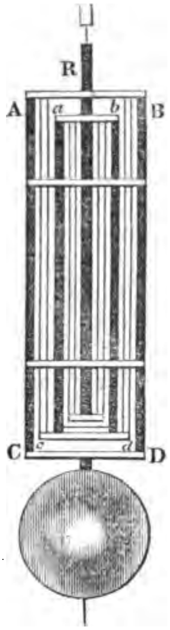
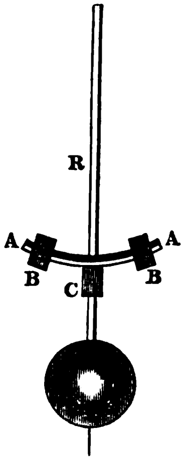


***Fig.* 26.**

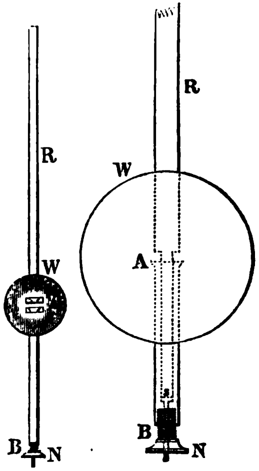


fixed to the rod R, which, when it expands, pushes down the small lower cross-bar; but the brass rods resting upon it expand upwards and carry the bar *a b*, and similarly with the remaining parts, the lengths being first arranged that the expansions both upwards and downwards may be equal.

A very simple compensating pendulum is described by M. Biot, as the invention of M. Martin, a clock-maker. The peculiarity consists in having a compound bar, A A (*fig.* 26), of brass and steel, fixed upon the pendulum rod, which carries two sliding weights B B. When the temperature increases and lengthens the rod, it also expands the compound bar, in which the brass is below the steel bar; and as brass expands more than steel, the whole bar will become curved as is represented in the figure, lifting the weights, and consequently effecting the compensation. With this instrument M. Biot made astronomical observations for the space of a year, without discovering any error consequent on the expansion of the materials.

Captain Kater describes a pendulum which he constructed, the rod being of white deal, a substance little affected by changes of temperature. This rod, after being well baked and varnished, was found to be impervious to moisture. This is a most indispensable condition, for the absorption of the moisture continually floating in the atmosphere would deprive the rod of its fitness for the purpose of a pendulum. The compensation required for the small expansion of this rod is effected by a cylinder of zinc A B (*fig.* 27). In the larger figure the arrangements of the compensation are shown: the lower part of the wooden rod R is cut of a smaller size to allow the zinc tube to slide upon it; at the extremity of the rod a brass cap *s* is fixed, carrying a screw. The zinc tube is about 7 inches long and three quarters of an inch square, a thick plate of brass being attached to the lower end to receive the screw B, which regulates the position of the tube. This screw is hollow, to allow the small screw on the rod to pass through it and carry the nut N. By this nut the height of the zinc tube and bob on the rod is regulated, as the bob W rests upon the top of the zinc tube at A. The expansion of the deal rod R is therefore neutralised by the upward expansion of the zinc tube which carries the bob.

***Fig.* 27.**



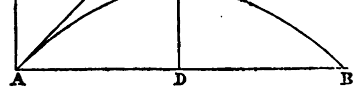
(71.) The principle upon which the pendulum acts is one of the greatest importance with respect to all machines, but more particularly when they are intended for rapid motion. It is the tendency of a body to settle in a position wherein its centre of gravity is lowest: such a state is called stable equilibrium. Many objects in nature and art exhibit the properties of pendulous motion, although there may be no likeness in form to the pendulum. Those curiosities of nature in different parts of this country, which have sometimes been ascribed to the labours of the Druids — the rocking stones — afford instances so curious as to call in the aid of imagination among the unphilosophical to explain their origin. Standing on a very small base, an enormous mass of rock, weighing many tons, can be made to oscillate by the exertion of one or two persons, and, after a few vibrations, it returns to its original state of rest. In machinery, unless the parts of a machine be arranged with a strict regard to the conservation of stable equilibrium, in whatever circumstances the machine may be placed, great irregularity of motion may ensue when the machine is at work. It is then that all defects of construction show themselves most plainly. The motion of a carriage or engine on a railway supplies an illustration of these remarks. The distance between the flanges or guards of the wheels is always less than the distance between the rails, to allow of free movement; and when at considerable velocities, the engine or carriage may be observed to oscillate between the limits thus allowed with great rapidity, causing a vibratory motion, both horizontally and vertically, in the carriage, as the centre of gravity is above the wheels, and the force exerts itself on the flanges pushing against the rail; consequently the centre of gravity of the carriage, and the carriage itself, has a tendency to turn over. This is a particularly important point, because the rapidity of these vibrations increases as the square of the velocity at which the carriage moves. From this we learn two most necessary precautions—that the centre of gravity, or the carriage itself, should be as near the ground as possible; and that the wider the base on which the carriage rests, the more steadily and safely will it move.

Chapter VI.—*Motion of Projectiles.—Resistance of the Air to Bodies moving through it.*

(72.) Free motion, in a line not perpendicular to the earth's surface, is peculiarly affected. If a stone be thrown into the air, not perpendicularly, we may notice that it does not move in a straight line. At a short distance from the hand it commences to describe a curve, in consequence of the action of gravity, which finally draws the stone to the ground. This result, however, may be seen, upon a little reflection, to arise from the nature of the forces in action. In the first place an impulse is given to the stone, communicating an amount of uniform motion which it would for ever retain; but the instant it leaves the hand, the force of gravity acts upon it: this force, according to the principles already given (art. 45), will continue to urge the body from the direction of motion first given, and finally overcome. This subject is called the Theory of Projectiles: by means of the rules which it establishes, the calculation of the distance which a projectile would be thrown by any force, and the time of its flight, would be very simple were it not for one circumstance, which, when the theory is applied to practice, fails to give a true result. The theory does not consider that the projectile has to move through the air, which is a powerfully resisting medium when bodies move rapidly. We shall, in the present chapter, consider the laws of projectiles, and the resistance of the air to moving bodies.

(73.) A slight observation is sufficient to show that the curve line formed by a body moving freely in the air is not part of a circle. An inclined fountain of water affords a good illustration, as the water forms a continuous line. The curve thus traced in the air is called a *parabola*, one of the conic sections; and every body, unless projected perpendicularly, will move through the air in a parabolic path. *Fig.* 28 shows one form of the curve in which the projectile moves from the point A, in the direction of the line A N; but gravity, after some time, prevents further upward motion, and the body describes a similar curve downwards to E.

Fig. 28.



(74.) By calculation the following rules are found to determine the range and time of flight of a body projected at any angle of elevation N A E, D C being called the height, and A E the range.

1. The *range* of a projectile is equal to the product of the square of the velocity of projection, and the sine of twice the angle of elevation, divided by the force of gravity.

2. The *time* of flight of a projectile is equal to the product of twice the velocity of projection, and the sine of the angle of projection, divided by the force of gravity.

In these rules the velocity given to the projected body and the angle of elevation or projection are supposed to be known. The calculation of any particular case becomes very simple with the aid of logarithms, in using which the rules are as follow:—

The log. of the *range* = log. of square of projectile velocity + log. sine of twice the angle of projection — log. 32.

The log. of the *time* of flight = log. of twice the velocity of projection + log. sine of angle of projection — log. 32.

Thus, suppose a cannon-ball to be projected with a velocity of 2500 feet per second, the angle of elevation or projection being 30 degrees. Then, the range which the ball would have, by the laws of projectiles, may be found by the following calculation:—

Log. 625,000 = $(2500)^2$ . . .	6.7958800
Log. sin. 60°	9.9375306
	6.7334106
Log. 32	1.5051500
Log. range = 169,140 feet .	6.2282606

The ball would have a range of above 32 miles.

(75.) According to theory, the projectile will have the greatest range when the sine of twice the angle is greatest, which is where the sine is equal to the radius; and twice the angle will therefore be 90°, or the angle of elevation 45°. A body projected at an angle of 45° with the horizon will consequently have the maximum or greatest range. In calculation, we have only to divide the square of the velocity by the force of gravity to obtain the range of a body projected at an angle of 45°, and its time of flight will be equal to twice the velocity divided by the force of gravity.

(76.) When the velocity of projection is not known, it may be easily found by the perpendicular height which the body will attain by the action of the given force; for, from the nature of gravity, the velocity which must be impressed upon any body, to make it ascend to any height, will be equal to the velocity it would acquire in falling down that height. The preceding rules may then be used as follows:—

1. The *range* of a projectile is equal to twice the height to which it would ascend perpendicularly by the action of the given force, multiplied by the sine of twice the angle of projection.

2. The *time* of flight is equal the square root of twice the height to which the projectile would ascend perpendicularly by the action of the given force divided by the force of gravity, the square root, thus obtained, being multiplied by twice the sine of the angle of projection.

Thus, if a force is used which can impel a ball a perpendicular height of 10,000 feet, the range at an angle of projection of 45° would be 20,000 × sin. 90°; but the sine of 90° is equal to radius, which is considered as unity; therefore, 20,000 feet will be the range, and the time of flight

$$\sqrt{\frac{20,000}{32}} \times 2 \times \frac{1}{\sqrt{2}};$$

the latter fraction being the length of the sine of 45°; upon calculation it will be found that the time is nearly 35 seconds.

(77.) The parabolic path of projected bodies was first demonstrated by Galileo, and is, as we have seen, easy to determine; but the enormous discrepancy, between the range assigned by the rule and that which is actually obtained in practice, at once shows that the simple law is insufficient for practical purposes. In the above illustration the projectile is found to have a range of nearly 32 miles, a distance unattained by any method of projection hitherto employed. It has been found that a musket ball, having an initial velocity of 1700 feet per second, has a range of about half a mile; but, calculating upon our last rules, a fall of 45,156 feet being required to produce the velocity stated, the range should be about 17 miles, or 34 times farther. This difference is owing to the resistance of the atmosphere, which is very great at high velocities. We shall, therefore, proceed to consider this important subject.

(78.) In the motion of any body through a medium, some attention must be paid to the relative densities of the body and the medium; we experience little resistance in walking through the air, but a considerable exertion is necessary to wade through water. In the case of a body acted on by the force of gravity, we find that it descends through air or water by its excess of specific gravity; if, on the contrary, its specific gravity is less than that of the medium, it will ascend; thus cork ascends in water, and hydrogen gas in common air. Under such circumstances the body is not effectively urged with the whole force of gravity, but it is equal to the force of gravity minus

$$\text{gravity} \times \frac{\text{spec. grav. medium.}}{\text{spec. grav. body.}}$$

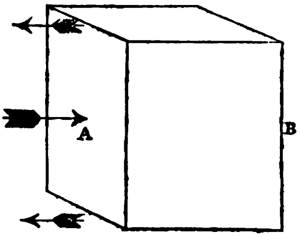
Suppose it were required to find the real accelerating force acting on an iron ball moving in the air; the density of air is to that of iron as 1 to 6000, therefore the real force is—

$$32 - 32 \times \tfrac{1}{6000} = 31\tfrac{5968}{6000}.$$

Hence it may be seen that, in many cases, the difference of density between the medium and the body moving through it is not worthy of consideration, except with light substances.

(79.) If a body be acted on by a force, according to the laws of uniform motion, it would begin to move with a proportional velocity, and if there be a resisting power, such as the atmosphere, its retarding effect upon the motion of the body, by the principles of Chap. III., will be found by dividing the opposing force by the mass of the moving body, and the result thus obtained must be subtracted from the quantity of motion of the body to find the effective quantity of motion which the body will have, being subject to this resistance; this effective quantity of motion is therefore equal to the original quantity of motion of the body $-\dfrac{\text{resisting force}}{\text{mass of the body}}$. We have thus to find the amount of this resisting force; and, in doing so, the amount of surface presented must be taken into account. Suppose the body A B to commence moving in the direction of the small arrows in a resisting medium, which will exert an opposing force in the direction of the arrow at A; the surface A is the object of the resistance. To obtain the amount of the resistance in pounds, it must be compared with the force of gravity, which is easily effected by taking the height required to obtain the body's velocity. Sir Isaac Newton demonstrated that the resistance to a surface moving through the air with any velocity is equal to the weight of a column of air whose base is equal to the moving surface, and its height equal to the height which a body must fall to acquire the known velocity. Then, according to the rules formerly stated (art. 47), the height required to obtain any velocity is equal to the square of that velocity divided by twice the force of gravity. Therefore the resistance which the body experiences in moving through the air is equal to the surface multiplied by the density of the air, and the square of the velocity of the moving body, divided by twice the force of gravity. The retarding force will be found by dividing the resisting force by the weight of the body, as in the last article*.

Fig. 29.



(80.) This subject is very much overlooked in practical mechanics, for the effect of the atmosphere upon the motions of bodies at slow velocities, and acting for a short time, are small and imperceptible; but, as the resistance

---

* These formulæ may be deduced with the aid of symbols. Let A be the area of the surface, M its mass, D the density of the air, v the velocity of the moving body, and dt an infinitely small portion of time during which the body moves with the velocity v. Then the small space which the body moves through in dt is $ds = v\,dt$, consequently, by multiplying with the area, A $v\,dt$ is the amount of displacement of the medium, which will receive a momentum equal to A $v\,dt \times v$, or A $v^2\,dt$. The moving body will lose a very small portion, dv, of its velocity from the inertia of the medium, and M dv will be the quantity of motion lost; then M dv = A D $v^2\,dt$, because the momentum gained by the medium must be equal to that lost by the moving body. We have here obtained the resistance, which is then as the square of the velocity. If we call R the retarding force, we find—

$$R = \frac{dv}{dt} = \frac{A\,D\,v^2}{M}.$$

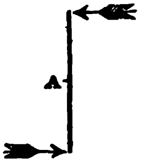
increases as the square of the velocity, the subject is important where rapid motion is required; thus, if a body was resisted with a force of 1 lb. when moving at a velocity of 1 foot per second, it would meet with a resistance of 16 lbs. at 4 feet per second. The question of resistance has been studied principally in consequence of its connection with warlike operations; the rapid motion of a cannon ball being so great, it is, as we have seen, very much affected, not only with the resistance of the air in front, but also by reason of the partial vacuum which is left immediately behind the ball. The air ceases to follow the ball when at a velocity of about 1340 feet per second; and the atmospheric pressure of nearly 15 lbs. on every square inch must be added to the resistance to give the whole amount of opposing force. Thus, a ball of 36 lbs. weight, and $6\frac{2}{3}$ inches in diameter, moving with a velocity of 1600 feet per second, meets with a resistance of about 417 lbs.; but, this being a velocity at which the air cannot follow, the pressure arising in consequence will be $478\frac{1}{2}$ lbs.; the ball, therefore, will have to sustain an opposing force of nearly 900 lbs.

(81.) The investigation of the subject is one of great mathematical difficulty, the circumstances attending it being of so varied a character. Many attempts have been made to establish some laws of the resistance to a projectile, but without success. For further information on this interesting subject we must refer the reader to treatises on pneumatics and hydraulics.

CHAPTER VII.—*On Rotation.—Rotation about a fixed Axis.—The Centres of Oscillation, Percussion, and Gyration.—Illustrations of Rotation.—Centrifugal Force.*

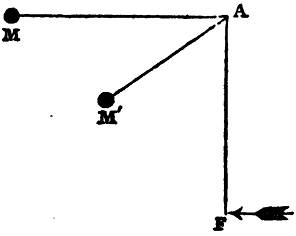
(82.) WHEN an unfixed body is struck at any point *not* in the centre of gravity, there will result two motions, one by which the whole body is carried forward, and another by which it appears to turn round some point, as an axis; the former motion has been termed a motion of translation, and the latter of rotation. This motion of rotation is unexplainable by our preceding considerations; it is an effect of quite a different character from that which would follow from the rules already given, for it cannot be shown that rotative motion is a resultant of any two or more forces, as is the case with rectilinear motion, as the laws of the resolution of forces do not apply to the case of two forces acting opposite to each other, as in *fig.* 30; if A be considered a long body, as a stick, a force applied at each end will produce a rotation about the point A, the stick having, at the same time, a motion of translation, unless it be fixed at the point A, which will then be stationary. We shall now proceed to investigate the nature and properties of rotation about a fixed axis.

Fig. 30.



(83.) For the purpose of making the inquiry as simple as possible, we shall suppose a force to act on a heavy particle of matter attached to a fixed axis by means of a radial arm without weight, whose effect may, therefore, be neglected. Suppose M M (*fig.* 31) to be two bodies, called a system, which turn round the fixed axis A, always remaining in the same position with respect to each other, their relative angular velocity will be as the length of the arms A M, A M′, according to a geometrical principle; and the velocities of each rotating particle will be found by multiplying the angular velocity of the system by the distances of each particle from the axis, that is, their radii: thus the velocity of M is equal to the angular velocity × A M, and the velocity of M′ is equal to the angular velocity × A M′. A force acting at the extremity of the radius F will communicate a certain amount of angular velocity to the whole system; and the

Fig. 31.



quantity of motion or momentum of M will be (art. 33.) equal to the angular velocity × A M × M, which is also a measure of the force impressed; as, however, it is not applied at M but at F, the force impressed is to the momentum of the mass M as the distances of the points where the force F is applied, and the mass M fixed, from the axis,—or force : angular velocity × A M × M : : A M : A F, which follows from the principle of the lever, and the force × A F = ang. velocity × A M × A M × M. The same may be shown of the particle M′, or any number of particles rotating round A as an axis; thus a regular body may be considered as a multitude of small particles rotating round an axis.

From the above investigation may be found the law of the resistance to motion, which is offered by particles at various distances from their axis. This resistance is called the *moment of inertia* of a particle. In the lever, the force applied multiplied by the length of the arm gives the momentum or power of producing motion round the stationary point or fulcrum; that momentum in the present case is Force × A F, and the moment of inertia of the particle M is A M × A M × M, or $(\text{A M})^2 \times \text{M}$, and of the particle M′ = A M′ × A M′ × M, or $(\text{A M}')^2 \times \text{M}$.

Therefore the moment of inertia of a body or system, with reference to any axis, is equal to the sum of the particles which compose it, multiplied by the square of the distance of each particle from the axis*.

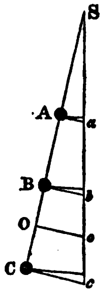
(84.) The angular velocity which may be given to a revolvable body by any force is also found from the last article, where the angular velocity of M is equal to $\frac{\text{Force} \times \text{A F}}{\text{A M} \times \text{A M} \times \text{M}}$, or the moment of the force impressed, divided by the moment of inertia of the body. The angular velocity is here given in parts of the radius A M; but, to convert it into degrees. we may use the length of an arc equal to the length of the radius A M, which by trigonometry† is found to be 57°.3; a degree is, therefore, equal to $\frac{1}{57.3}$ or .0174 of the radius.

(85.) The principal objects in the investigation of the effects of rotatory motion are the determining of certain points in any rotating body, called the centres of *oscillation*, *percussion*, and *gyration*.

(86.) In describing the nature of the pendulum, we supposed the pendulum rod to be without weight, because of the effect it would produce on the time of the vibrations; and remarked that the length of the pendulum was to be measured, from the point of suspension or axis, downwards to a certain point called the centre of oscillation, or the point to which, if all the mass of the pendulum were collected and suspended by means of a rod without weight, the oscillations would be performed in the same time as in its real form; it is, therefore, of great importance to determine this point.

Suppose S C (*fig.* 32) to be a rod without weight, to which the particles A B C are attached; that the times of vibration of each particle may be equal, the accelerating forces should be as their distances from the point of suspension, whereas they are really equal, because lines drawn at right angles to the rod S C are parallel, and by the laws of the inclined plane the forces will be equal on equally inclined planes. Let S O represent the force inciting the point O, in the length of the rod, to oscillation, the superior acceleration of the particles A B, above that point, will be shown by A O, B O; but the particle C, being farther than O from the axis, will retard the motion of O by the quantity C O. According to the principle of the lever, the effect of each particle is according to its distance from the point of suspension, and the accelerating forces must be equal to the retarding forces, that is—

Fig. 32.



---

[*] See Chap. VIII., *Mathematical Illustrations*, for an investigation of the phenomena of rotation.

[†] See Treatise on *Trigonometry*, Ex. 3, Art. 10.

$$A \times AO \times SA + B \times BO + SB = C \times OC \times SC.$$

In this result we have not obtained S O, or the distance of the centre of oscillation, separate from the other quantities, which we wish to do; it is, however, easily eliminated by considering that $AO = SO - SA$, $BO = SO - SB$, and $OC = SC - SO$, which gives S O in each instance. Then putting these values in the above result—

$$A \times SA \times (SO - SA) + B \times SB \times (SO - SB) \neq C \times SC \times (SC - SO),$$

which, on multiplying the quantities together, becomes—

$$A \times SO \times SA - A \times SA^2 + B \times SO \times SB - B \times SB^2 = C \times SC^2 - C \times SC \times SO.$$

Transposing $A \times SA^2$, $B \times SB^2$, and $C \times SC \times SO$, we have—

$$SO(A \times SA + B \times SB - C \times SC) = A \times SA^2 + B \times SB^2 + C \times SC^2.$$

Whence we find, lastly—

$$SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A \times SA + B \times SB + C \times SC}.$$

The distance is thus found, in which the numerator evidently expresses the moment of inertia of each particle, and the denominator the product of the mass of each particle and the distance of its centre of gravity from the axis. It may therefore be stated as follows:—

The centre of oscillation of any system is found by dividing the sum of the products of each particle and the square of its distance from the point of suspension, by the sum of the product of each particle and its distance from the point of suspension.

(87.) From this we can determine the centre of oscillation of a body. In a slender uniform rod, suspended at one end, this point is at the distance of two-thirds of the length; in a sphere, suspended at the circumference, it is two-fifths of the radius of the sphere.

(88.) The centre of oscillation of a body may be found experimentally by two methods. In one the body is suspended so as to vibrate freely; and by trial is found the length of a simple pendulum which will vibrate in the same time as the body. Another and better method is to notice the number of vibrations made, in any length of time, by the suspended body; then its length may be found from the law that the lengths of pendulums are inversely as the squares of the number of vibrations of the pendulum; taking a seconds pendulum for comparison, and an hour for the length of time, the proportion is as follows:—

$$(\text{No. of vibrations of the body})^2 : 3600^2 :: 39.138 : \frac{3600^2 \times 39.138}{(\text{No. of vibrations})^2}.$$

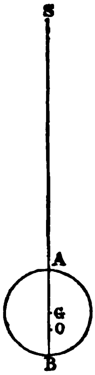
Therefore the distance of the centre of oscillation from the point of suspension will be $\frac{507228480}{(\text{No. vibrations})^2}$ inches. If the observation was continued for a minute, it would be $\frac{1408968}{(\text{No. vibrations})^2}$ inches.

(89.) It is a very important object in the investigation of this subject to determine the centre of oscillation of a sphere, suspended by a rod, or a pendulum. If the rod be of little weight in comparison with that of the bob or ball, the centre of gravity of the whole will remain at the centre of the ball nearly; and the moment of inertia of the sphere is two-fifths of the mass multiplied by the square of half the diameter, or $\frac{2}{5}$ mass $\times AG^2$ (*fig.* 33); but as we require the moment of inertia with respect to the axis at S, the moment of inertia of the rod, and half the diameter of the ball, that is, $SA + AG = SG$, then the moment of inertia of the whole, with reference to the axis at S, is $=$ mass $\times SG^2 + \frac{2}{5}$ mass $\times AG^2$, which must be divided by the product of the mass and the distance, S G, of the centre of gravity from the axis, according to art. 86, whence the distance of the centre of oscillation from the point of suspension is

$$= \frac{\text{mass} \times SG^2 + \frac{2}{5}\,\text{mass} \times AG^2}{\text{mass} \times SG} = SG + \frac{2}{5}\left(\frac{AG^2}{SG}\right).$$

Or, as the distance of the centre of gravity below S is represented by S G,

Fig. 33.



the distance of the centre of oscillation below the centre of gravity is two-fifths of the square of half the diameter of the ball divided by the distance of the centre of gravity from the point of suspension.

As an example of the use of this calculation, suppose we have a pendulum-rod 36 inches long, of very little weight, with a heavy ball, 6 inches in diameter; then A G = 3, or A G^2 = 9, and S G = 36 + 3, using which, according to the above rule, we find the centre of oscillation $\frac{2}{5}\left(\frac{9}{39}\right) = \frac{18}{195} = \frac{6}{65}$ of an inch below the centre of gravity; this added to the quantity S G, or 39 inches, makes the whole distance of the centre of oscillation from the point of suspension $39\frac{6}{65}$ inches; which, as we have previously remarked, is the proper measure of the length of a pendulum.

(90.) When the weight of the rod has to be taken into account, the calculation becomes more troublesome; the following rule is the result of such an investigation, in which the weight of the parts is used instead of their mass:—Multiply the weight of the ball by the square of the distance of its centre from the point of suspension, and two-fifths of the square of its own radius, adding one-third the product of the weight of the rod and the square of its length; and divide the whole by the weight of the ball multiplied by the distance of its centre from the point of suspension, added to one-half the weight of the rod multiplied by its length*.

Suppose we had a pendulum composed of a rod 36 inches long and 2 lbs. weight, and a bob or ball 6 inches in diameter and 10 lbs. weight, the centre of oscillation of such a compound pendulum would be—

$$\frac{10\,(39^2 + \frac{2}{5}\,3^2) + \frac{1}{3}\,(2 \times 36^2)}{10 \times 39 + \frac{1}{2}\,(2 \times 36)} = \frac{15213.6}{426} = 37.7 \text{ inches.}$$

The length of the rod and the radius of the ball are the same as in the last example; but, in consequence of the weight of the rod, the centre of oscillation, instead of being 39.1 inches distance—nearly the proper length to vibrate in seconds—it is found to be only 37.7 inches from the axis.

(91.) There is a curious property of the centre of oscillation, by which it and the point of suspension are convertible; that is to say, a pendulum vibrating in any certain time will vibrate in the same time if it be suspended from the centre of oscillation: the point which was previously the point of suspension then acting as the centre of oscillation. In general, the distance of the centre of oscillation from the centre of gravity is inversely as the distance of the point of suspension from the same centre.

(92.) When a body, or system of small bodies, such as a long rod, strikes against any thing, the effect is generally a jarring or vibration through the length of the rod; it may happen, however, that the blow is given so that no tremor takes place, the whole force apparently being expended on the object struck; the point of the rod which must strike to produce this effect is called the centre of *percussion*. This point is situated in a straight line from the axis passing through the centre of gravity, and farther than the latter point from the axis. It may be found by the following rule:—

The centre of percussion of a body is equal to the moment of inertia of the body divided by the product of its mass and the distance of the centre of gravity from the point of suspension.

(93.) This point is, therefore, the same as the centre of oscillation, except

---

[*] The formula can be more readily expressed by putting W = weight of the ball, *w* = weight of the rod, L = length of the rod, *r* = radius of the ball, and L + *r* = R. Then,

$$S\,O = \frac{W\,(R + \frac{2}{5}\,r^2) + \frac{1}{3}\,w\,L^2}{W\,R + \frac{1}{2}\,w\,L}.$$

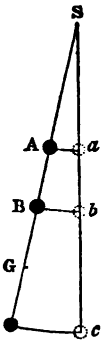
where the body be of great width in proportion to its length.

In machines, where the work is intended to be done by percussion, it is highly important to attend to this property of moving bodies. In a hammer, the handle is generally, and always should be, as light as possible, in order that the centre of percussion may be near the centre of the iron mass of the hammer; in estimating the centre of percussion, the elbow or shoulder-joint becomes the axis. That ponderous instrument called the forge-hammer, as generally constructed, affords an instance of the effects produced by a striking instrument not arranged with reference to this property; it consists of a long bar of iron turning on an axis passing through one end, the other, after being lifted up by a cam-wheel, is allowed to fall freely on the substance placed upon the anvil beneath it; as the blow is not given at the centre of percussion, a part of the force is spent in attempts to tear away from the axis. Some pendulums are made to affect the clockwork at the bob instead of the upper part, as is the usual method; in such a case it is highly necessary to be careful that the pendulum may strike with the centre of percussion, which will be the centre of oscillation; so delicate an instrument as the pendulum would soon exhibit the derangement produced by it.

(94.) It is a point of great practical importance to determine the point or line in a rotating body where, if the whole weight of the body were collected, the rotatory power would be the same as in the real form of the body or instrument. Such a point is called the centre of *gyration*.

(95.) To find a rule for calculating this point, suppose the weightless line S C (*fig.* 34), on which is fixed a number of small particles, as A, B, C, to move through the small arc A *a*, B *b*. C *c*, their velocities will be proportional to the lengths of these arcs, or, what is similar, to S A, S B, S C; as the arcs of circles are proportional to their radii, the momentum of these moving particles is A × S A, B × S B, C × S C; but, from the principle of the lever, their relative effects will be according to their distances from the fulcrum or axis, or A × S A × S A, B × S B × S B, C × S C × S C. We may take a point in the line S C, as G, at which, if all the particles, A, B, C, were collected, the rotatory effort would be the same; therefore—

Fig. 34.



$$(A + B + C)\, SG^2 = A \times SA^2 + B \times SB^2 + C \times SC^2,$$

$$\text{whence } SG_2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A + B + C}; \text{ or}$$

$$SG = \sqrt{\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A + B + C}}.$$

Therefore the centre of gyration of a system of bodies is found by dividing their moment of inertia by the sum of their masses, and taking the square root of the quotient.

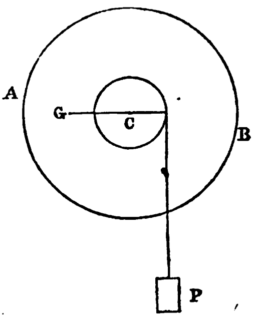
(96.) In a regular straight rod, such as the spoke of a wheel, the distance of the centre of gyration from the axis is the square root of one-third multiplied by its length, or .577 times its length.

In a circle turning about its centre, the distance of the centre of gyration is equal to the square root of one-half multiplied by the radius, or .707 the radius of the circle.

In the periphery of a circle, or the rim of a wheel, the centre of gyration is at a distance equal to the radius.

(97.) There is a useful method of determining the centre of gyration experimentally. Suppose the wheel and axle D C (*fig.* 34*), to be movable about a fixed axis, passing through the centre of gravity at C, and P a weight which gives motion to the whole; then allowing the weight to descend for any length of time, and measuring the space through which it has descended, we have the following rule :—

Fig. 34*.



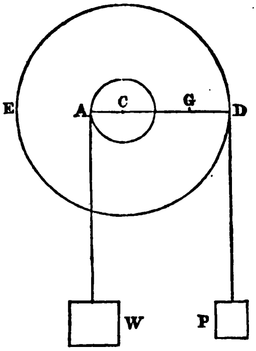
Subtract the product of the space descended by the weight and its moment of inertia from the product of the force of gravity, the moment of inertia of the weight, and the square of the time of descent; and divide by the product of the weight of the wheel and axle, and the space descended by the weight; the square root of the quotient will give the distance of the centre of gyration from the centre of gravity.

Thus, suppose a wheel and axle of 10 lbs. weight to be moved by a weight of 4 lbs., acting at the circumference of the axle, whose radius is 2 inches, and falling through 12 feet in three seconds; then C G, the distance of the centre of gyration from the axis, is

$$\sqrt{\frac{32 \times 4 \times 4 \times 9 - 12 \times 4 \times 4}{10 \times 12}} = \sqrt{\frac{4416}{120}} = \sqrt{36.9} = 6.07 \text{ inches.}$$

(98.) The following examples will illustrate some of the properties of rotation about a fixed axis.

Fig. 35.



1. Suppose it were required to find the amount of accelerating force urging a revolving wheel D E (*fig.* 35), when acted on by a force at D, and doing work by means of an axle A C. If the force at D, or the circumference of the wheel, is a weight P, pulling up another weight W, suspended from the axle, the effect of these weights will be proportional to their distances, C D, C A, from the axis; the weight W resists the force P by the power it would have at D, C serving as a fulcrum, and by the principle of the lever this resistance is equal to $\frac{W \times AC}{CD}$, therefore the effective accelerating force acting at D is the weight *minus* this resistance. But the wheel and axle must have their moment of inertia, as also the weights P and W; the moment of inertia of the wheel and axle will be the product of its weight and the square of the distance of the centre of gyration from the axis; and the weights P and W, multiplied by the squares of their respective distances from the axis, will give their moment of inertia; the two being added together, the total moment of inertia is

$$= \text{wheel's weight} \times CG^2 + W \times AC^2 + P \times CD^2.$$

The force acting at the point D will be P *minus* the resistance of W, which multiplied by the square of the distance from the axis at which it acts, or C D², is

$$CD^2\left(P - \frac{W \times AC}{CD}\right) = P \times CD^2 - W \times AC \times CD,$$

and the accelerating force at the circumference of the wheel is

$$\frac{P \times CD^2 - W \times AC \times CD}{\text{wheel's weight} \times CG^2 + W \times AC^2 \times CD^2};$$

Or, multiply the weights moved by the radii of the axle and the wheel, and subtract the sum from the moving weight multiplied by the square of the wheel's radius, and divide the remainder by the moment of inertia of the wheel and axle and the weights.

To find the force which accelerates

the weight W it must be remembered, that it is to the force accelerating the weight P as the respective distances of the weights from the axis, or

accelerating force at A : accelerating force at D :: A C : C D.

The accelerating force at D, as found above, will therefore have to be multiplied by A C and divided by C D to give the accelerating force at A; which is

$$\frac{P \times CD \times AC - W \times AC^2}{\text{wheel's weight} \times CG^2 + W \times AC^2 + P \times CD^2};$$

That is, subtract the weight moved multiplied by the square of the radius of the axle from the moving weight multiplied by the radius of the wheel and the axle, and divide by the moment of inertia of the wheel and the weights.

From this determination of the accelerating force acting on a revolving system, we may find the velocity a descending or ascending weight will attain in any time, and also the space moved through by each weight; the amounts obtained from the above rules being multiplied by the quantities given in art. 47, for finding the space and velocity; thus, according to the rules there stated,

velocity = 32 × time of falling.
space = 16 × square of the time of falling.

Each of these being multiplied by the accelerating force at P or W will give the velocity attained and the space moved through by P or W in any time.

Suppose it were required to find the space through which a weight of 2 lbs., hanging from the circumference of a wheel of 6 inches radius, would descend in 4 seconds, in drawing up a weight of 5 lbs. suspended from the axle, which is 2 inches radius, the weight of the wheel and axle being 3 lbs., and its centre of gyration $\frac{2}{3}$ of the radius, or 4 inches from the axis.

By the first rule the accelerating force urging the moving weight P would be

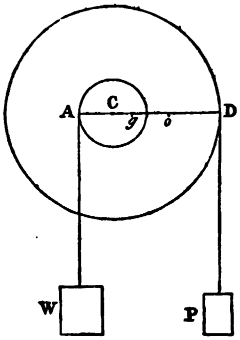
$$\frac{2 \times 36 - 5 \times 2 \times 6}{3 \times 16 + 2 \times 36 + 5 \times 4} = \frac{3}{35},$$

or $\frac{3}{35}$ of the force of gravity; multiplying this by the rule for finding the space, we have $16 \times 16 \times \frac{3}{35} = 21\frac{7}{8}$ feet, the space descended by P in 4 seconds, and at the end of that time would have acquired a velocity of $32 \times 4 \times \frac{3}{35} = 11$ feet per second. The height ascended by the weight W may be similarly calculated.

2. It is of importance to know, when a wheel turns on an axis by the action of any weight, the amount of pressure exerted upon the axis; when the system is at rest, the pressure would be equal to the weights of the wheel and weights suspended from it; but, in motion, this pressure is in part taken off the axis.

Suppose the wheel, *fig.* 37, to revolve by the action of the weight P, drawing up another weight W; *o* being the centre of oscillation and *g* the centre of gravity. In this case the system revolves with the same angular velocity as if P and W were placed at the centre of oscillation, and, as the accelerating forces are as the velocities produced in a given time, we obtain the proportion—

Fig. 37.



$$Co : Cg :: P + W : \frac{Cg}{Co} \times (P + W).$$

The last term of the proportion gives the accelerating force at the centre of gravity *g*. Now the distance of the centre of oscillation from the axis, according to art. 86, is found by dividing the sum of the weights in the system, multiplied by the square of their respective distances from the axis, by the sum of the weights multiplied by their respective distances from the axis; we have, in the present instance, two weights, and, as they are on opposite sides of the axis, the distance of their centre of oscillation, C *o*, will be—

$$\frac{P \times CD^2 + W \times AC^2}{P \times CD - W \times AC}.$$

To find C*g*, or the distance of the centre of gravity of the system from the axis, multiply each weight by its distance from the axis, taking the difference of the two, as they are on opposite sides of the axis, and divide by the sum of the weight; hence

$$Cg = \frac{P \times CD - W \times AC}{W + P}.$$

$$\text{Then } \frac{Cg}{Co} = \frac{(P \times CD - W \times AC)^2}{P \times CD^2 + W \times AC^2} \times \frac{1}{W + P};$$

as it was found by the proportion shown above that the accelerating force is $\frac{Cg}{Co} \times (P + W)$, if we multiply by $P + W$, the accelerating force acting on the centre of gravity will be

$$\frac{(P \times CD - W \times AC)^2}{P \times CD^2 + W \times AC^2};$$

or the square of the difference of the weights multiplied by their distances from the axis, and divided by their moment of inertia. The more powerfully the force acts upon the centre of gravity to make it descend, the less weight will be incumbent upon the axis of motion, so that the real weight upon the axis during motion will be

$$P + W - \frac{(P \times CD - W \times AC)^2}{P \times CD^2 + W \times AC^2}.$$

Suppose it were required to find the pressure upon the axis of a wheel of 2 feet radius, to which two weights, as P and W in the above figure, weighing 10 lbs. and 20 lbs. respectively, the axle being 6 inches radius. The sum of the weights is $10 + 20 = 30$ lbs. Then

$$30 - \frac{(10 \times 24 - 20 \times 6)^2}{10 \times 576 + 20 \times 36}$$

will express the pressure upon the axis when the wheel is in motion; performing the operations indicated, the above expression is equal to

$$30 - \frac{14400}{6480} = 30 - 2\tfrac{2}{9} = 27\tfrac{7}{9} \text{ lbs.}$$

If the weights act at an equal distance from the axis, the expression for the weight upon the axis will become

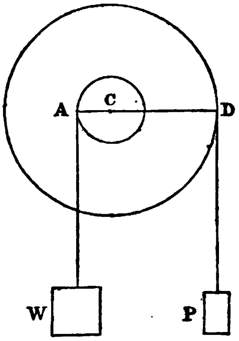
$$P + W - \frac{(P - W)^2}{P + W};$$

or the pressure will be equal to the sum of the weights minus the square of the difference of the weights, divided by their sum. Thus, in the above example, if the weights act one on each side of the pulley, they will of course act at equal distances from the axis, and the pressure will be

$$30 - \frac{(30 - 20)^2}{30 + 20} = 30 - \frac{100}{50} = 28 \text{ lbs.}$$

3. It is an interesting problem to find what the proportions of the radii C D, C A (*fig.* 37), of a wheel and axle must be, in order that a given force P may raise a weight W, or perform a work in the shortest time. We can here give the results only of such an investigation which requires the assistance of mathematical principles, which cannot be introduced here. The radius C D of the wheel may be thus found, the radius C D of the axle being considered equal to 1:—

Fig. 37.



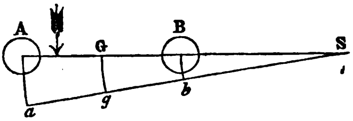
To the square of the weight, divided by the square of the power, add the weight divided by the power added to one-half the weight of the wheel, and to the square root of the sum add the weight, divided by the power.

For example: suppose the wheel whose radius is required, to be 10 lbs. weight, the diameter of the axle being 1 inch, the weight lifted or W = 50 lbs., and the power P = 15 lbs. Then—

$$C\,D = \sqrt{\frac{50^2}{15^2} + \frac{50}{20}} + \frac{50}{15} = 3.75 + 3.33 = 7.1 \text{ inches nearly.}$$

(99.) When a force actuates a body which, having no fixed axis, is at liberty to move in any direction, two consequences may be observed—there is either a simple progressive motion of the whole body, or a rotation about some point as an axis, and a progressive motion also. The former effect results when the direction of the force passes through the centre of gravity of the body, and the latter, when the direction of the force passes in any other direction through the body. The subject is one of great interest in the physical sciences, as it explains several astronomical phenomena, such as the precession of the equinoxes and the nutation of the earth's axis in consequence of the attractive forces of the sun, moon, and planets, besides numerous questions in the mechanical sciences generally.

Fig. 38.



(100.) For the purpose of showing the nature of this compound motion, suppose the two particles A, B (*fig.* 38), to be connected by the line A B, the centre of gravity of the whole being at G, and we require to determine the effect produced by a force acting at any point, as C. According to the property of the lever, the effect produced on the particle A is to the effect produced on the particle B, as B C : A C, which will consequently be the ratio of the spaces A *a*, B *b*, described in any time, if the bodies be equal; if they be not equal, the velocities with equal forces being inversely as the masses (art. 32), the ratio of the spaces described will be $\frac{B\,C}{A} : \frac{A\,C}{B}$. Taking A *a*, B *b*, in the ratio of B C : A C, draw a line *a b*, joining their extremities, and produce it to meet the line A B produced in S; then S is the point about which the whole system revolves for the instant during which it describes the space between the lines; its distance from the point of application C of the force may be found from the property of similar triangles, whence B S : A S : : B*b* : A*a*; or as $\frac{A\,C}{B} : \frac{B\,C}{A}$, or as A × A C : B × B C; again, as B S = C S — C B, and A S = A C + C S, the proportion may be given—

$$C\,S - C\,B : A\,C + C\,S : : A \times A\,C : B \times B\,C.$$

Multiplying them together we have

$$C\,S \times B \times B\,C - B \times C\,B^2 = C\,S \times A \times A\,C + A \times A\,C^2.$$

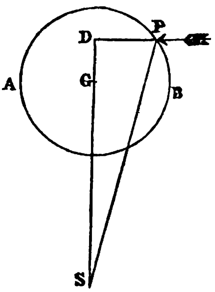
From this we obtain

$$C\,S = \frac{A \times A\,C^2 + B \times B\,C^2}{B \times B\,C - A \times A\,C},$$

which is the expression for the distance of the centre of percussion of a body from its axis. While the particles A and B move through their respective arcs A *a*, B *b*, the centre of gravity, G, of the whole has moved through the space G *g*; the system thus acquires a progressive as well as a rotatory motion. The point S is called the centre of *spontaneous* rotation, and varies in position according to the point at which the body is struck. When the system is struck at G, the leverage of both particles being equal or in inverse proportion to their masses, an equal amount of motion ensues in each particle, and there is merely a progressive motion of the whole.

(101.) As we have seen that the centre of spontaneous rotation is to be found by the same method as the centre of percussion or the centre of oscillation, we may readily determine the two motions of a body when the force applied and the mass of the body are known. Suppose, for instance, a force P (*fig.* 39), to act at P upon the ball A, its direction P D would not be through G, the centre of gravity; a motion will, therefore, ensue for an instant round some point S. The progressive velocity communicated by the force is equal to the force divided by the mass of the body; and the rotatory velocity may be found by dividing the force applied by the product of the mass of the body,

Fig. 39.



the ratio of the circumference of a circle to its diameter, and the distance of the centre of spontaneous rotation from the centre of gravity of the body, or $\frac{P}{A \times 6.28 \times S\,G}$.

For instance, if a ball 12 inches in diameter, and 20 lbs. weight, were struck by a body of 1 oz. weight, moving at the rate of 60 feet per second; the direction P D of the force meeting a radius at a distance from the centre G of $\frac{1}{2}$ of the radius, what would be the progressive and rotatory velocity of the ball? The comparative momentum of the impinging force may be called 60 (art. 33), and the ball being 240 oz. weight, its progressive velocity will be $\frac{60}{240} = \frac{1}{6}$ of a foot per second. To find the rotatory velocity, the distance S G must first be determined; but as it is the same as the centre of oscillation, it will be two-fifths of the square of the radius divided by the distance between the point D and the centre of gravity G; or $S\,G = \frac{2}{5}\left(\frac{\text{rad.}^2}{D\,G}\right)$; but as D G is supposed to be equal to half the radius, $S\,G = \frac{4}{5}$ radius, which will be $\frac{2}{5}$ of a foot. The rotatory velocity will be

$$\frac{60}{240 \times 6.28 \times \frac{2}{5}} = \frac{60}{303} = \frac{2}{10} \text{ of a foot}$$

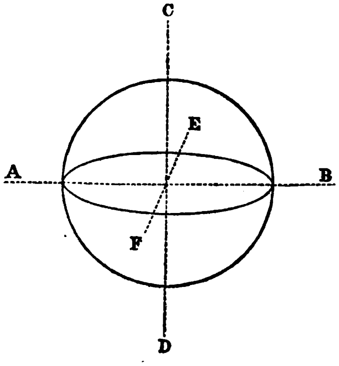
per second nearly.

(102.) It may be generally observed in a free rotation that the axis is temporary, for after the body has rotated about one axis for a short time it will move about some other axis passing through it, which is in consequence of the centrifugal force created by the motion of the body not acting equally on each side of the axis. This movement of the axis will continue until one is obtained around which the centrifugal forces will be equal, and the axis manifests no tendency to change from that position; such an axis is called a *permanent axis* of rotation. In every body there are three such axes, called the *principal axes of rotation*, about any one of which motion will continue, as the centrifugal forces are in equilibrium.

(103.) The determination of these axes by calculation would be a troublesome process, but they may be found in any body *symmetrical* about an axis passing through it, from the circumstance that this axis will be one of the three principal axes, the centrifugal forces being in equilibrium; also it is found that the three axes are at right angles to each other.

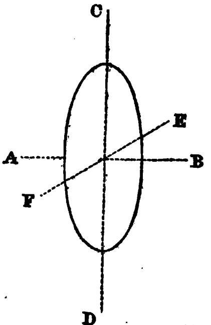
Thus a sphere is symmetrical about any diameter, and the centrifugal forces are equal if the sphere be homogeneous; therefore the three principal axes in a sphere are any three diameters at right angles to one another, as A B, C D, E F (*fig.* 40).

Fig. 40.



In a circle one of the axes is a line A B drawn at right angles to its plane (as

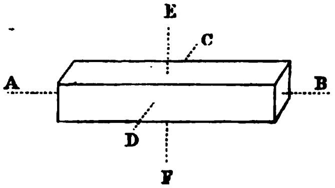
Fig. 41.



the axle of a wheel), the other two being any two lines drawn in the direction of its surface, and at right angles to one another, as in *fig.* 41.

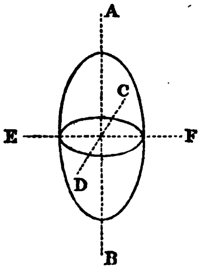
In a rectangular parallelopiped the axes are three lines drawn parallel to the sides and equidistant from them. The three principal axes of a cylinder may be similarly drawn. *Fig.* 42.

Fig. 42.



In an ellipse the principal axes are the axes of that figure, the longest axis being the major axis of the ellipse, and the other two any two minor axes at right angles to one another, as in *fig.* 43.

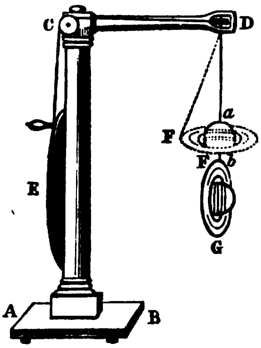
Fig. 43.



(104.) It has been found that bodies do not tend to turn about any one of these axes indifferently, for in the case of the axis A B, *figs.* 41, 2, 3, unless they rotate accurately round that line, which is scarcely possible, the axis will be relinquished for that which is the *shortest* axis of the three; this axis is the only one of the three into which there is a tendency of the body to settle. Any mass rotating freely will, therefore, adopt a number of spontaneous axes until it settles finally in the shortest which can be obtained. This singular law of rotating bodies is exemplified in a most striking manner in the rotations of the planets. The earth has been proved by various methods to be of an oblate spheroidal form, revolving about its shortest diameter.

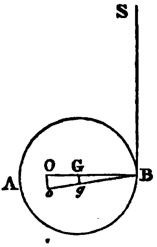
These facts may be pleasingly illustrated by means of the instrument shown in *fig.* 44. Upon the stand A B is fixed the pillar A C, carrying a horizontal arm C D; at D there is a small pulley turned by the multiplying wheel E, which is moved by the handle near its circumference. From the pulley D a thread is suspended, to which is attached the object intended for experiment. In the figure we have placed a representation of the planet Saturn, attached to the string at the point F, which is one of the longest diameters of the whole body. On turning the wheel it will not rotate about the axis F G, but the shortest axis *a b*, as is shown in the figure by the dotted lines, if the rotation be sufficiently rapid.

Fig. 44.



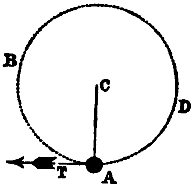
(105.) In the descent of bodies along inclined planes by rolling, the effect is different from that which would result from a sliding motion; a ball or a cylinder will roll under such circumstances because friction prevents its sliding, and as the line drawn from the centre of gravity of the ball perpendicularly downwards does not pass through the point of contact, the ball continually tumbles over or rolls. The effect of gravity in such a case as compared with its effect upon a body moving freely may be found in the following manner. Suppose the circle A B, *fig.* 45, to have a thread wound round the circumference, one end being attached to a fixed point at S; if the circle be allowed to move, it will descend by unwinding the thread, but not so rapidly as if it were free; it is thus made to rotate. At every instant of its downward motion it is turning about some point, as B, in its circumference, which is thus a spontaneous axis, and is similar to a pendulum, having its centre of oscillation O, which moves as though the whole mass of the circle were placed there, and suspended from the point B; also the motion at this point will be the same as that of a body moving freely. The velocity of the centre of gravity is, however, the proper measure of the velocity of the body's motion; so that the ratio of the velocities of these two points will be equal to the ratio of the velocities of a body moving freely by the action of gravity and the body unwinding the string; that is as O *o* : G *g*, or from the relations of the sides of similar triangles, as B O : B G; but B O and B G are the distances of the centres of oscillation and gravity from the temporary point of suspension B; therefore the velocity of a body falling freely is to the velocity of the body unwinding the thread as the distance of the centre of oscillation to the distance of the centre of gravity from the temporary axis of rotation at the circumference of the circle.

Fig. 45.



(106.) We have had occasion to notice (in art. 102) the effect of *centrifugal* force upon rotating bodies; it is a force called into action by revolving motion, and its tendency is to force the body from the curve line in which it moves into a rectilinear path, as is shown in *fig.* 46; the dotted circle A B D represents the path in which a particle A is made to revolve about the centre C, from which it will continually endeavour to fly off in the direction indicated by the arrow, or the tangent A T to the curve. A stone tied to one end of a piece of string, the other being held by the hand, affords, when whirled in the air, a simple illustration of the nature of centrifugal force; as the velocity of its revolving motion increases, the centrifugal force increases; the tension or pull on the string being equal to the centrifugal force. The opponent force, or that which retains the revolving body in its curvilineal path, is called the *centripetal* force. In the arrangements of all heavy instruments and machines, where a considerable rotative velocity is required, it is of importance to know the amount of centrifugal force brought into action, or the centripetal force necessary to preserve the machine from injury.

Fig. 46.



(107.) The calculation of the centrifugal force exerted on any body revolving with a known velocity, and at a known distance from the axis, is easily effected. The following is a simple method:—

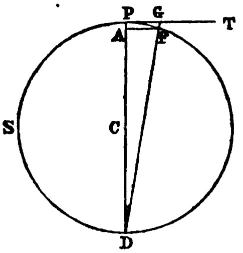
Find the height through which a body must fall to acquire the velocity with which the body in question rotates, and this proportion is obtained: the radius of the circle described by the revolving body is to twice the height producing the velocity as the weight of the body is to the centrifugal force. According to the rules given in art. 47, the height or space moved through to acquire any given velocity is equal to the square of that velocity divided by twice the force of gravity, so that twice the height is equal to $\frac{(\text{velocity})^2}{32}$; making use of this the above proportion will be

$$\text{radius} : \frac{(\text{veloc.})^2}{32} :: \text{weight of body} : \frac{(\text{vel.})^2 \times \text{weigh}}{32 \times \text{radius}}$$

The last term therefore expresses the centrifugal force.

(108.) When two or more bodies are revolving on their centres, the relation between their centrifugal forces is as their radii directly and as the squares of the times of their revolution inversely; or the centrifugal force acting on any body A is to that acting on any body B as $\frac{\text{radius of A}}{(\text{time of A})^2}$ is to $\frac{\text{radius of B}}{(\text{time of B})^2}$.

Fig. 47.



(109.) From the nature of central forces we may find the time of revolution and the centrifugal force acting on any body revolving at any distance from the centre of the earth; the centripetal force in this case being gravity. Suppose P, *fig.* 47, to be moving in the circle P D S, its tendency, according to the property of inertia, would be to continue its rectilinear motion in the direction P T, but by the centripetal force it is brought to F, through the very small arc P F, in the time that it would have reached G; the body may thus be said to have fallen through the space G F or P A, the latter being considered parallel and equal to G F, while it moved through the distance P G; the lines P A and P G may therefore be taken to represent the relative amounts and directions of the forces acting in the directions P T and P C. We shall here consider the arc P F so small that the line P F may not perceptibly differ from a straight line, and P F A will be a triangle; then drawing a line from F to D, the point where the diameter from P meets the circumference, we have the similar triangles P F A, P D F, and consequently the proportion—

$$PD : PF :: PF : PA.$$

P A, or the space fallen by the moving particle, is thus equal to $\frac{PF \times PF}{PD}$; and the distance P F described in the same time will be equal to $\sqrt{PD \times PA}$, or the square root of the product of the diameter of the orbit and the distance moved through by the action of gravity.

These results will serve for calculation. The space P A may be taken to represent the space a body falls by the action of gravity in the first second of time or 16 feet; then the small space will be $\sqrt{16 \times \text{diameter}}$; or, as the diameter is equal to twice the radius, it will be more convenient to make the above equal to $\sqrt{32 \times \text{radius}}$; which expresses the velocity of P's orbital motion. The time of revolution may be easily found; as the velocity of P we have obtained gives the rate per second, the whole time of revolution will be to one second as the velocity of P is to the circumference of the circle, which is equal to the diameter or twice the radius multiplied by 3.1416; or using these more calculable quantities

$$\sqrt{32 \times \text{radius}} : 2 \text{ radius} \times 3.1416 :: 1'' : \left(\frac{2 \text{ rad.} \times 3.1416}{\sqrt{32 \times \text{rad.}}}\right)''.$$

By the last term the time of revolution may be calculated. This term may, however, be simplified thus:—

$$\frac{2 \text{ rad.} \times 3.1416}{\sqrt{32 \times \text{rad.}}} = \sqrt{\frac{4 \text{ rad.}^2}{32 \times \text{rad.}}} \times 3.1416 = 3.1416 \times \sqrt{\frac{4 \text{ rad.}}{32}};$$

or, the time is equal to 3.1416, multiplied by the square root of the quotient of twice the radius divided by the space fallen freely in the first second. Thus, in order that a body should revolve round the earth at its surface, the radius of the earth being 3,956 miles or 20,887,000 feet, it would require a velocity of about 26,000 feet per second, and the time of revolution would be 5,084″, or about 1 hour, 24 minutes, 44 seconds.

(110.) By the aid of the above considerations we are able to find the relative amount of the centrifugal and centripetal forces at the earth's surface. In the first problem, the relative amount of the central forces, we can find what the time of revolution ought to be in order that the forces may counterbalance each other; then by art. 108 the forces are to one another as the squares of the times of revolution inversely. We have already found the time of revolution at the earth's surface, when equilibrium is obtained, to be 5,084 seconds, but the earth revolves in 23 hours, 56 minutes, 4 seconds, or 86,164 seconds; then, if we call the force of gravity 1, we find

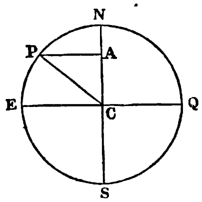
$$(86{,}164'')^2 : (5{,}084'')^2 :: 1 : \tfrac{1}{289}.$$

Therefore the centrifugal force at the

equator of the earth is $\frac{1}{289}$ of the attractive force or gravity.

The centrifugal force is not the same in all parts of the earth, and the above ratio of the two forces is limited to the equator. In explaining the causes of variation in the time of vibration of pendulums (art. 67), we had occasion to remark that the force of gravity, or the centripetal force, is greater as we approach the poles or stationary points of the earth; at the north and south poles the centrifugal force is nothing; one of the causes of the increased effect of gravity is therefore the decrease of this opponent force. In *fig*. 48, let N and S represent the north and south poles of the earth, E Q the equator; then the latitude of the place P will be the arc E P, and measured by the angle E C P; taking A P as the radius of another circle, we find by art. 108 the relation of the centrifugal forces at E and P is as E C to P A, which gives the relative centrifugal force in the direction A P; but as we require to have it in the direction C P, or the vertical at the point P, we have the following proportion, calling the radius E C = 1 :—

Fig. 48.



$$1 : A P : : A P : A P^2.$$

According to the principles of trigonometry, A P represents the *cosine* of the angle E C P, whence $A P^2 = E C^2 \cos.^2 E C P$; or the centrifugal force at the equator is to the centrifugal force at any latitude as the radius of the earth to the square of the cosine of that latitude; the force of gravity also diminishes as the square of the cosine of the latitude. We say this however, considering the earth as a sphere; but as the earth is compressed at the poles, by reason of its rotatory motion, .0033 or $\frac{1}{289}$th of its equatorial diameter, the decrease of gravity is not truly according to the above law. It has also been found, in the use of the pendulum to determine the intensity of gravity at different latitudes, that local circumstances, such as geological constitution or the vicinage of hills, affect the intensity of gravity.

(111.) The following illustration exhibits a simple method of calculating the tension resulting from the action of centrifugal force. Suppose the ball A (*fig*. 46), to be revolving in the dotted orbit or circle A B D, then the centrifugal and centripetal forces are equal at the circumference of the earth when the time of revolution is $3.1416 \sqrt{\frac{2 \text{ rad.}}{16}}$ (art. 110), and the greater centrifugal force is to the lesser centrifugal force as the greater to the lesser radius directly, and as the square of the greater to the square of the lesser time of revolution inversely; or (T being the greater and t the lesser time) as $\frac{1}{T^2}$ to $\frac{1}{t^2}$ (art. 108). Supposing the greater radius and greater centrifugal force each equal to unity, the proportion just stated will be

$$\frac{16}{2 \times 3.1416^2} : \frac{A C}{(\text{time of } A)^2} : : 1 : \frac{2 \times 3.1416^2 \times A C}{16 \times (\text{time of } A)^2}.$$

The last term thus gives the centrifugal force in the circle A B D, whose radius is A C.

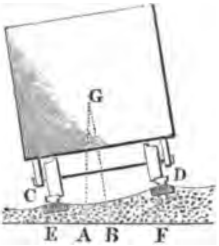
For example; if A C = 10 feet, and the time of A's revolution = 2 seconds, its weight being 5 lbs., the centrifugal force would be

$$\frac{2 \times 3.1416^2}{16 \times 4} = \frac{197.7}{.64} = 3\tfrac{1}{12} \text{ nearly.}$$

That is the string, or whatever held the ball A in its orbital motion, would experience a tension of $3\frac{1}{12}$ times the weight of the ball, which in the present example would be $15\frac{5}{12}$ lbs.

The powerful effects of centrifugal force in locomotion are well known; a carriage turning the corner of a street cannot be driven with very great velocity lest it should overturn; and the experienced horseman almost instinctively leans inward under similar conditions. This principle is imitated in railways where curves occur. From the great speed with which a train moves, the centrifugal force on a curve would be so great as to throw the train from the rails unless some method were adopted to neutralize this force. The plan adopted is that of raising the outer rail, as is shown in (*fig.* 49), D being the outer rail, or the farthest from the centre of the circular curve, and C the inner rail. The centre of gravity G of the carriage is thus thrown inwards, or towards the inner rail, by the distance B A, which neutralizes the centrifugal force tending to turn the carriage over the rail D. In this case it is required to find the elevation of the outer rail D, in order that the centre of gravity may be thrown enough inward to keep the carriage or train in a state of stability when at its greatest speed on the curve. It has been found that for a curve of a thousand feet radius the outer rail should be about 3 inches higher than the inner rail, when the train moves at the rate of 30 miles an hour. Notwithstanding the precaution thus taken, it may be observed, when trains run along curves at high velocities, that there is a great deal of vibration, especially when the gauge or width between the rails is narrow.

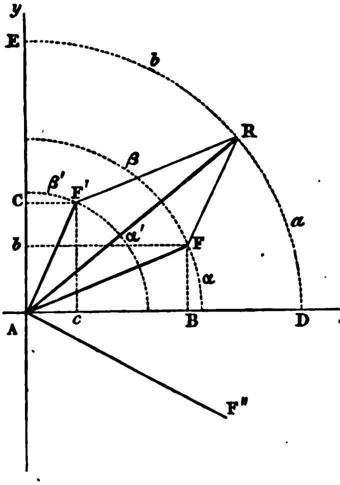
Fig. 49.



CHAPTER VIII.—*Mathematical Illustrations.*

(1.) *The parallelogram of forces.*—This proposition is one of great importance in the science of Mechanics, being found under various points of view in very many of the problems in statics and dynamics.

The most general view which can be taken of the subject is that wherein several forces are supposed to act simultaneously upon a material particle, no limitation being made to the direction in which each force shall act, these forces are then referred to three planes at right angles to one another to obtain the necessary expressions for the calculation of their resultant. We shall, however, in the succeeding investigation limit the direction of the forces to one plane, by which the problem is much simplified; referring the reader to some of the works mentioned at the end of this treatise for a demonstration of the more general proposition.



Let F F′ be two forces acting upon a particle at A, in the directions A F, A F′, situated in one plane. Draw through the origin A the lines A x, A y, at right angles to each other; then each of the forces F F′ may be decomposed into two—one in the direction of the axis A x, and the other in that of A y. If we represent the angles F A B, F′ A B, which the forces F F′ make with the axis A x by α, α', and the angles F A C, F′ A C, made by the same forces with A y by β, β', by the principles of trigonometry the component forces of F, which are A B, A b, may be denoted by

$$F \cos. \alpha, \ F \cos. \beta.$$

Similarly the components of F′, or A c, A C, will be

$$F' \cos. \alpha', \ F' \cos. \beta'.$$

Adding these components as referred to the axes A x, A y, and expressing their sums by X and Y, they become

$$F \cos. \alpha + F' \cos. \alpha' = X. \quad (1)$$

$$F \cos. \beta + F' \cos. \beta' = Y. \quad (2)$$

The forces being thus reduced to two, acting at right angles to each other, their resultant R will be the diagonal of the parallelogram formed by their quantities. Therefore

$$R = \sqrt{X^2 + Y^2}. \quad (3)$$

By this equation the magnitude of the resultant may be found; to determine its direction, let a and b be the angles R A D and R A E which it makes with the axes A x, A y; then

$$X = R \cos. a, \quad Y = R \cos. b.$$

From which we obtain the function of the angles

$$\cos. a = \frac{X}{R}; \quad \cos. b = \frac{Y}{R} \quad (4)$$

If the number of forces in the investigation are more than two, their components with respect to the axes A x, A y, have merely to be added to the equations (1) and (2).

If the angle between any two forces be greater than a right angle, some of the component quantities will be negative, as occurs with the force F″; in this case denoting the angle B A F″ by a'', and the angle made with A y or E A F″ by β'', the components of F in these directions will be

$$F'' \cos. a'' - F'' \cos. \beta.$$

(2.) *Uniformly Accelerated Motion.*—In order to determine the relation of time, space, and velocity in this description of motion, we may suppose a uniform force F acting on any material particle for a length of time T composed of a number of minute portions of time t, t, &c.; the whole number n of these portions will therefore make up the whole time, or $n\,t = T$; then the force acting at each of these small portions of time multiplied by the time of motion will give the velocities at the end of each time.

$$t, \quad 2\,t, \quad 3\,t, \quad 4\,t, \quad 5\,t \ldots\ldots n\,t.$$
$$F\,t, \; 2\,F\,t, \; 3\,F\,t, \; 4\,F\,t, \; 5\,F\,t \ldots\ldots n\,F\,t.$$

And these velocities being supposed uniform during each of the times, the spaces corresponding will be

$$Ft^2, \; 2Ft^2, \; 3Ft^2, \; 4Ft^2, \; 5Ft^2 \ldots\ldots nFt^2,$$

because the velocity multiplied by the time gives the space described (art. 12). The sum of these quantities is—

$$Ft^2(1 + 2 + 3 + 4 + 5 \ldots\ldots n) = Ft^2\frac{n(n+1)}{2} = \frac{Ft^2n^2}{2} + \frac{Ft^2n^2}{2n};$$

and as $n\,t = T$, the last expression $= \dfrac{FT^2}{2} + \dfrac{FT^2}{2n}$.

This, therefore, is the sum of the spaces described with a number of uniform velocities, and is greater than the space described by a body uniformly accelerated, for it must be evident that a particle would move through a less space with an increasing velocity than it would if it had the terminal velocity during the whole time of its motion; so that the space s described with a uniform acceleration is less than the last expression, but it will approach it nearer as the small times t are smaller, by which their number n becomes greater, and when n is infinitely large

$$s = \frac{FT^2}{2} + \frac{FT^2}{2n};$$

we may observe here, that as we have supposed n to become an infinitely large number, the second fraction will be exceedingly small, and may be disregarded, whence

$$s = \frac{FT^2}{2}, \text{ or } \tfrac{1}{2}FT^2.$$

Again, according to the note to art. 12, the velocity of a body moving through any space s in a certain time t, is $= \dfrac{s}{t}$. Suppose the uniformly accelerated particle to describe the small space ds in the small time dt, the velocity will be

$$v = \frac{ds}{dt}, \qquad (5)$$

which supposes the rate of motion to be uniform; if the velocity increase by uniform acceleration during the small time dt, the velocity will be $v + dv$ at the end of the time $t + dt$, or dv is the increment of velocity during the small time dt, and, as the force is constant, it will give the additional velocity dv during each small portion of time, and dv multiplied by the number of small times dt contained in any length of time will give the velocity attained during that time; if the time be 1, dv will have to be multiplied by $\dfrac{1}{dt}$; thus

$$F = \frac{dv}{dt}. \qquad (6)$$

We have now obtained two equations of motion, $ds = v\,dt$, and $F\,dt = dv$, whence

$$F\,ds = v\,dv. \qquad (7)$$

By substituting g (= 32.19 feet) for F we can obtain expressions of the force of gravity; for by equation (6) $g = \frac{dv}{dt}$, whence $dv = g\,dt$, and integrating $v = g\,t$; also, by equation (5), $ds = v\,dt$, or using the value just found for v, $ds = g\,t\,dt$, which by integration gives $s = \frac{1}{2}gt^2$, which is similar to the result obtained by the first method; from this equation, and $v = g\,t$, we have the following equations for calculating space, velocity, time, and force:—

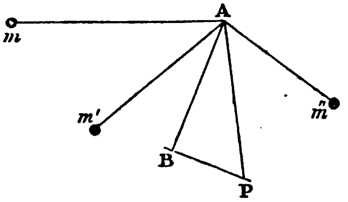
$$s = \tfrac{1}{2}g\,t^2 = \tfrac{1}{2}t\,v = \frac{v^2}{2g}.$$

$$v = g\,t = \frac{2s}{t} = \sqrt{2gs}.$$

$$t = \frac{v}{g} = \frac{2s}{v} = \sqrt{\frac{2s}{g}}.$$

$$g = \frac{v}{t} = \frac{2s}{t^2} = \frac{v^2}{2s}.$$

(3.) *Rotation about a fixed axis.*—Suppose the particles m, m', m'', to be attached to a fixed axis A, and stationary with respect to each other, their radii being r, r', r''; any motion communicated to the system would produce the same angular velocity θ in each particle, but the axes described being as the length of the radii, the respective velocities would be $r\theta$, $r'\theta$, $r''\theta$, of the particles m, m', m''. Their quantities of motion would therefore be $mr\theta$, $m'r'\theta$, $m''r''\theta$, which will be proportional to the force impressed upon the whole system. Let f be a force acting at P in the direction P B, which would equilibrate the force mr at m; then if a line be drawn from the axis perpendicular to P B by the principle of the lever—



$$f : mr\theta :: \mathrm{A}m : \mathrm{AB} :: r : \mathrm{AB}.$$

From which we find

$$f \times \mathrm{AB} = mr^2\theta.$$

The same may be shown of any other forces, f', f'', equilibrating the moments of the particles m', m'', or $m'r'\theta$, $m''r''\theta$; so that, taking the sum of the forces $f + f' + f'' = \mathrm{F}$, we have

$$\mathrm{F} \times \mathrm{AB} = \theta(r^2m + r'^2m' + r''^2m'').$$

Supposing the particles m, m', m'', equal

$$\mathrm{F} \times \mathrm{AB} = \theta\,\Sigma\,mr^2.$$

The quantity $\Sigma\,mr^2$ indicates the *sum* of the moments of all the particles in the system; for which sign we use that of integration, whence $\Sigma\,mr^2$ will become

$$\int r^2 dm.$$

In the last equation $\mathrm{F} \times \mathrm{AB}$ represents the moment of the force F with respect to the axis A, and $\int r^3 dm$ the *moment of inertia* of the system with respect to the same axis; the same equation also furnishes an equation for the angular velocity θ,

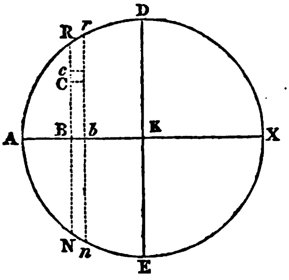
$$\theta = \frac{\mathrm{F} \times \mathrm{AB}}{\int r^2 dm}.$$

The calculation of the moment of inertia is illustrated in the following examples:—

To find the moment of inertia of a straight rod.—Let l be the length of the rod, being attached to a fixed axis at one end, and x any point in its length. Then the moment of inertia of the point x will be $\int x^2dx$, which, to extend to the whole length, must be integrated between the limits of $x = 0$ and $x = l$, or $\int_0^l x^2\,dx = \frac{1}{3}l^3$, which is the moment of inertia of a straight rod.

To find the moment of inertia of a circle rotating about its centre.—In a circle whose radius is r take a point at any distance x from the axis, which will thus describe a circle whose circumference is $2\pi x$ ($\pi = 3.1416$), and surface πx^2, the latter expression being equivalent to the mass of the circle, as it is supposed to be very thin; then the differential of πx^2, or $2\pi x\,dx$, will express the element dm, and $\int r^2dm = \int 2\pi x^3\,dx = \frac{1}{2}\pi x^4$, which when x becomes equal to r, $= \frac{1}{2}\pi r^4$, the moment of inertia of a circle revolving like a wheel about its centre. If the circle be of any thickness, it becomes a cylinder, and the result we have obtained must be multiplied by the length of the cylinder; if the length be l, the moment of inertia of a cylinder is $\frac{1}{2}\pi l r^4$.

To find the moment of inertia of a sphere revolving about a diameter.—In the accompanying figure, D E is a section of a sphere, A X being a diametral axis. Suppose the sphere to be divided into a number of thin plates, one of which is shown at B*b*, and may be called dx; C a point at the distance r from the axis A X, C*c* being an elementary portion, which we shall call dr, so that B*c* $= r + dr$. The circumference at the point P describes a length equal to $2\pi r$ during a revolution, and dm will in this case be $= 2\pi r^3\,dr\,dx$, which, integrated with respect to $r =$ BR and $r = 0$, gives $\frac{1}{2}\pi y^4\,dx$ for the moment of inertia of the cylinder or small portion whose thickness is dx, where $y =$ BR. To apply this to the whole sphere, we shall take the equation to the circle*, which gives $y^2 = 2rx - x^2$, r being the radius A K of the circle or sphere, whence $y^4 = 4a^2x^2 - 4ax^3 + x^4$; substituting this for y^4, we have $\frac{1}{2}\pi\int y^4\,dx = \frac{2}{3}\pi a^2 x^3 - \frac{1}{2}\pi d x^4 + \frac{1}{10}\pi x^5$, which, taking x between the limits $x = r$ and $x = 0$, shows the moment of inertia of the hemisphere D A E to be $\frac{4}{15}\pi r^5$. This amount being multiplied by 2 gives the moment of inertia of the whole sphere $= \frac{8}{15}\pi r^5$.



4. *To find the centre of oscillation.*—According to the investigation given in art. 86, the distance O of the centre of oscillation from the axis may be found from the general formula—

$$O = \frac{\int r^2\,dm}{\int r\,dm}.$$

Thus the centre of oscillation of a slender rod, whose length is l, will be

$$\frac{\int l^2\,dl}{\int l\,dl} = \frac{\frac{1}{3}l^3}{\frac{l^2}{2}} = \frac{2}{3}l.$$

As the centre of percussion is identical with that of oscillation, except where the body has considerable width in proportion to its length, the same formula will apply.

5. *To find the centre of gyration.*—By art. 95 the distance G of the centre of gyration from the axis is equal to the square root of the moment of inertia divided by the sum of the particles in the body or system; whence we have the following formula:—

$$G = \sqrt{\frac{\int r^2\,dm}{dm}}.$$

Thus, in a straight rod whose moment of inertia is $\frac{1}{3}l^3$,

$$G = \sqrt{\frac{\frac{1}{3}l^3}{l}} = \sqrt{\tfrac{1}{3}l^2} = l\sqrt{\tfrac{1}{3}}.$$

In the preceding chapters of this treatise we have endeavoured to give a general and simple view of dynamical science, although of necessity much that is valuable is omitted on account of the mathematical expressions which would necessarily have to be introduced; we have also omitted those parts of the science of motion which connect themselves more particularly with astronomy, the treatise on that sublime science comprehending a full statement of those subjects.

The science of dynamics is of comparatively recent date, nothing having been done before the time of the celebrated Galileo. Aristotle, who lived about 320 years before the Christian era, had indeed attempted to give some laws of moving bodies, but they were generally erroneous. From that time nothing was done to advance the knowledge of moving forces for many centuries; the few who studied mechanics after the time of Archimedes (who wrote upon statics) either implicitly believed the dogmas of the philosopher of Stagira, or had not sufficient perseverance to investigate for themselves those phenomena which must have continually demonstrated to them the incorrectness of their great master's views. Pappus, Cardan, the Marquis Ubaldi, and Simon Stevin, subsequently wrote upon several of the principles of equilibrium, the last-mentioned philosopher discovering that very important proposition the parallelogram of forces; which, however, he left to succeeding inquirers to apply generally. We now arrive at the period of Galileo's labours in physical science. In a little work on Statics, published in 1592, he laid the foundation of the principle now known as that of virtual velocities; he afterwards gave his theory of variable motions, a subject which had been untouched by all the philosophers who lived before his time; he thus found the rate at which a body was accelerated in falling towards the surface of the earth. He discovered also the principle of the pendulum, it is said through accidentally observing a chandelier swinging in a church at Pisa; and founded the theory of projectiles, proving that the curve described in the air by the projected body is a parabola. From this time the science of mechanics was continually enlarged, either by private investigation or through the continual efforts of the scientific men of succeeding times to puzzle and annoy one another. Mersenne, in 1635, proposed for solution the problems of determining the centres of oscillation and percussion in a system of bodies capable of moving about a fixed axis. Huyghens correctly determined both questions; in solving the former, he brought out a theorem which subsequently formed an important principle in mechanics; this proposition was, that if a compound pendulum has described the downward half of its arc of oscillation, and the separate bodies of which it is composed were suddenly disunited, each performing the remainder of the arc of oscillation with the velocity it had attained in descending, the common centre of gravity of these several parts will rise to a height equal to that which would have been described by the centre of gravity of the compound pendulum. Huyghens is celebrated as the discoverer of the isochronism of a pendulum when vibrating in a cycloidal curve; also, he was the first who studied curvilinear motion. Our countrymen Wallis and Sir Christopher Wren, and Huyghens, about this time investigated the laws of collision, each arriving at the truth independently of the others. In 1686, James Bernouilli took up Huyghens' disputed theorem, and, with the assistance of the Marquis de l'Hôpital, established the principle in a moving system of bodies called the preservation or conservation of living forces (*conservatio virium vivarum*), which is this:—If we take the sum of the products of the masses multiplied by the squares of their velocities, it will be found the same at every instant of time. At this period (1687), Newton published his *Principia*, which related more particularly to the motions of the heavenly bodies; investigating their orbits with reference to the new theory of gravitation. He demonstrated the principle of the preservation of the centre of gravity, by which we understand that the condition of the centre of gravity of a body remains unaffected by the mutual action of those bodies; so that whether they act on each other by any material interconnection as a lever, or by the attraction of gravitation, the centre of gravity will always be at rest or move in a right line. In the same work are given investigations concerning the oscillation of pendulums, resistance of fluids to solids moving in them, and the theory of projectiles. D'Alembert wrote, in 1743, on mechanics, and gave the proposition called by his name, which is, that if a number of bodies tend to move with velocities and in directions which they are compelled to change through mutual action, the motions may be divided into two classes—one, which the bodies take, and the other such as would of themselves have kept the bodies in equilibrium. About the beginning of the last century Liebnitz commenced a discussion which excited the combative tendency of the scientific world for many years; the great subject of dispute was, whether the force of a moving body should be considered proportional to the simple velocity, or its square, the English philosophers maintaining against the Continentals that the simple velocity should be taken. Nobody gained the victory. Professor Segner, in 1755, discovered the beautiful principle that a rotating body possessed three axes which are the principal axes of rotation; and Coulomb commenced his inquiries on the subject of torsion, a principle which he elegantly employed in several of his scientific investigations. Lagrange, in 1788, published his "*Mécanique Analytique*," in which he used the important principle of virtual velocities, but, as it requires considerable explanation to make it intelligible, we cannot introduce it here. Since the time of Lagrange little has been done for theoretical mechanics; M. Poinsot, in 1834, communicated to the Académie Royale des Sciences the results of his investigations for simplifying the explanation of the phenomena of rotation, and other branches of mechanical science. In conclusion, we must advise our readers, if they desire to become acquainted with the great principles of this science, to make themselves somewhat familiar with the nature of mathematical reasoning, and particularly with the principles of algebra, without which little progress, comparatively speaking, can be made in our knowledge of the physical sciences.

---

* See *Algebraical Geometry*, art. 65.

CONSTRUCTION OF MACHINERY.

Chapter I.—*Elements of Machines.—Wheelwork.—Pitch Circle and Pitch.—Forms for Teeth—their strength.—Friction Wheels.—Shafts and Spindles — their strength. — Simple and Differential Movements. — Gearing Apparatus.*

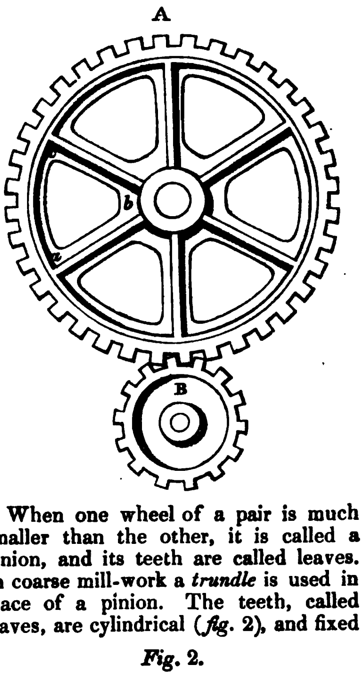
Machines, generally, may be separated into a number of working parts, or elements, which are common to all; and, in constructing any machine, it is obviously of consequence that the elementary parts of which it is composed should be formed and arranged in a manner calculated to make the whole work together without unnecessary loss of power or wear of the material. In this chapter it is intended to give a concise view of the contrivances adopted, from calculation and experience, for the purpose of transmitting forces in a regular and effective manner.

(1.) *Wheelwork* forms a most important class of mechanical elements, from its universal utility as a means of conveying forces continuously. Wheels are adapted in different ways; the principal being those which act by means of *teeth* and *simple friction.*

The term *gearing* is generally applied to toothed wheelwork; and, according to the direction in which the teeth are arranged, they are called *spur* or *bevel gear.*

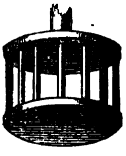
In the common form of spur wheels the teeth are placed radially on the circumference, the rim being attached to the central part, or boss, by arms, which, in large cast-iron wheels, are strengthened by a thin web, as *a b c* (*fig.* 1), passing along the sides of the arms and the inner surface of the rim. When the wheels are small (as pulleys), the boss and rim are sometimes conjoined by a membrane or plate of metal, as in the figure B, and thence called plate wheels.

Fig. 1.



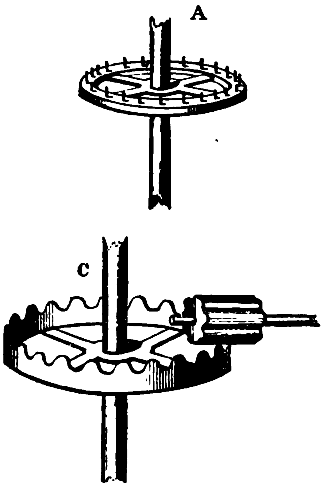
When one wheel of a pair is much smaller than the other, it is called a pinion, and its teeth are called leaves. In coarse mill-work a *trundle* is used in place of a pinion. The teeth, called staves, are cylindrical (*fig.* 2), and fixed in two discs, between which the teeth of the wheel act upon the staves. The form of teeth in this kind of wheel has been preferred on account of the smoothness of their action, as well as their strength and the small amount of friction incurred; the trundle has been called also lantern, and wallower. Pin-wheels,

Fig. 2.



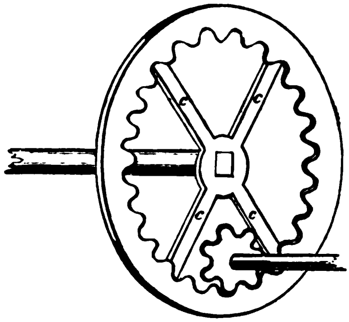
A (*fig* 3), are of a similar character to the trundle, but not possessing its strength; in small work, or on particular occasions, they are frequently found a convenient form of wheel.

Fig. 3.



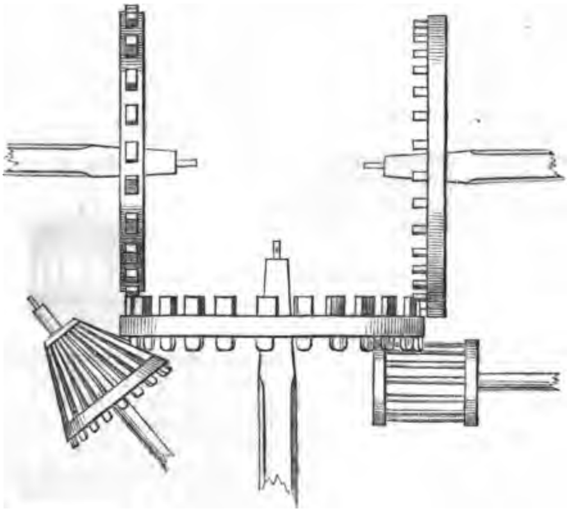
The teeth are sometimes cut out of the rim of the wheel, as at C, which is then called a *crown*, or contrate wheel. Wheels where the teeth are cut on the inner surface of the rim are called *annular* wheels (*fig*. 4), the radial arms, *c c*, being fixed behind the annulus, so as to allow the pinion to work. In this form of wheel there is less friction than in the ordinary spur wheel.

Fig. 4.



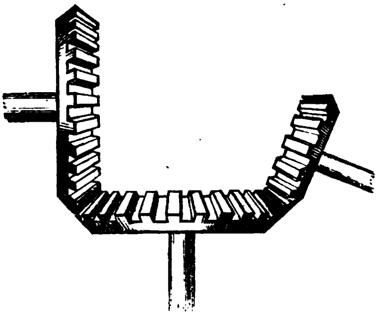
In the preceding cases the axes of the wheel have been considered parallel; it, however, frequently happens that the axes are inclined to each other. In ancient machinery the teeth of wheels, whose axes were not parallel, were disposed as in the following collective illustration (*fig*. 5). The crown wheel and pinion (*fig*. 3), which may be considered as a face-wheel and trundle, is used in clock and small work to enable an axis to give motion to another at right angles to it. The general practice, however, is to form the teeth of both wheels upon conical surfaces, which is termed *bevel*

Fig. 5.



gear. A part or frustum only of the cone is required; and it may be shown that two cones, whose apices meet, will work together evenly*. The inclination of the conic surfaces varies according to the direction of the given axes, as in *fig*. 6. In all cases of a pair of common bevel wheels, the axes cannot be carried past each other; if this be required, it is simply obviated by the use of an intermediate double bevel, by which the relative position of the two principal axes may be varied to a considerable extent.

Fig. 6.



The teeth of wheels communicate motion with a sliding or rubbing of their surfaces, and consequently create friction. This is not the case with wheels which work by the mere unevenness of their surfaces in contact. The latter touch, practically speaking, in one point only, which is in the line joining their centres, as in *fig*. 7; and, unless their surfaces slip, there is no remarkable rubbing of their surfaces; therefore we may infer, that in all cases where the points of contact of two wheels is in this line there will be no sliding nor friction. Wheels of this kind, when covered with rough leather, or roughened on their surfaces of contact, are very useful in small machinery, or where no great amount of force is to be transmitted; otherwise the constancy of their action cannot be depended on. Dr. Hooke contrived a form of toothed wheel in which the contact is constantly in the line of centres, and, consequently, work without sliding. This wheel is composed of a number of concentric wheels, with teeth formed in the ordinary manner; they are placed together upon the same axis, and arranged so that the teeth succeed each other by steps, the last tooth of one row being made to answer to the first tooth of the next row, as may be seen in *fig*. 8, where six wheels are represented as combined. If another compound toothed wheel, analogously arranged, be made to work with such a wheel, the contact of any pair of teeth will be momentally, and therefore almost only in the line of centres. If, instead of a number of steps, there be inclined planes, the wheel may be considered to have an infinite number of teeth, and the contact will be strictly in the line of centres only. Such a form is occasionally used; though an objection arises from the lateral pressure resulting, which increases as the inclination of the planes to the axis of motion.

Fig. 7.

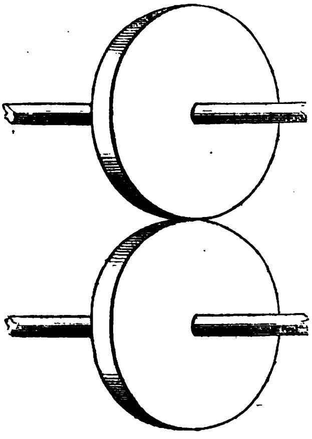
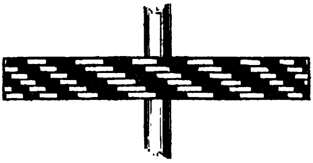


Fig. 8.

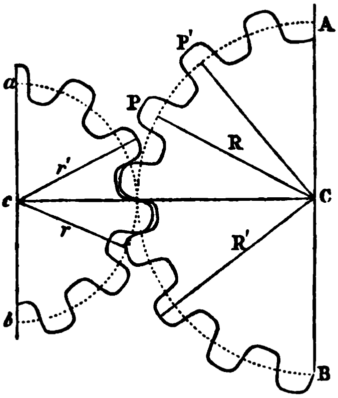


In forming a wheel, a line is first drawn, called the *pitch circle*, as A B, *a b*, (*fig*. 9,) of the working diameter required for the wheel, which is the basis of succeeding operations. This circle is the working circumference of the wheel; for if two cylinders, having

---

* See Treatise II. chap. 7.

Fig. 9.



their circumferences equal to the pitch circles of any two wheels, were to act upon each other by rolling contact (or simple friction), their action would be equivalent to the action of the two wheels working by teeth. Indeed teeth may be considered as a development merely of the rough surface of the wheel acting by friction; for, in the latter case, the minute projecting parts upon the surface fall into the small spaces in the face of the wheel which it drives, and thus compel it to revolve. The pitch circles of two wheels which are intended to work together are thus drawn so as to touch. They are divided into as many parts as there are teeth required in the wheels, each of these parts being equal to a tooth and a space, or to the distance, P P′, between the centres of two contiguous teeth. P P′ is called the *pitch* of the wheel. The line C c is called the line of centres; R, r, the pitch radii; and R′, r′, the real radii of the wheel. The wheel which moves another is called the *driver*, and the driven wheel the *follower**.

The pitch of a wheel may be of any convenient value, provided it be not so small that the teeth become too weak to bear the forces they are destined to transmit. In practice, however, a series of pitches are generally used for cast-iron wheels; those for the larger description being 1, 1¼, 1½, 2, 2½, 3, inches; and for the small wheels, ¾, ⅝, ½, ⅜, ¼, of an inch. A pitch smaller than ¼ of an inch is seldom required for cast-iron wheels.

The number of teeth in any wheel of given diameter and pitch, or the pitch, or diameter, when the number of teeth are stated, may be easily found; for as the circumference of a circle is equal to the diameter × 3.1416, and the pitch is equal to a tooth and a space, it is $= \frac{\text{diameter} \times 3.1416}{\text{number of teeth}}$. From this we may find the other quantities when required; thus,

$$\text{No. of teeth} = \frac{\text{diameter} \times 3.1416}{\text{pitch}}.$$

$$\text{Diam. of wheel} = \frac{\text{pitch} \times \text{No. of teeth}}{3.1416}.$$

For example, if we intended to make a wheel two feet in diameter, and with 100 teeth, the pitch would be

$$\frac{24 \times 3.1416}{100} = .754,$$

or ¾ of an inch pitch very nearly.

Also, if the pitch was required to be 1 inch, using the same diameter, the number of teeth would be

$$\frac{24 \times 3.1416}{1} = 75.$$

If we required teeth of such a size that the pitch would be ½ an inch, and the number of teeth 120, the diameter of the wheel must be

$$\frac{.5 \times 120}{3.1416} = 19 \text{ inches}.$$

A more ready method is adopted for finding the pitch where it is continually required, by dividing the diameter of the pitch circle of the wheel into as many parts as there are teeth required. This diametral pitch is some fraction of an inch, and the denominator of this fraction will thus be an integral number. A wheel, for instance, whose diameter is 10 inches, and number of teeth 60, will have a diametral pith of $\frac{10}{60}$ or $\frac{1}{6}$. The most useful values of this denominator, which we may call P, are 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 20; and a wheel in which the denominator of the diametral pitch is 6, is said to be a six-pitch wheel. The following table shows the circular pitch corresponding to the above measures of the diametral pitch:—

---

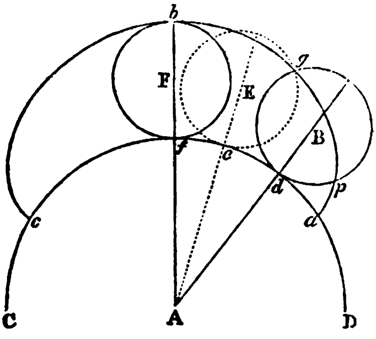
* In the illustrations of mechanical movements, the *driving* part and its direction of motion will be indicated by an arrow, and the *driven* parts by an arrow without a feather; thus, the driver ➳, and the follower ⟶.

Value of P.	Circular pitch.
3	1 inch.
4	$\frac{3}{4}$ "
5	$\frac{5}{8}$ "
6	$\frac{1}{2}$ "
7	$\frac{7}{16}$ "
8	$\frac{3}{8}$ "
9	
10	$\frac{5}{16}$ "
12	$\frac{1}{4}$ "
14	
16	$\frac{3}{16}$ "
20	$\frac{1}{8}$ "

The circular pitch is obtained by multiplying the diametral pitch by 3.1416. Thus a wheel of 20 inches diameter, with 80 teeth, would have a diametral pitch of $\frac{20}{80}$ or $\frac{1}{4}$; or it would be a four-pitch wheel, and the circular pitch $\frac{1}{4} \times 3.1416 = \frac{3}{4}$ very nearly, as in the table, in which the values are given to the nearest sixteenth of an inch.

The most simple idea of a toothed wheel is that of a wheel having a number of straight pieces or spokes projecting from its circumference or rim, in order to catch similar projections upon the circumference of another wheel; but a little thought and acquaintance with mechanical science will show that this ought to be a subject for consideration. As the value of the toothed wheel depends upon the shape of the teeth, many eminent mathematicians have been led to investigate the motion of two wheels revolving in contact, in order to determine the curve which should be given to the face of the teeth, so that they may convey the forces most effectually, and produce a constant velocity in the driven parts. The most simple curves which have been found to answer the conditions of the problem are the *epicycloid* and *involute*.

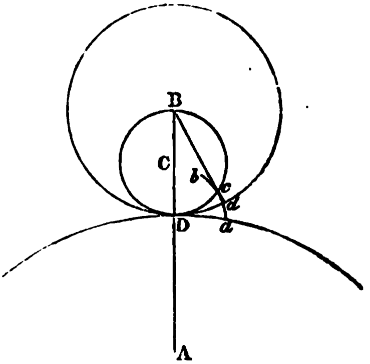
Fig. 10.



The epicycloid is a curve traced by a point on the circumference of a circle, rolling over the circumference of another circle. The curve *a p b c* (*fig.* 10) is an epicycloid, drawn by the pencil *p* on the circumference of the circle B (called the *generating circle*), as it rolls upon the circumference of the circle A. From the nature of this operation, it follows that the arcs *a d*, *d p*, are equal[*].

Again, suppose the circles A and B (*fig.* 11) to be capable of moving freely

Fig. 11.



on their centres A and B, their circumferences touching at D, and *a c p*, a small epicycloid arc described by the small circle C; a radius B *c d* drawn from the centre of the circle B, through the describing point *c* of the small circle, will touch or be a tangent to the curve *a b* at *c*; and if we suppose the point *d* to have been at D, when *d* and *a* would coincide, and, by the revolution of the circles, A and B to have moved into the present position, the circles will have described the arcs D *a*, D*d*, in the same time. The angle D B *c*, or D B *d*, thus described, is at the circumference of the circle C, but at the centre of the circle B, which is double the diameter of the circle C; therefore the arc D c is the measure of an angle double that measured by the arc D d*. Again, the radius D C is half the length of the radius D B; therefore the arcs D c and D d are equal. As the circle C is the generating circle of the epicycloidal arc $a d p$, the arcs D a and D c are equal, and the arc D a equal to the arc D d; or the ratio of the angular velocity of the circles A, B, remains constant.

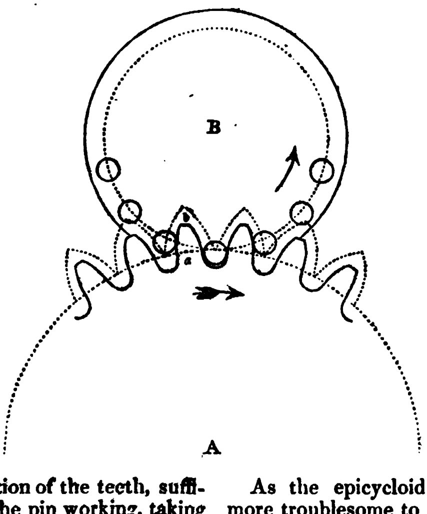
---

[*] The epicycloid may be described by the following method:—Draw several circles, as B, E, F, of equal radius, touching the circumference of the circle D C, and take the arcs *d p*, *e g*, *f b*, upon the generating circle, equal in length to the arcs *d a*, *e a*, *f a*, respectively; then through the points *p*, *g*, *b*, thus obtained, draw a curve *a p g b*. This curve will be part of an epicycloid. If an arc *a f* be taken, equal in length to one-half the circumference of the generating circle, one-half of the epicycloid will be determined, and the remaining half may be drawn similar to it.

We may now apply these forms to the shaping of teeth of wheels. In the construction of the epicycloid given in *fig.* 10, we may observe that the circle B describes the curve $a p$ with a pin p, by rolling on the circumference of the circle A. If both circles be supposed to be fixed, and capable of revolving upon their axes, a tooth or projection from the circumference of the circle A, and of the length $a p$, would, if the wheel A revolved, push a very small pin p, fixed on the circle B, from the line of centres A B, to the point p, the small pin, from the principle of the curve, sliding evenly upon the side of the tooth, and the rotatory velocity of the circle B constant†.

This construction applies to wheels or racks working with trundles or pin wheels. In practice, the staves or pins must of course be of some considerable diameter to give them sufficient strength, and a slight modification of the teeth formed on the above principle is requisite; also, as the wheel may be required to move the follower in a direction opposite to that already supposed, the teeth must be curved on both sides. A wheel, with teeth thus fitted for practice, is shown in *fig.* 13, where the dotted lines show the teeth, supposing the pins in the follower to be of insensible diameter; by the reduced teeth it may be seen that it is necessary merely to cut away a portion of the teeth, sufficient to allow of the pin working, taking care to preserve the epicycloidal form.

Fig. 13.



The teeth of the wheel are described upon the pitch circle of the wheel, and the centre of the staves are placed upon the pitch circle of the follower, so that a space below the pitch circle of the wheel must be cut out to admit the pins between the teeth at the line of centres.

As the epicycloid is a curve much more troublesome to draw than a circle, arcs of circles are used, which do not materially differ from it. Teeth are simply described by this method as follows:—From the centre of the pin, which is supposed to be on the line of centres and consequently at the point where both pitch circles meet, describe an arc $a b$, which will be the working part of the tooth, and sufficiently proximate to the required form. The part below the pitch line, being unimportant, may be figured as an arc of a circle.

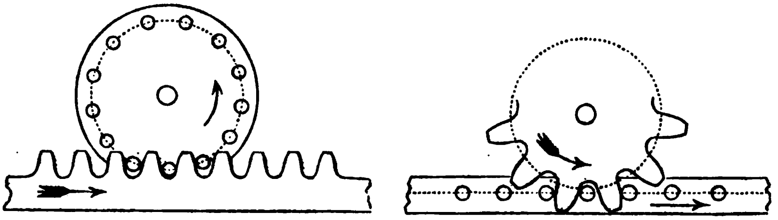
---

* From Euc. 20, 3; or Geom. prop. 14, § 2, iii.

† Some writers on this subject have improperly remarked that the fact of two wheels, describing equal arcs in equal times, proves that the required conditions are fulfilled; but equal arcs may be described in equal times, and yet the relative velocity during those times may

If this curve be applied to the rack and pinion, when the former drives, the pinion will have pins or staves, and the rack cycloidal teeth, for the surface upon which the curve is described is no longer a circular but a straight line; and a circle rolling upon a level line or base will trace, with a point in its circumference, a cycloid. When the pinion drives, the rack must have pins, but the pinion will not here have epicycloidal teeth; for, as the teeth must be described by a point in the pitch line of the rack rolling upon the pitch circle of the pinion, the curve thus drawn will be an *involute* of the pitch circle. (See p. 55.) These forms are shown in *fig.* 14.

Fig. 14.



In an annular wheel and pinion, the pinion, if it be the follower, will have pins, and the teeth of the annulus will of course be shaped according to the curve described by the pitch circle of the pinion when rolling within the pitch circle of the wheel. This curve will be a hypocycloid. When the pinion drives, the pins on the annular wheel must be fixed on its face, and the teeth of the pinion formed by the rolling of the wheel's pitch circle upon the pitch circle of the pinion.

When both driver and follower are of the same diameter, the smallest numbers of teeth and pins which can be employed are eight in each wheel, the pins or staves being not less than half the pitch. Under the condition last mentioned, the following table shows the least numbers of pins in a wheel driven by a pinion, and a pinion driven by a wheel, with the corresponding number of teeth:—

Teeth in pinion.	Pins in wheel.	Teeth in wheel.	Pins in pinion.
7	14	14	7
6	12	12	6
5	15	25	5
4	24		

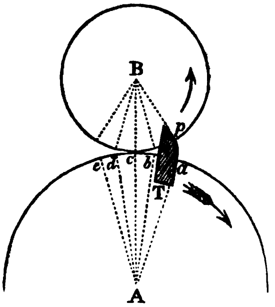
A rack may be driven by a pinion of four teeth; if the rack drives, the pinion must have at least five pins.

The epicycloid generated in the manner shown in *fig.* 11, is very generally applied to the formation of the teeth of wheels. A pair of wheels, with teeth drawn after this manner, is shown in *fig.* 15, the pitch circles of the wheels being supposed to touch at D in the line of centres; that part of the tooth outside the pitch circle, as D*a*, is called the *face*, and that within, as D*e*, the *flank*; the flank of a tooth is a radial line, and the face D*a* an arc of an epicycloid formed by the generating circle D*h*B; which is of a diameter equal to one-

---

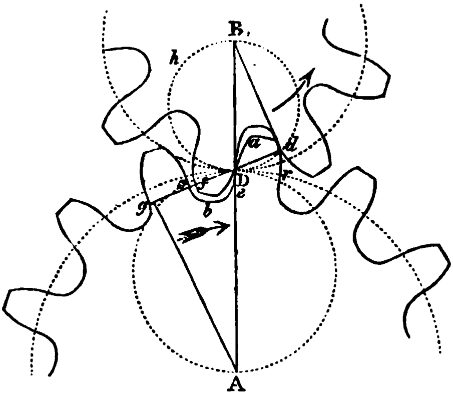
be changeable. This may be simply illustrated by fixing on the circumference of a wheel a tooth T (*fig.* 12), the face *ap* being

Fig. 12.



cut of an epicycloidal form, according to the principles given above; and arranging another wheel with a projecting pin *p*. If the wheel A revolve, the tooth T will push against the pin *p*, and turn the wheel B with a constant velocity, as may be practically demonstrated by drawing a number of radii on each wheel, as A*a*, A*b*, A*c*; and each successive pair will coincide with the line of centres B A at the same time, in the course of revolution.

Fig. 15.



half the diameter of the wheel B; the teeth of the wheel B are similarly formed, their generating circle being D*g* A, equal in diameter to half the diameter of the wheel A. In the motion of such a pair of wheels the contact commences between the flank of the tooth at *c* of the lower wheel, and the extremity of the tooth *f* of the follower, and ends when the teeth arrive at the position *a*, *d*, when the end of the tooth *a* presses against the flank of the tooth *d*. It may be seen that the line of contact from *c* to D is the generating circle D*g* A, and from D to *d* the generating circle D*h* B.

This method of describing epicycloidal teeth has one disadvantage, that a wheel will not work fairly with another of lesser or greater diameter than the wheel with which it was made to work; this plainly appears from the construction. But it frequently occurs that a wheel is intended to drive several wheels of various diameters, and an inequality of motion must arise from such a combination. To obviate this inconvenience it has been proposed to employ a *constant* generating circle, so that wheels of different diameters, but whose teeth are traced by a constant describing circle, will correctly work together.

When toothed wheels drive, or are driven by, pinions with teeth, there is a limit to the relative number of teeth which can be employed, varying according to the length of the arc of action or contact D*r* from the line of centres. When the action of the teeth commences at the line of centres, and the preceding tooth is breaking contact, the arc of action is equal to the pitch; if it be ¾ or ⅔ of the pitch, there is an arc of action equal to ¼ or ⅓ of the pitch before the line of centres. Thus we have the following table*:—

	No. of teeth in pinion.	Least No. of teeth in wheel.	
		Wheel drives.	Pinion drives.
Arc action = pitch.	5	Impossible	Impossible
	6	...	176
	7	...	52
	8	...	35
	9	...	27
	10	a rack	23
	11	54	21
	12	30	19
	13	24	18
	14	20	17
	15	17	16
	16	15	...
Arc of action = ¾ pitch.	3	Impossible	Impossible
	4	...	35
	5	...	19
	6	...	14
	7	31	12
	8	16	10
	9	12	10
	10	10	10
Arc of action = ⅔ pitch.	2	Impossible	Impossible
	3	...	36
	4	...	15
	5	...	13
	6	20	10
	7	11	9
	8	8	8

---

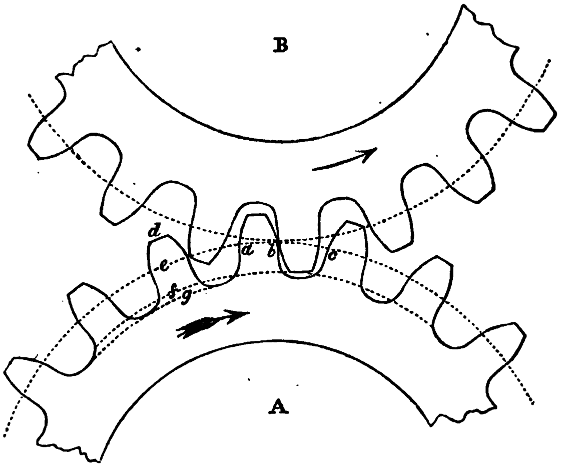
* Willis's Principles of Mechanism.

When a wheel drives a pinion, the latter should have at least eight teeth or leaves, and when a pinion drives a wheel the former should have not less than six teeth, eight or nine working better. The ratio of the number of teeth on a wheel and pinion should never exceed the ratio of one to six; if a wheel makes any number n of revolutions per minute, and the pinion be required to make a certain number of revolutions N per minute, the proportion $n : N :: 1 : \frac{N}{n}$ gives the ratio of their velocities, which is $1 : \frac{N}{n}$ if this be not greater than $1 : \frac{6}{1}$ or 1 to 6, a single wheel and pinion will be sufficient.

In practice, teeth are not so long as in our figures; but the length of the tooth is an important circumstance. The part of the tooth which extends beyond the pitch circle is sometimes called the *addendum*, and is that to which the peculiar curve is given; in general, an addendum of $\frac{3}{10}$ of the pitch is employed. The following figure exhibits part of the form of a pair of wheels of common construction, *a c* is the pitch, or a tooth and a space, *d e* the addendum; the usual proportions of the different parts of the arrangement are as follows:—

d e, or the depth to the pitch line = $\frac{3}{10}$ pitch.
d f, or working depth . . . . = $\frac{6}{10}$ „
d g, whole depth . . . . . . = $\frac{7}{10}$ „
a b, thickness of a tooth . . . = $\frac{5}{11}$ „
b c, breadth of space . . . . . = $\frac{6}{11}$ „

Fig. 16.

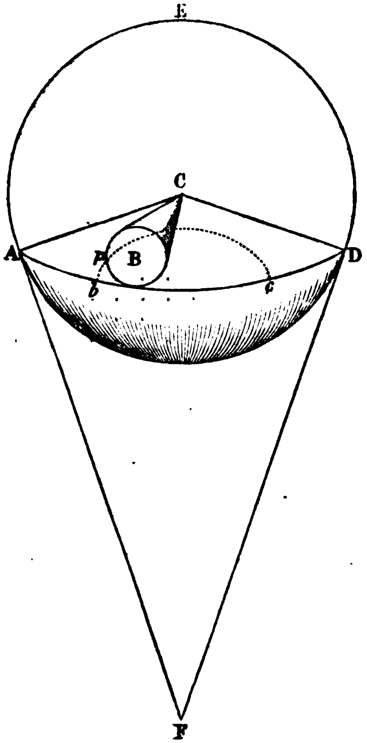


By this it appears that the breadth of the tooth is $\frac{1}{11}$ of the pitch less than the space, to allow the teeth to move freely in the space; also, $\frac{1}{10}$ of the pitch is allowed, to prevent collision between the tops of the teeth and the bottoms of the spaces. For wheels having less than fifteen teeth, an addendum of $\frac{3}{10}$ of the pitch is not sufficient. In some of the limiting cases given in the table (p. 52), the addendum required would be from about $\frac{6}{10}$ to $\frac{4}{10}$ of the pitch. In clock and watch machinery, the addendum allowed to the driver is .36 or $\frac{3}{8}$ of the pitch, and to the follower .24 or nearly $\frac{1}{4}$ of the pitch.

We shall now notice the application of the curve to the formation of the teeth of bevel wheels, which, as we see from p. 47, are frusta of cones which are supposed to roll upon one another, their apices meeting in one point.

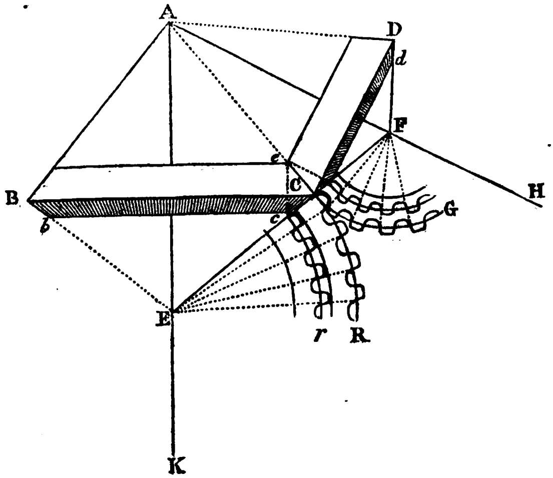
It appears, in *fig.* 17, that if the cone B roll upon the cone A C D, the apices of both meeting at C, the curve *c p b* will be traced by a point *p* in the circumference of the cone B, coinciding with the surface of the sphere A D E; this curve is called a *spherical epicycloid*, and is the form required for the teeth. The problem is, however, much simplified by considering that the portion of the curve which is necessary to form a tooth being very small, the tangent A F to the sphere may be taken for the curved surface of the sphere, not sensibly differing; and the teeth may be described upon the surface of the tangent cone A F D instead of the surface of the sphere. The same may be done with respect to the other cone B. By developing the surfaces of these tangent cones, that is, drawing circles equal in radius to the sides of the cones, teeth may be drawn upon the circles as for spur wheels. This simple method was applied by Tredgold*, and is shown in the following figure. B C, C D are frusta of the cones B A C, C A D, upon which the teeth are purposed to be made; perpendicular to A C, the line of contact, draw the line E F, meeting the axes of the cones at E and F; B E C and C F D will be the tangent cones; then with the radii E C and F C describe the circular arcs C R and C G; these arcs will be the development of the surfaces of the two tangent cones, upon which the teeth may be figured, as though intended for a spur wheel; the arcs C R, C G of course representing parts of the pitch circles. Patterns may be cut out of thin copper or other pliable substances, agreeably to the forms determined for spur wheels, and attached to the conical surfaces B *p*, D *d*, for a guide in cutting the teeth. The form of the interior extremity of the teeth may be found by a procedure analogous to that already shown. From *e* draw *e c* parallel to A E, and with the radius E *c* the pitch circle *c r* may be described.

Fig. 17.



The application of the *involute of a circle* to the formation of teeth has been mentioned in a preceding treatise†. The involute of a circle is a curve described by the end of a string unwinding

Fig. 18.



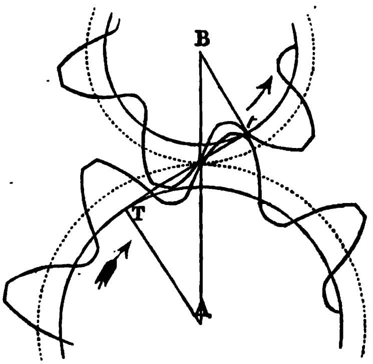
---

* Buchanan's Treatise on Millwork, page 58 (1841).

† Mechanics, Tr. II., page 29.

from the circumference of a circle. By this construction the whole tooth is formed of one curve, as in *fig*. 19. Dr. Young appears to prefer this form to the epicycloid*. A pinion with involute teeth will work with a rack having teeth straight-sided and inclined to their pitch line. Dr. Young says that the proper form for the teeth of a pinion driving a rack is the involute; they work steadily, as the leaves so curved exert a small downward pressure upon the rack. To the use of teeth formed upon this principle there is, however, an important objection, which "is founded upon the obliquity of their action, by which a much more considerable divergent pressure is thrown upon the axes than in the other forms of teeth. The action of epicycloidal teeth is, in fact, perpendicular to the line of centres at the instant of crossing it, but that of involute teeth is constantly in the direction of the common tangent of their bases (as T*t*, *fig*. 19), and is therefore oblique to the line of centres. This injurious property is balanced by the advantages that a variation of the distances of their centres does not destroy the action of the teeth, and that any two wheels of the pitch will work together; but this last property . . . . is possessed by some arrangement of the epicycloidal teeth (p. 52). In smaller machinery, constructed rather for the modification of motion than for the transmission of force, this oblique action ceases to be objectionable, and the other advantages of involute teeth will recommend them in preference to all others."†

Fig. 19.



The curves which have been described in the preceding pages, as most proper to be given to the teeth of wheels, are only approximated in practice. The comparative facility with which an arc of a circle may be drawn is so great, and the difference between such an arc and the true curve in the length of a tooth so practically imperceptible, that circular arcs are used for the purpose. In doing so, however, it is of great importance to determine correctly the radius of curvature and the position of the centre for the circular arc. Euler, who wrote an intricate investigation of the subject, was the first who proposed arcs of circles as sufficiently accurate for the forms of teeth. By following these suggestions Mr. Willis has succeeded in constructing an instrument for drawing the teeth of wheels by arcs of circles, which he calls somewhat improperly the *odontograph* (or tooth-drawer), of which the following diagram represents the principal part; it is half the size of the original, and may be made of card or thin metal. The side C T makes an angle with A B, so that B T C = 75°, which was found from calculation to be the best adapted to give a proper position to the centres for the arcs, and each of the divisions on the edge of the instrument is $\frac{1}{20}$th of an inch.

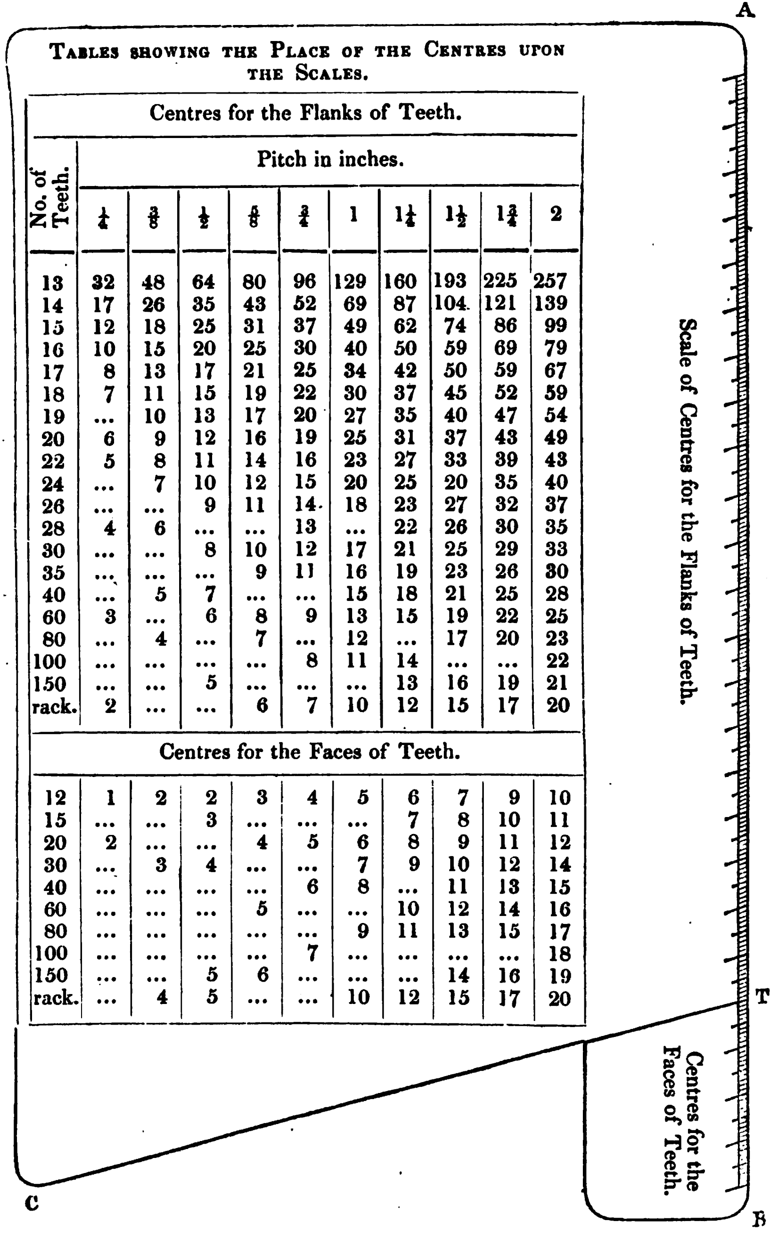
The use of the odontograph may be seen from the following example :—

---

* Lectures on Nat. Phil., page 135 (1845).

† Willis's Principles of Mechanism, p. 116.

Fig. 20.


A
T
B
C
Scale of Centres for the Flanks of Teeth.
Centres for the Faces of Teeth.

TABLES SHOWING THE PLACE OF THE CENTRES UPON THE SCALES.

Centres for the Flanks of Teeth.

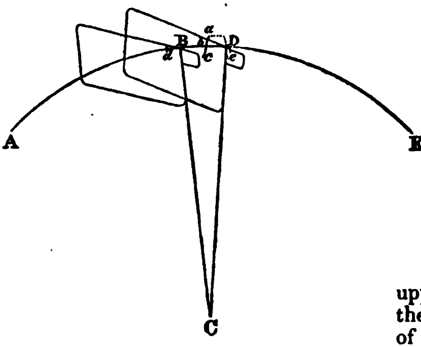
No. of Teeth.	Pitch in inches.									
	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2
13	32	48	64	80	96	129	160	193	225	257
14	17	26	35	43	52	69	87	104	121	139
15	12	18	25	31	37	49	62	74	86	99
16	10	15	20	25	30	40	50	59	69	79
17	8	13	17	21	25	34	42	50	59	67
18	7	11	15	19	22	30	37	45	52	59
19	...	10	13	17	20	27	35	40	47	54
20	6	9	12	16	19	25	31	37	43	49
22	5	8	11	14	16	23	27	33	39	43
24	...	7	10	12	15	20	25	20	35	40
26	...	...	9	11	14	18	23	27	32	37
28	4	6	...	...	13	...	22	26	30	35
30	...	...	8	10	12	17	21	25	29	33
35	...	...	...	9	11	16	19	23	26	30
40	...	5	7	...	...	15	18	21	25	28
60	3	...	6	8	9	13	15	19	22	25
80	...	4	...	7	...	12	...	17	20	23
100	...	...	...	...	8	11	14	...	...	22
150	...	...	5	...	...	...	13	16	19	21
rack.	2	...	...	6	7	10	12	15	17	20

Centres for the Faces of Teeth.

No. of Teeth.	¼	⅜	½	⅝	¾	1	1¼	1½	1¾	2
12	1	2	2	3	4	5	6	7	9	10
15	...	...	3	...	...	...	7	8	10	11
20	2	...	...	4	5	6	8	9	11	12
30	...	3	4	...	...	7	9	10	12	14
40	...	...	...	...	6	8	...	11	13	15
60	...	...	...	5	...	...	10	12	14	16
80	...	...	...	...	...	9	11	13	15	17
100	...	...	...	...	7	...	...	...	...	18
150	...	...	5	6	...	...	...	14	16	19
rack.	...	4	5	...	...	10	12	15	17	20

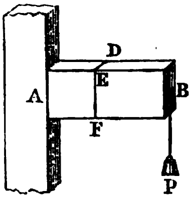
Suppose it were desired to draw the teeth for a wheel of 40 teeth and 1 inch pitch. Describe an arc of the pitch circle A B (*fig.* 21), and take B D equal to the pitch, bisecting it at *b*; then to draw the flank *b c*, apply the edge C T of the instrument to the radius C B, the point T being on the pitch circle at B; and by the table giving the centres for the flanks opposite 40, and under the column for 1 inch pitch, we find 15; therefore the point *a*, which is at 15 on the scale for the flanks, will be the centre from which the arc *b c* must be described with the radius *d b*. For the face *a b* we find in the table of centres for the faces against 40, and in the column for 1 inch pitch, the figure 8; then applying the edge T C of the instrument to the radius C D, take the point *e*, upon the scale for the faces, at the eighth division, and with the radius *e b* describe *a b*. The other side of the tooth may be similarly drawn by reversing the instrument.

Fig. 21.



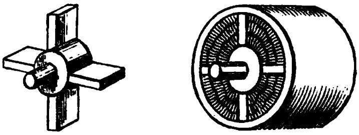
In setting out the teeth of wheels, attention must be paid to the work for which the wheel is intended. It is generally a point to have the pitch as fine as possible; but as the thickness of the teeth is determined by the pitch, it is evident there will be a limit to its fineness. This thickness must be such as to prevent the possibility of any tooth breaking in the performance of its work. A tooth may be considered as a beam fixed at one end, and its strength calculated accordingly. Thus let A B (*fig.* 22) represent a tooth which is strained by the weight attached at the end B; this weight will have a tendency to break the tooth at some part, suppose E F; the resisting force will be the cohesion of the particles; and, by the principle of the lever, the power P produces a greater effect in overcoming the cohesion the greater the length of the arm F B, the point F being the fulcrum. The greatest strain will be along the upper part E D. By the principles of the strength of materials* the strength of the tooth is as the square of its depth, or as $E F^2$, and from what we have said, inversely as its length. It is also as the breadth E D, which some consider should be to the length as 2 to 1. When in motion, the stress upon the teeth is inversely as the velocity; so that a tooth moving at the rate of 6 feet per second has a stress one-half as great as though it moved 3 feet in the same time. It is advisable, however, not to exert upon a tooth much more than one-tenth of the force which it is able to sustain.

Fig. 22.



(2.) *Shafts* and *spindles*, the technical names for the axes or axles of wheel-work, are made generally of iron. Where wooden shafts are used, a *gudgeon*, or arbor, or spindle, has to be attached to the ends, upon which the shaft turns. There are two methods of fixing these gudgeons to the end of a shaft. One is called the *cross-tailed* (*fig.* 23) gudgeon, made of cast-iron, and let into the end; a hoop is then driven on the end of the shaft to make the whole firm. Sometimes the hoop is cast along with the gudgeon, which is a preferable arrangement. Another plan of fixing the

Fig. 23.

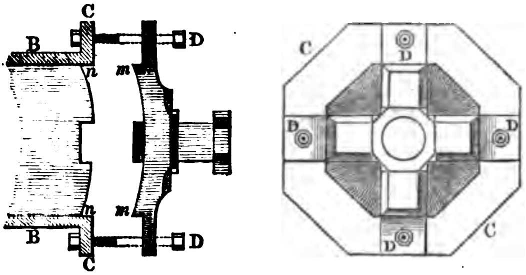


---

* See the subsequent chapter on this subject.

gudgeon is to have a cast-iron box, seen in section B B (*fig.* 24), firmly fitted to the end of the shaft, having a flange C C, and four notches *n*, *n*, to admit the cross-arms of the gudgeon when placed in their proper position. These arms are fixed by four bolt screws D, D, D, D, to strengthen the hold of the cross-arms. There are four projections, two of which are shown at *m*, *m*, fitting into the spaces *n*, *n*, and which, with the screws, hold the gudgeon firmly to the shaft.

Fig. 24.



Cast-iron shafts are either hollow or solid cylinders. The former are lighter than solid cylinders, and stronger than a solid cylinder of the same weight and length; but the trouble and expense of making them is so great that, unless of very large diameter, the shafts are cast solid. Solid shafts are made of a feathered or a square form, the latter being universally used on account of its simplicity, and is found as useful as any other description of solid shaft. Gudgeons and shafts are made as small as is consistent with the strength required to avoid unnecessary weight of material, and consequent strain and friction. Wrought iron possesses much more cohesive power, and is consequently more serviceable for gudgeons or axles than cast iron; but the facility with which the latter kind can be moulded into the required form, and its cheapness, recommend it generally in practice. The strength of a solid cylinder to support a weight (as a wheel) being as the cube of its diameter, and a cylinder of 1 inch in diameter is considered capable of sustaining a hundredweight, we have the following table (according to Buchanan) of the weights sustained by solid cylinders of cast iron, of different diameters, used as shafts or spindles; in the third column are given the requisite diameters of gudgeons of wrought-iron, corresponding to the same weights.

TABLE OF GUDGEONS.

Diameter in inches.	Weight sustained.	Diam. of wrought-iron gudgeon of same strength.
	lbs.	
½	14	
¾	47	
1	112	.64
1¼	218¾	1.06
1½	377½	1.26
1¾	600¼	1.51
2	896	1.71
2¼	1276¾	1.91
2½	1747¼	2.15
2¾	2329½	2.35
3	3024	2.57

(3.) *Simple and Differential Movements.*—In combining the elementary parts of machines, it is frequently required to effect an alteration in the direction or velocity of motion, as from a circular to a rectilinear motion, and from a uniform to a variable velocity. This is effected in various ways by simple combinations, which are highly interesting features in modern machinery. Few were known and required in the earlier times of mechanical invention; but the variety of machines constructed of late years caused a great number of these elementary combinations to be devised.

In a former treatise several methods of changing the direction of motion were given*. We shall here describe some of the later and most useful contrivances for this purpose. Those combinations which are intended to communicate motion simply, or to produce a change, constant or variable, in the *direction* of movement, may be classed as follow:—

---

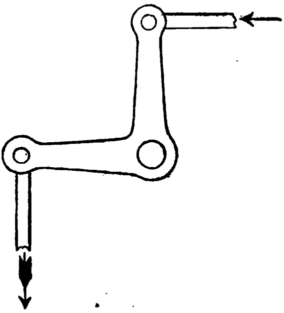
* Mechanics, Tr. II., chap. 13.

1. Rectilinear movement producing rectilinear or curvilinear movement.

2. Curvilinear movement producing rectilinear or curvilinear movement.

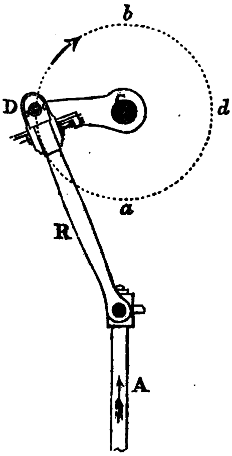
1. *Rectilinear producing rectilinear or curvilinear movement.* — Amongst this class of movements the lever and its combinations are simple and universal means of changing the direction of motion. Its action is, however, not continuous, as in the case of a wheel. The bell-crank lever (*fig.* 25) is a very useful contrivance, by which an alternate rectilinear motion is produced at right angles to another rectilinear movement. This was used by Watt in his earlier steam-engines, to regulate the supply of steam to the cylinder of the steam-engine.

Fig. 25.



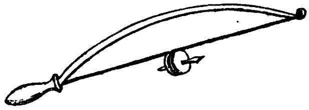
A rectilinear may be converted into a circular movement by means of the crank and connecting rod (*fig.* 26), which is a very useful, and generally adopted, invention. The upward motion of the rod A causes the end D of the crank-rod R to describe a semicircle from *a* to *b*, and the subsequent downward motion makes the same end D complete a revolution. It may be deduced, from the nature of this action, that there are times when the crank-rod R is exerting no available power upon the crank, which occurs when the crank is in the positions C *a* and C *b*, or in a line with the crank-rod; and that there is most circumvolving power exerted, as regards the crank, when the crank is in a line with D *d*. Any effort made by the crank-rod in the direction *b* A will therefore injuriously affect the stability of the axis C, and tend to tear it from its bearing or pillow. Again, the crank-rod R is incapable of conveying all the force which is applied to it when in any other position than the line *b* A; and its worst position in this respect is that shown in the diagram, or its opposite point *d**. Much discussion has arisen on this subject from the universal use of the crank and rod in the steam-engine as a means of converting the reciprocal movement of the piston-rod into a circular motion. Many contrivances have been made with a view to reduce or to annihilate this irregularity of action; but few have been adopted, in consequence of either possessing no capabilities superior to the crank and rod, or on account of their complexity. This latter is a most powerful objection in practical mechanics, though on many occasions too much disregarded by inventors; the crank and rod have therefore continued to hold their place as a most useful converser of motion.

Fig. 26.



A rack and pinion will produce a circular from a rectilinear movement, and *vice versâ.* A common bow drill affords an example of the same description of conversion. It is a very simple and serviceable instrument in small manufactures. A cord, stretched tightly by a steel bow, is wound once round the

Fig. 27.



---

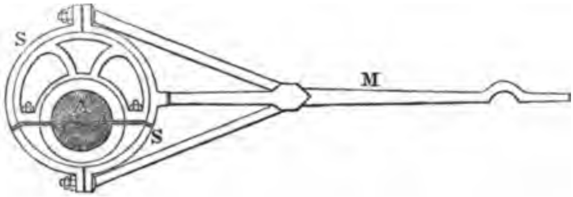
* See Dynamics, chap i. art. 27,

drill stock (*fig.* 27), and when motion is given by the hand to the bow, the drill revolves very rapidly.

2. *Curvilinear producing rectilinear or curvilinear movements.*

There are a great number of contrivances for these purposes, amongst which the *eccentric* wheel holds a prominent place, as being at once simple, elegant, and capable of much variation, so as to produce the effect required. The common eccentric, used in the steam-engine for regulating the supply of steam to the cylinder, is composed of a wheel (*fig.* 28) turning upon a shaft or axis A, which does not pass through its centre; a strap S S fitted to work easily on the circumference of the wheel; and the arm, M, fixed to the strap, which works with an alternate rectilinear motion.

Fig. 28.



Another form of the eccentric is shown in *fig.* 29, where it is supposed to give rectilinear motion to the frame A B, while revolving about the centre C. In these cases the motion of the rod on the frame is not uniform. It commences with a small velocity, which rapidly increases and attains a maximum; it then similarly decreases until the remainder of the semi-revolution is completed, when the rod is for a moment stationary. Where a uniform velocity is required, the form of eccentric shown in *fig.* 30 may be used. For its construction see velocity movements (page 65). This description of conversion may be readily obtained by the use of pins working in slots. *Figs.* 31, 32, represent two forms: in the first the wheel W has a pin fixed in it, moving in the slot S S, cut in the vertical lever, which gives rectilinear motion to the rack beneath it by means of a toothed

Fig. 29.

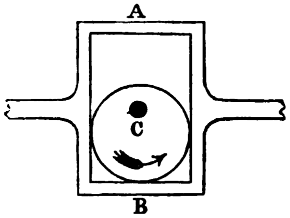


Fig. 30.

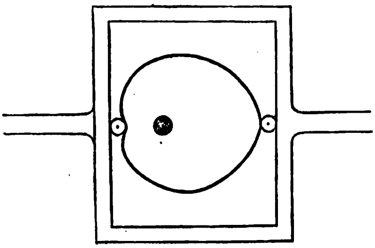


Fig. 31.

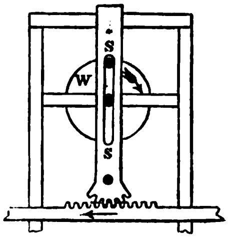
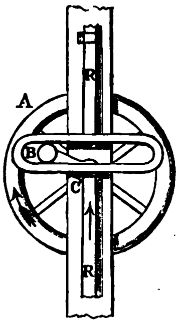


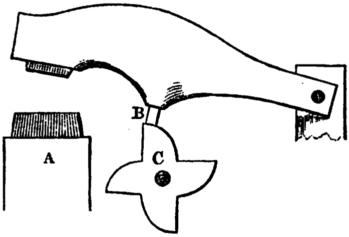
Fig. 32.



arc of a circle, or imperfect pinion. The second form is sometimes used in small steam machinery; if R R be supposed to be the piston rod, by its alternate motion it will push the crank B C up and down; and as the latter can move only in a circular path, which is allowed for by the slot, it will cause the fly-wheel A to revolve. Otherwise, if the wheel A be turned by handle, or otherwise, it will give an alternate rectilinear motion to the rod R R.

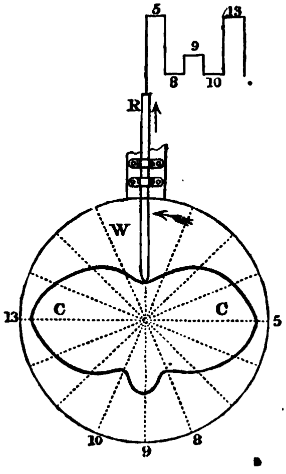
In some machinery, as the tilt and forge hammer, and stampers, a weight is required to be successively raised and let fall upon the object to be hammered. For such purposes the *cam* is a very useful movement. The common forge hammer is worked by a cam C (*fig.* 33) composed of four teeth, or wipers, which in revolving push against the foot or helve B of the hammer, alternately lifting the ponderous mass and letting it fall suddenly on the substance placed upon the anvil A.

Fig. 33.



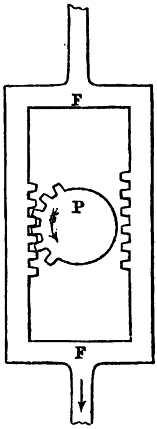
Where a varying rectilinear motion is required, the principle of the cam arrangement, shown in *fig.* 34, is very useful. The wheel W, turned by some motive power, carries on its face the system of cams C C, upon the edge of which rests the rod R, receiving a rectilinear motion. The up and down paths, described by the end of the rod during the revolution of a wheel with cams similar to those in the figure, is exhibited in the accompanying vertical lines. A velocity, either constant or variable, may be given to the rod by properly varying the radii of the different parts of the curve. (See p. 65.)

Fig. 34.



A continued alternate motion is frequently required in machinery, and many ingenious contrivances have been applied to planing, printing, and other machines for this purpose. A very simple traverse is obtained by using a frame F F (*fig.* 35), lined on two opposite inner sides with teeth as far as is requisite, with which the leaves of a pinion P, which is toothed round nearly half of the circumference, work alternately. If too much of the circumference of the wheel is toothed, the extreme teeth will push at the same time against the teeth of both racks, and would break some of them; the extreme tooth on one side of the pinion should break contact with the working rack at the moment or a little before contact begins between the other extreme tooth on the other side and the opposite rack; that is, supposing no cessation of motion is required, which would be for a length of time, according to the arc described by

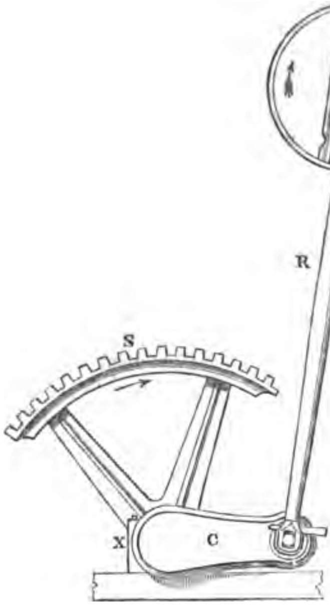
Fig. 35.



the pinion between the moments of breaking and making contact.

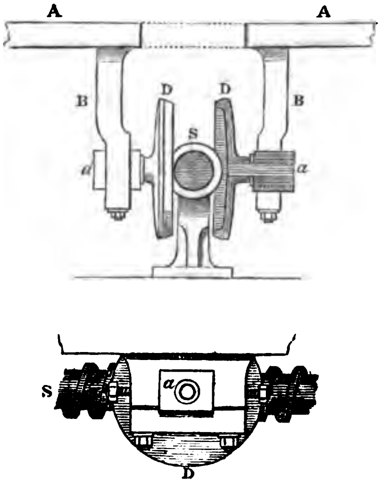
A useful reversing motion is represented in *fig.* 36. The wheel A is the driver, acting upon one end of the crank-rod R, which can be adjusted by means of a sliding bush in the groove *a*, at any distance from the centre of the wheel; this rod works the crank C, on the axis X, of which the sector S is fixed. The wheel revolving with the end *a* of the crank-rod, will produce an upward and downward motion of the crank, varying in extent according to the distance from the centre of the wheel at which the end *a* of the rod is fixed. The sector S will thus describe the arc of a circle backwards and forwards, and may move a horizontal table either by teeth or chains. This arrangement is used in one of Messrs. Carmichael's planing machines.

Fig. 36.



The contrivance shown in the following figure is one patented by Mr. Whitworth some time since, and used by that gentleman in some of his planing machines. It consists of two discs, D D, capable of revolving round their axes *a a*, which rest in the bearings B B, firmly fixed to the under side of the work table A A. The circumference of each disc has a flange extending facially, which works between the threads of a screw S. This screw is made to revolve in alternate directions by a simple reversing motion, and, while so doing, its thread pushes against the flanges of the discs, which revolve and carry the table along on its bearings; all friction is here

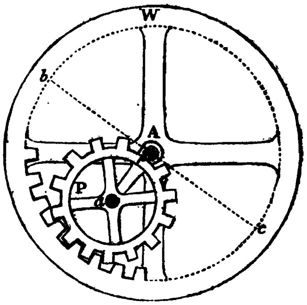
Fig. 37.



removed to the axes of the discs, and to avoid the friction arising from the velocity of the screw-thread being greater at different distances from the centre of the screw, the thread of the latter and the flanges of the discs are bevelled.

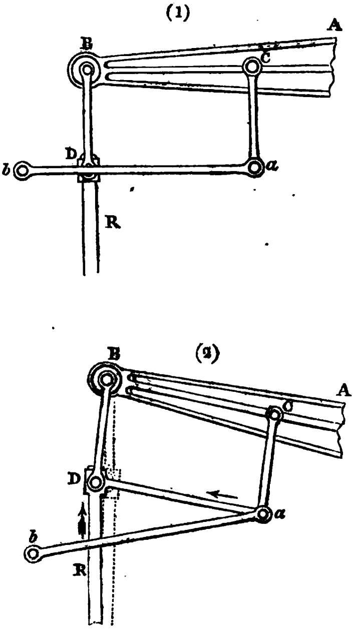
An elegant combination was proposed by Mr. White, for producing a rectilinear from a circular motion, and since used by Prof. Wheatstone in his photometer. It consists of a pinion, P (*fig.* 38), working with an annular wheel, W, which is twice the diameter of the pinion. The wheel being fixed and the axis A turned, the pinion P will have a motion of translation, and, as it works loosely on its axis, *a*, the contact of its teeth with those of the wheel will cause it to revolve on its axis, any point, as *p* on its circumference, describing the right line *b c*. The necessity of this result is demonstrated in the second Treatise on Mechanics, p. 58.

Fig. 38.



The combination of rods called the parallel motion, used by Watt in his steam-engines, is shown in the two diagrams (*fig.* 39), the arm A B representing a part of the beam of a steam-engine, and R the piston-rod. In the horizontal position (1) the point B will be in the same straight line with the rod R; but, when the beam has been pushed upwards, it would draw aside the upper end of the rod into the oblique position seen in the dotted lines (2), but for the arrangement B D, C*a*, *a* D, *a b*, the three first rods with part of the beam forming a parallelogram, and each rod freely rounding the joints at its extremities. The rod *a b* is attached to the corner *a* of the parallelogram and a fixed beam at *b*, so that, when the beam tends to pull the piston-rod inwards, the rod *ab* simultaneously pulls the lower side of the parallelogram outwards, and thus insures a nearly rectilinear motion to the piston-rod; the real path described by the point D being a curve somewhat similar to the figure 8.

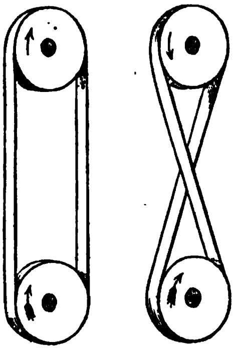
Fig. 39.



The screw and lever is used with great effect in machinery, such as coining, stamping, button-making machines, and printing presses, for converting a circular motion into a rectilinear; by this method great power and regularity may be obtained.

For the conversion of circular into rectilinear motion, the direction being similar or different, toothed wheels and friction-wheels are generally used. In the latter case, when a pair is some distance apart, the connection is effected by means of a band of leather or gutta percha. The band thus works by simple friction, and is a safer method of communicating motion than toothed wheels, for, should any obstruction to the motion occur, the band slips and prevents a fracture; but with toothed wheels the motion must continue, if the motive power be sufficient, until the resistance be overcome or the machine broken; the band is also particularly serviceable where the power is required to be suddenly connected with and disconnected from the machinery. When the motion communicated is to be in a direction parallel with that of the driver, the strap is simply laid over the friction-wheels or drums, as in *fig.* 40 (1); when in the opposite direction, the band is crossed (2); the latter method of disposing the band is the most economical, as may be demonstrated*, for the tension of the band is not required to be so great as in the first arrangement, there being much more surface exposed to the action of friction, and the power necessarily expended in overcoming the friction of the axles will consequently be less. The band wheels should have their circumference slightly rounded, as in the figure, in order to retain the band on the wheel, as it is a property of bands running over wheels or pulleys to incline towards that part which is of the greatest diameter. An endless band can be made to transfer motion in every possible direction with respect to the driving pulley or drum; but in practice very long bands, when working a number of pulleys, are found to be troublesome. In printing machinery small endless bands or tapes are used to carry the sheets of paper to be printed over several drums or cylinders to receive impressions from the form of types prepared for that purpose. In some printing machines the number and arrangement of the tapes are surprising. (See next chapter.)

Fig. 40.



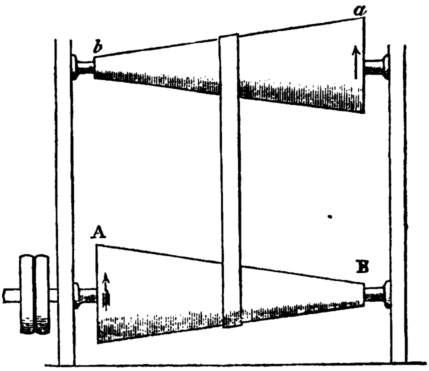
(1) (2)

In machinery where no great amount of power is transmitted the buff-wheel, or a wheel truly turned and covered on the circumference with buff leather, will drive another wheel similarly armed very smoothly.

2. *Velocity Combinations.*—We shall here describe a few of the contrivances used for giving variable velocities and intermittent movements.

Combinations of toothed wheels are commonly used for producing different rotatory velocities; thus, when a wheel of 72 teeth drives another of 36 teeth, the latter will move with double the rotatory velocity of the driver. The relative sizes of a pair of wheels of any description required to produce a given ratio of velocity, may be most readily determined; as the circumferences of circles are as their diameters, the latter must be in the ratio of the velocities sought. If a wheel, for instance, of 3 feet diameter, revolving once in two seconds, be the communicator of any motive force, and we desire to drive an axis at the rate of 3 revolutions in a second, the ratio of their velocities is 1 : 6, or $\frac{1}{6}$, which will be the ratio of their dia-

Fig. 41.



---

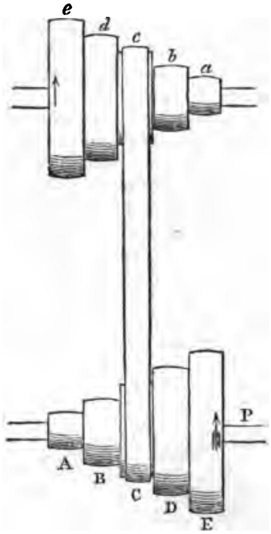
* See Moseley's Mech. Principles of Engineering, p. 245.

meters; so that a pinion of $\frac{1}{6} \times 36 = 6$ inches diameter must be used. The same rule applies to wheels working by simple friction or bands. The velocities thus given by circular wheel-work is constant during the whole time of action, but if a varying velocity is wanted eccentrics or cams are used, the principles of which, being highly interesting, will be subsequently explained.

A method of gradually varying velocity is the pair of friction cones used by rope-spinners (*fig.* 41). It consists of two cones connected by a crossed strap; the lower cone, A B, being driven by some motive power, communicates a velocity which varies with the position of the strap, if it be at the end A, or base of the lower cone, it will pass over that end *b* of the driven cone which is of the smallest diameter, consequently the velocity of the upper cone will be a maximum; if the strap be shifted to the middle position, as in the figure, the velocity of both cones will be equal; and if the ends A *a* be used the velocity of *a b* will be a minimum.

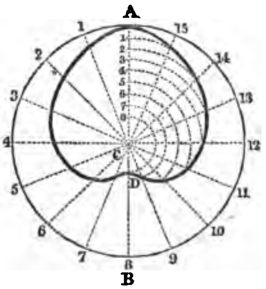
In the lathe and other machines these cones are reduced to a number of pulleys, as in *fig.* 42, and are called *speed* pulleys. The arrangement in the figure would be said to have five "speeds," each reduced cone consisting of five gradatory pulleys, the surfaces of which are convex to prevent the strap from slipping off when at work. The velocities obtained by these speed pulleys are readily calculated. Suppose the pulleys *a*, *b*, *c*, *d*, *e*, and A, B, C, D, E, to be respectively 2, 3, 5, 7, 9 inches in diameter, and the axis P to revolve 30 times in a minute, then the ratio of the diameters of any pair, multiplied by the revolutions per minute of the driving pulley, will give the number of revolutions per minute made by the follower or upper pulley. Thus when A and *e* work together the former will revolve $\frac{2}{9} \times 30 = 6\frac{1}{3}$ times in a minute; *b* driven by D will revolve $\frac{3}{7} \times 30 = 12\frac{6}{7}$ times; *c* by C, both being equal, $\frac{5}{5} \times 30 = 30$ times; *b* by D, $\frac{7}{3} \times 30 = 70$ times; and *a* by E, $\frac{9}{2} \times 30 = 135$ times, per minute.

Fig. 42.



The eccentric movements mentioned in the preceding pages, afford an infinite variety of variable velocity combinations. According to what has been already remarked, the velocity of the parts of a revolving wheel will be proportional to their radii or to their diameters, so that if we take the case of a common eccentric wheel, the relative rate of the rod's motion at any moment is according to the length of the radius acting at that moment. Thus the common eccentric wheel gives a variable velocity to the rod, because the radii drawn from the centre of motion to the circumference do not decrease or increase regularly. If a cam or eccentric be constructed so that the radii shall vary regularly, it will give a uniform velocity to the rod. This explains the formation of the eccentric shown in *fig.* 30. Draw any circle, as A B, and divide it into a number of equal arcs; and having determined on the eccentricity of the wheel, or the distance of the traverse, which will of course be equal to the difference between the least and great-

Fig. 43.

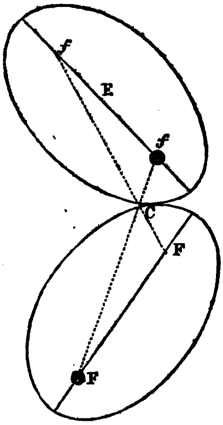


est diameters, measure the former on one of the radii, as C 8, divide the remaining part of the radius A 8 into as many parts as there are divisions in one half of the circle, and draw the lines 1—15, 2—14, and so on; the line drawn from A through the points thus obtained will be the curve required; by drawing similar lines on the other half of the circle, the heart-wheel may be completed. It is obvious from the construction of this eccentric wheel, that the rate of motion of a traverse moved by it may be uniform; again, by giving a sufficient variation to the length of the radii, a great variety of rectilinear motions may be obtained.

Mr. Wright ingeniously uses several kinds of eccentrics in his patent pin-making machine.

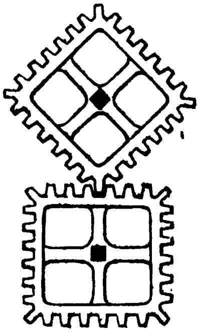
A pair of elliptic wheels, their axes being in one of their foci, will work together and produce a velocity varying as the eccentricity E *f* (*fig.* 44), the ratio of their velocities at any moment will be $\frac{Cf}{CF}$, or the ratio of their acting diameters.

Fig. 44.



A method of transmitting motion in a manner similar to the preceding was used in some printing machines; the wheels were toothed, but not really eccentric (*fig.* 45).

Fig. 45.



A very useful arrangement for obtaining an adaptative velocity consists in applying a buff-wheel to work on the face of another, as in *fig.* 46; the velocity of motion of the face wheel A will be 0 at the centre, and a maximum at the circumference, so that all velocities between 0 and that maximum may be given to the driven wheel by varying the distance of the point of its contact from the centre of A. M. Morin has made use of this combination, at the suggestion of Mr. Poncelet, for some experiments on the traction of carriages; to the axis *a* was attached wheelwork, by which means the revolutions of B were registered, and of course the force of traction known, as the lower or face wheel A was moved backwards and forwards according to the force with which it was pulled by the horse or other motive power. The instrument was called the *compteur*. The Rev. H. Moseley has taken up the same principle and applied it to an instrument for measuring the work

Fig. 46.

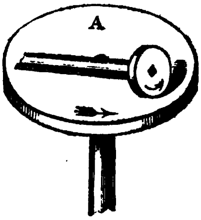
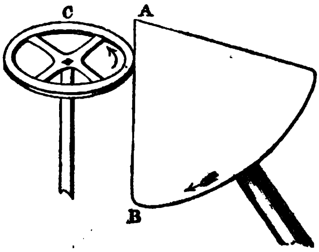


Fig. 47.

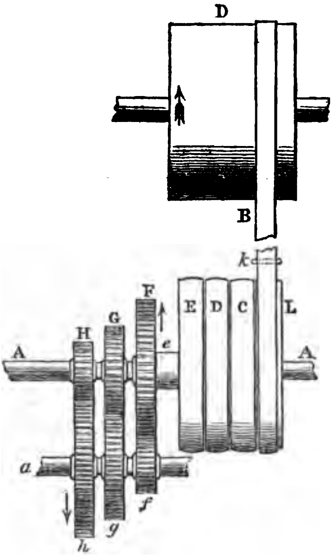


performing by a steam-engine, which is thence called a steam-engine indicator, also to a machine for mechanically performing some operations in mathematical analysis, the integration of given differentials. In the former instrument a cone (*fig.* 47) is used in place of the face wheel A, by which means the rotatory velocity of the wheel C may be more gradually varied from 0 (at A) to the maximum (at B).

In combinations for readily obtaining different velocities, a series of drums or pulleys, capable of being worked by one band, is very often used. The following examples illustrate this practice.

Fig. 48 represents an arrangement for transmitting motion with three different velocities or speeds; the drum D is able to drive the system of pulleys on the lower shaft with one band B; these four pulleys are not attached to the same axis; L is a loose pulley, used merely to keep the band ready for use; the pulley C is fixed to the axis A; D is fixed to a hollow axis, working over the axis A; E is fixed to another hollow axis, seen at *e*, working over the other two axes. The axis A carries also the pinion H, and the axes of D and E carry respectively the pinions or wheels G and F; immediately below these wheels are a series of wheels, *h*, *g*, *f*, similar in size, and driven by them.

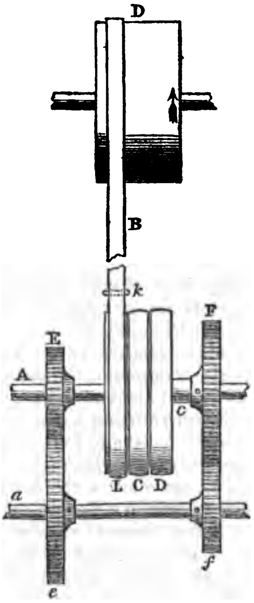
Fig. 48.



When it is required to use any one of these three speeds, the band must be slipped from the loose pulley on to one of the pulleys C, D, E; if a speed less than that of the driving axis be wanted, the pulley C is to be used, which, carrying the pinion H working with the wheel *h*, will give the axis *a* a speed less than that of A, in proportion to the ratio of the diameters of the wheels H, *h*; similarly G, *g*, being equal in size, will give a velocity to the axis *a* equal to that of A, and F, *f* (the pulley E bearing the band) will increase the velocity of the motion transmitted.

The next illustration shows a method of transmitting motion with two speeds.

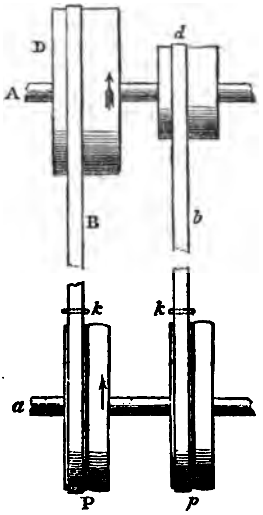
Fig. 49.



The pulley L, driven by the strap B from the drum D, is loose; the pulley C is fixed upon the spindle A, which also carries the pinion E; D is a pulley fixed upon a hollow spindle *c*, capable of moving freely upon the spindle A, and carrying the wheel F; another pair of wheels, *e*, *f*, is carried by the axis *a*. If the band be slipped on to the pulley C, motion is given to the pinion E, and thence to the axis *a*, which will have a less velocity than A; if the pulley D is used, F will be put in motion and drive the pinion *f*, whose rotatory velocity will thus be greater than that of F.

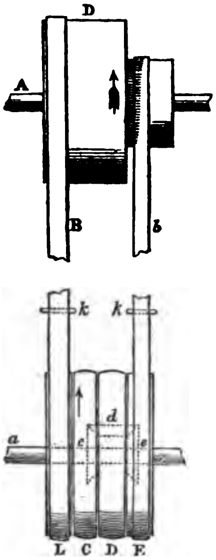
Two speeds are also obtained without the medium of toothed wheels, by using two belts and two drums, as in *fig.* 50; here the larger drum D drives the pulleys P, one being loose, as before, and the smaller drum *d* drives the pair *p*; if the diameter of this drum be half the diameter of D, both pairs of pulleys being of equal diameter, the velocity or speed of the axis *a* may be made equal to or half that of the axis A.

Fig. 50.



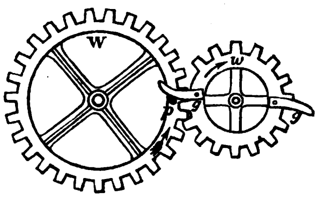
The following combination affords two speeds, with a compound movement. The drum D, *fig.* 51, drives the pulleys L, C, D, E, the first being loose; C is fixed upon the axis A; D and E are loose. At *c* a bevelled wheel is attached to the pulley C, which can work with another *d*, held by D; a third bevelled wheel is fixed to E, which is driven by the strap *b*. When the band is pushed on to the pulley C, a simple speed is given; but if D be used a double velocity will be given to the wheel *c*, and thus to the axis *a*, the bevelled wheel *e* being fixed; for, in that case, the wheel *d* turns once during a revolution of the pulley D. If the pulley E is driven by the drum and belt *b* in the same direction as the pulley D, the bevelled wheel *d* will not revolve once during a revolution of the pulley D, by the distance the pulley E has moved round; also, if the band *b* is crossed, E will revolve in a direction opposite to that of D, and the wheel *d* will turn round more than once during a revolution of its pulley, by the distance that E has moved round during that time. We have thus a combination by which the transmitted velocity can be varied in any way that may be desired.

Fig. 51.



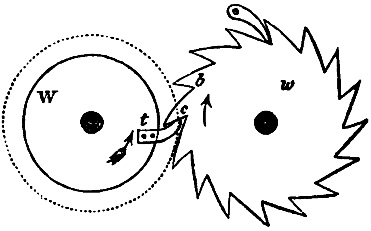
In some cases the action of some parts of machinery is required to be intermittent, or a certain alternation of rest and motion; this is effected, in toothed wheels, by leaving a portion or portions of the circumference of the driver untoothed, as is shown in the subsequent illustrations. In *fig.* 52 there are two portions of the driving wheel represented as without teeth, and, as one of these is double the length of the other, the follower *w* would rest during a time twice as long in one case as in the other. It is found that, on the return of contact, the teeth do not always

Fig. 52.



fall into their proper spaces in the other wheel, and there is a chance of the parts being fractured; a guide and pin prevents the possibility of such an accident; the pin *p* catching in the guide *g* pushes the follower until a tooth of the driver falls into a corresponding space of the follower. A very useful intermittent motion is obtained from the combination shown in *fig.* 53. The driving wheel W has one tooth *t*, working with the ratchet teeth of the wheel *w*; in the course of every revolution of W the wheel *w* will be pushed round a distance equal to one tooth; care, of course, must be taken that the single tooth *t* falls into a space at every return of contact, and that the teeth are of such a length that the action with one (*b*) may continue until another (*c*) tooth is brought up ready for the succeeding action.

Fig. 53.



Gearing.—The great inconvenience which would arise from suddenly destroying the motion of a prime mover of machinery, and the necessity of occasionally stopping some parts of a machine while others are required to continue in motion, has led to the invention of a series of contrivances by which the action of the prime mover may be engaged to or disengaged from a machine, which is thus said to be *in* or *out of gear*.

Where long shafts are arranged for gearing with one another, *couplings* are employed, by which the ends of any two shafts may be connected or disconnected at pleasure. There are two descriptions of couplings, according as they have one or two bearings; a simple coupling with two bearings is given in *fig.* 54. A being the end of the driving shaft, *a* its journal resting on the bearing C, and B the following shaft resting, by means of its journal *b*, on the bearing D, the coupling G *g*, which is thus said to have two bearings, consists of two circular iron plates P *p*, cast with a face tooth *e* and a corresponding space *d*; when the two plates are pushed together, the tooth on one falls into the space in the other, and the coupling is effected. It will be readily seen that, if the bearings are not accurately adjusted with one another, a part of the force transmitted must be expended in an endeavour to tear the journals from their bearings, or to break the coupling; on this account, and from the friction arising, couplings with one bearing have come into general use. The square coupling, *fig.* 55, is at once a very simple and effective arrangement; A being the driving, and B the following shaft, the driving shaft rests upon the bearing D, supporting the shaft B by a tooth T fitting in a corresponding space or hole in B; over these two ends the square box C is fitted and fixed by pins P P,

Fig. 54.

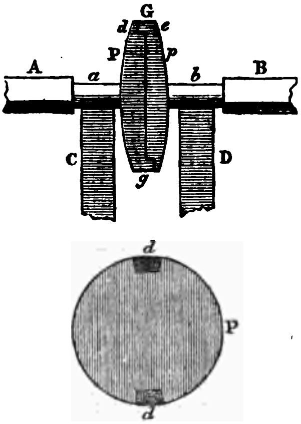
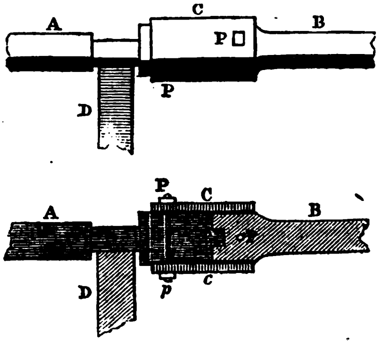


Fig. 55.

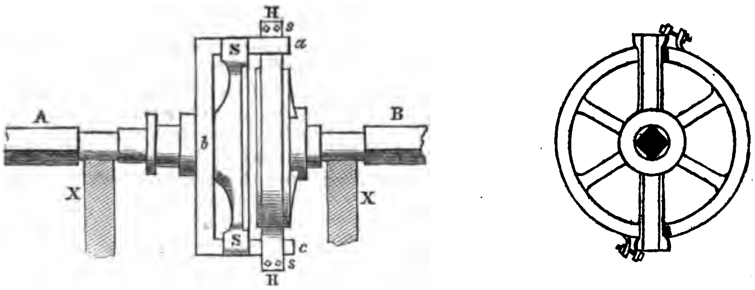


one passing through each shaft. In these contrivances it is required to convey the moving power with as little stress and wear of the parts as possible; the axes of the shafts should be carefully placed in the same straight line, and the power transmitted at as great a distance from the axis as can be conveniently allowed; this, indeed, follows from the property of the lever;—when a force is exerted at double the distance from the fulcrum, or centre, it will exert but half the strain upon that centre.

The sudden engagement and disengagement of machinery is frequently required in cotton, printing, and steam machinery generally. The old contrivances for effecting this object made the solid parts of the machine come suddenly into contact, and were consequently very objectionable, as no allowance was madê for the property of inertia of the quiescent mass; and we learn, from the nature of material bodies, that this natural indifference to motion must be gradually overcome. Some of the later contrivances are therefore made to act consonant with this principle. A gearing very generally used, on account of its simplicity, is the fast and loose pulley, examples of which are given in *figs.* 49—52; the band driven by the main shaft is held in readiness by the loose pulley, which, moving freely on its axis, exerts no effective rotatory force; by the fork *k* the band may be moved on to the next and fixed pulley, which is of an equal size, and the machine is then in gear, but it is not immediately driven at the same rate as the band moves, for, until a great portion of the inertia is overcome, the band will slip, and then the velocity of the driven parts will gradually attain the speed of the driver.

The friction-clutch is a beautiful contrivance for gradually transmitting an applied power. A and B (*fig.* 56) are, as before, the driving and following shafts or spindles, resting upon their bearings X, X. On the end of the shaft A is slipped a fork, called a bayonet, *a b c*, which is held either by the square end of the shaft or passes through the lateral arms S S, which are fixed to the axis. At the end of the following shaft B a drum is fixed, which is provided with flanges or ledges on each side to prevent a hoop or friction strap H H, from slipping off when the arrangement is in motion; this hoop is a spring tightened by the screw *s* so as to move upon the drum with considerable friction. When the bayonet *a b c* is pushed out so as to catch the hoop at *b*, *c*, the shaft A being in motion, it forces the hoop to revolve with the same velocity; but, as it can move upon the drum, it slips to some extent until by the friction it has gradually transmitted the same rapidity of motion.

Fig. 56.



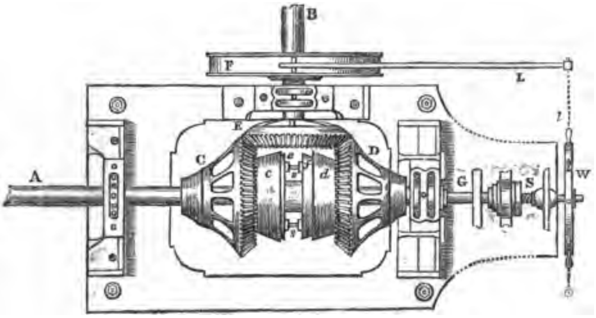
Where cog wheels are to be put in gear, it is generally effected by allowing the follower to fall on the driver; there is, however, a great chance of breaking the teeth of the wheels, as they frequently meet and produce a violent shock.

A friction and reversing gear is described by Mr. Higson (the patentee), in which a pair of cones is used to moderate the effect of the applied force*. The principal parts are represented in *fig.* 57; A is the shaft

---

* London Journal of Arts, Sciences, and Manufactures, vol. 29 (1846).

Fig. 57.



connected with the moving power, and B that acting on the machinery to be driven. On the driving shaft A two bevel wheels C D, and their projecting friction cones *c d*, are fitted to revolve loosely; these cones, *c d*, are hollowed so as to admit of the cones *d d* coming in contact with their inner surfaces and acting on them by friction; the cones *e f* are fixed upon the shaft A as regards rotatory motion, although they can be pushed backwards and forwards on the shaft, and thus come in contact with either of the hollow cones. If the pair of cones, which are held together by screws *s s* for adjustment, be pushed towards the wheel C, the cone *e* will come into contact with the cone *c*, the position shown in the diagram, and by friction cause it, with the bevel wheel C, to revolve, turning the superior bevel wheel E and its shaft; by pushing the cones towards *d*, and making contact with it, C will no longer be acted on, as it fits loosely upon the driving shaft, but D will work and turn the shaft B in a contrary direction. The friction cones, *e f*, are brought into contact with the cones *c d*, as may be required by the following contrivance:—A part of the shaft A is made hollow, so as to admit the smaller shaft G to slide within it; this shaft carries a key, which passes through a mortise in the exterior shaft A, and catches in a groove made in the boss of the cone-wheels; thus, when A revolves, the cones must revolve, but through the length of the mortise they can be moved longitudinally; the shaft G is worked by the screw S, on one end of which is fixed a wheel W, turned by handles on its circumference. This part of the apparatus is supported by a saddle, only parts of the three cross-bars of which are shown in the figure. For the convenience of quickly stopping the motion of the shaft B, a break, consisting of a wheel and friction-band F, is added; the band or strap presses on the wheel according as the lever L is depressed, and, to make it self-acting, a pin is placed upon the wheel W, so that, when the latter is turned and it moves the cones *e f* from contact with either of the hollow cones, the pin is at its lowest point, and there presses upon a lever *l*, which, being attached to the end of the lever L, pulls it downward, and tightens the friction-strap.

Chapter II.

Illustrations of Machinery.

The applications which have been made of elementary combinations, and of the principles of mechanical science, are so numerous at the present time that it would require a large volume to give a full description of the machines now used in the arts and manufactures. The principal features of interest in machinery, besides the accurate workmanship displayed in them, are their self-acting and self-adjusting capabilities; and the rapidity and accuracy with which the work is done. This is exemplified in machines for turning, planing, boring, drilling, punching, sawing, pile-driving, stamping, printing, copying, dividing, &c. With some of these machines little manual exertion is required further than to *feed* them, although it is impossible to distinguish them as requiring and not requiring manual assistance, being employed in either way according to circumstances; but the general application of steam as a motive power has led to the construction of what are termed self-acting machines to a very great extent.

In the present chapter we purpose giving a few illustrations of modern machinery, for which we shall describe the turning-lathe, planing machine, forge-

Fig. 1.

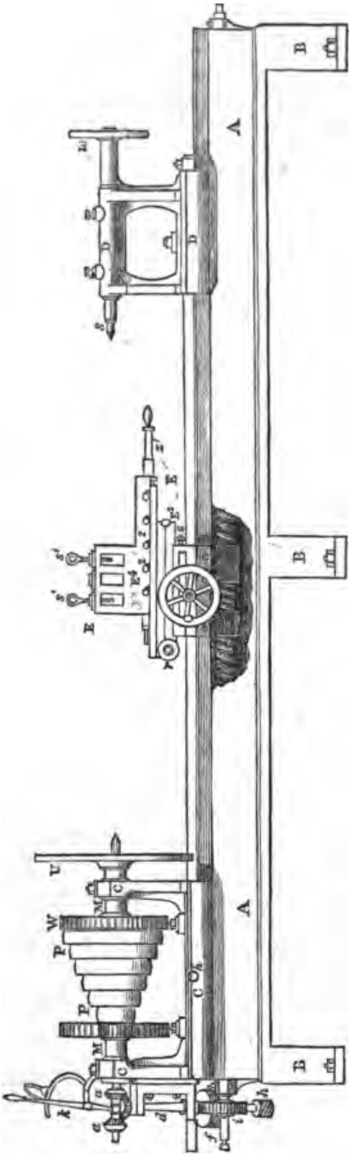
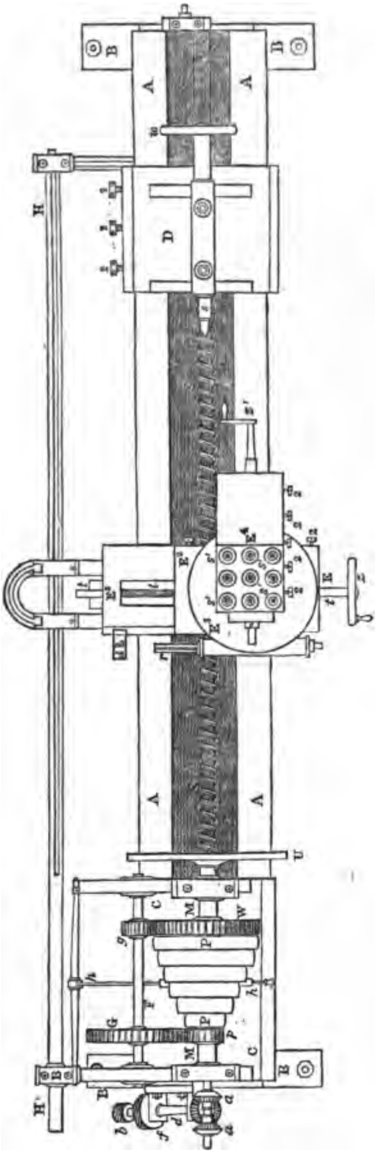


Fig. 2.

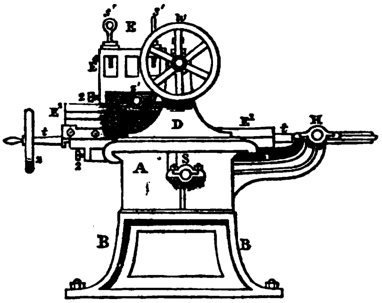


hammer, punching and plate-cutting, wheel-cutting and drilling, and printing machines.

I. *The Turning-Lathe.*—This tool holds a most prominent place among machinery, on account of its great utility. In the present construction of the machine, by which it is rendered both self-acting and self-adjusting, admirable arrangements are made for the purpose of giving the greatest accuracy and universality to its operations.

The turning-lathe represented in the accompanying diagrams is one of the most complete; it is adapted for plain and circular turning, screw-cutting, and boring. A A, *fig*. 1, is a sliding iron bed, resting upon the blocks B B, which are fixed to the stone flooring of the workshop. The bed is planed truly on the upper surface of its flanges, upon which rest the headstock C C, fixed to the bed by screws, the shifting headstock D D, which is fitted to slide along the bed, and can be made fast at any convenient distance from the headstock C C, and the slide-rest arrangement E E. The flanges of the bed are of a dovetailed form, as may be observed in the end view (*fig*. 3); one side of the moveable headstock is placed so as to suit the inclination of the flange, and at the opposite side a loose piece is put between the shoulder of the headstock and the flange, against which the loose piece is driven by screws 2 2 until the headstock moves easily but not loosely along the bed. The same adjustment may be seen in the other slides; the tightening screws being denoted by the numeral 2.

Fig. 3.

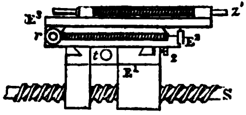


The moveable headstock, for tightening up the work to be turned when attached to a chuck U (or *holder**) on the mandrel M, may be further adjusted by a screw, worked by the wheel *w*, which moves the pointed spindle *s*; when in a proper position, the spindle is held fast by the screws above it.

The fixed headstock C C carries the mandrel M, upon which are the loose driving pulleys P P and pinion *p* (cast with the pulleys) and the fixed wheel W; and a spindle F, to which is fixed the wheel and pinion G *g*. By this arrangement two sets of motions are obtained; for the more rapid motions the wheel and pinion G *g* are put out of gear by two levers acting on the moveable bushes or bearings of the spindle F, the levers being worked by the rod and handle *h h*, and the pulleys and pinion on the mandrel are fixed by a key; when the slower motion is required, the pulleys are allowed to run loosely on the mandril, and the wheel and pinion G*g* is put in gear with the wheel and pinion on the mandril; the motion communicated to the pulleys by the moving power in this case does not immediately affect the mandril, but is transmitted by the pinion *p* to the wheel G, and by the pinion *g* to the fixed wheel W.

The slide-rest E E (*figs*. 1, 2, 3, 4, 6) is capable of different arrangements according to the manner in which the tool, which is fixed by screws to the upper slide, is to work. It rests upon the smooth surfaces of the bed-flanges, and is moved along the bed by a main screw S, which works in nuts fixed to the under side of a slide E¹; upon this rests another slide E², which can be driven across the bed of the lathe by a handle and wheel *z* on a shaft *t* partly screwed, working in a nut similar to the first slide and main screw; above this slide is a worm-wheel E³ turned by a

Fig. 4.



---

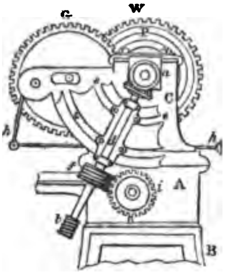
* The chuck called "Clement's Equilibrium" is shown in the figure; the two arms at the circumference catch some projecting part of the work, which rests upon the central pin, and drives it round.

worm on the shaft *r* (see *figs*. 4, 6), by which a rotatory motion of the cutter may be obtained for turning spherical surfaces. A fourth piece E^4 rests upon the wheel, carrying the tool, which is held in its place by some of the screws *s' s'*. This is also a slide, moved longitudinally by a handle *s'* on the end of a screw.

The following are the arrangements adopted in such an instrument for parallel, spherical, and face-turning, barreling, and screw-cutting:—

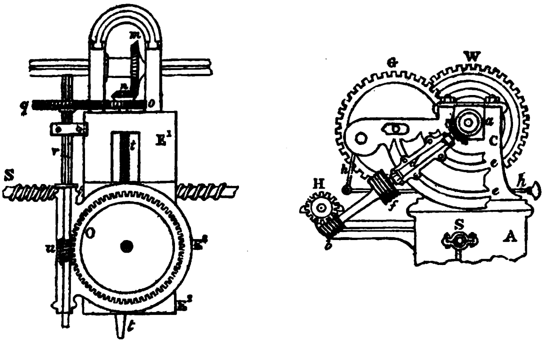
In parallel turning (as the turning of plain cylinders or rods) the mandril end of the lathe is adjusted as in *fig*. 5. One of the bevelled wheels *a a* on the end of the mandril is put in gear with the bevelled wheel on the shaft *d*, fixed in its proper position by means of two circular grooves or slots *e e*, in which the block carrying the shaft is moveable. A worm or endless screw *f* placed on the shaft works the wheel *i*, on the end of the screw S. The slide-rest and tool is thus moved along the bed of the lathe while the work is revolving between the chuck on the end of the mandril and the spindle in the headstock D; the rate at which the rest moves is regulated by the number of teeth in the wheel *i*, the other wheels being the same; three changes are provided, having thirty, forty, and fifty teeth in each wheel respectively; and the pitch of the screw S being one inch, the slide-rest will move with the abovementioned wheels $\frac{1}{30}$, $\frac{1}{40}$, and $\frac{1}{50}$th of an inch along the bed for every turn of the mandril. The rest may be moved to or from the headstock C C by the reversing combination *a a*; the pair of bevelled wheels are moveable along their shaft, and either can be put in gear with the wheel on the upper end of the shaft *a*, by moving the handle *k*.

Fig. 5.



For spherical turning, the arrangement of the end of the lathe and slide-rest shown in *fig*. 6 is adopted. The shaft *d* is shifted from the position in the last figure so as to allow the worm *b* to work, with a small wheel, on the end of the shaft H. This shaft moves the bevelled wheel and pinion *m*, *n*; the latter wheel is cast solid with the wheel *o*, and turns loosely upon its axis *t*, the lower slide of the rest is motionless, while the wheel *o* turns the spindle *r* by the wheel *q*; the spindle *r* is provided with a worm *u*, in gear with the horizontal wheel O, which supports the upper parts carrying the tool; the motion of the cutting edge of the tool will therefore be concentric with the wheel O, and not progressive. If the centre of O be immediately below the axis of the lathe, the tool in revolving will cut a spherical surface, the diameter of the ball thus formed depending on

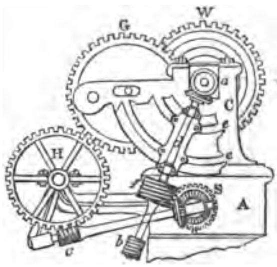
Fig. 6.



the distance from the centre of the wheel O at which the cutting edge of the tool is fixed. If the centre of O be not vertically below the axis of the lathe, a figure will be given to the work, the curve being concave or convex according to the relative positions of the different parts.

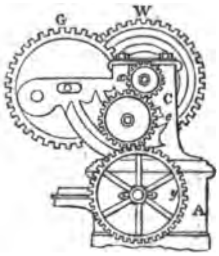
The contrivance for barreling (giving a swell in the middle of a rod or shaft similar to a barrel) is a combination of the two former, as might be supposed (*fig.* 7). A shaft is placed so that a bevelled wheel at one end may work with another similar wheel on the end of the screw S; while at the other end a small worm *c* moves a wheel on the end of the shaft H; thus, while the tool is moving along the bed of the lathe, it describes an arc by its rotation on the horizontal wheel O with a very slow motion.

Fig. 7.



In face-turning no motion of the large screw S is required, but the tool is to be carried gradually across the lathe. The end of the lathe will be adjusted as is shown in *fig.* 6; the wheel *q*, in the slide-rest apparatus, is taken off, and the bevelled wheel *n* is keyed to its shaft, which is a screw, and moves the slide with the tool across the bed of the lathe.

Fig. 8.

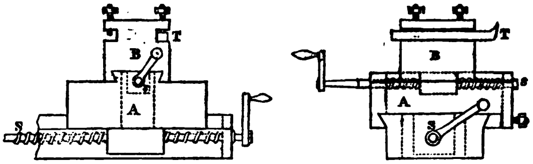


For screw-cutting the bevelled wheels *a a* are replaced by a spur-wheel *x*, gearing with another wheel on an axis which can be adjusted in the groove to convey the motion from the mandril to the wheel *y* on the end of the screw S. In this manner any proportion between the rate of the progressive motion of the slide-rest and the rotatory motion of the mandril (and cylinder to be cut) may be obtained, by properly proportioning the number of teeth in the wheels *x* and *y*, the velocity of the slide-rest being always equal to—

$$\frac{\text{No. of teeth in } x}{\text{No. of teeth in } y}.$$

The pitch of the screw S being one inch, if the wheel *y* have eighty teeth, a change of ten teeth in the wheel *x* on the mandril will produce a convenient difference of one-eighth of an inch in the motion of the slide-rest.

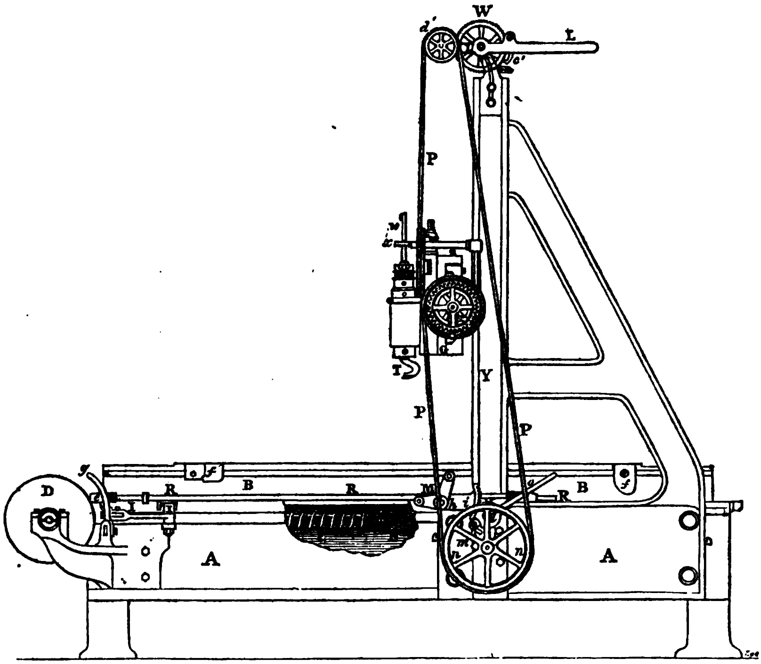
Fig. 9.



From the above description of the operations performed by the lathe described, it is evident that the slide-rest is a very important part of the machine. It is an introduction of late years, and has proved an invaluable instrument in practical mechanics; it has also been extended in principle to many other tool-machines with eminent success. In the ordinary rest the workman holds the cutting tool as firmly as possible on the edge of the rest, but in the slide-rest the tool is fixed to the upper slide, and requires no attention; and being worked by a screw, and moving against a shoulder or flange truly planed, the edge of the tool must cut very truly in whatever direction it is applied; in parallel turning a cylinder or rod may be turned of any length allowed by the size of the lathe, and preserve throughout an equal diameter, or, in face-turning surfaces, cut so truly as to adhere together when put in contact,—in fact as true as the face of the shoulder against which the slide bears; but such accuracy was not attainable in the manual use of the tool with a common rest. A slide-rest may be said in general to consist of two slides, as in the subjoined diagram, worked by screws, one of which is placed at right angles to the other; the lower slide A is moved by means of a nut, in which the screw S works, along the bed of the lathe; the upper slide B is worked by the screw *s*, in a similar manner, across the lathe; on the upper slide the tool T is fixed by screws. By such an arrangement the tool commands the surface of the lathe longitudinally and latitudinally.

II. *The Planing Machine.*—This tool is intended for planing the surfaces of iron and other metals, which is now done with very great accuracy and despatch. Before the introduction of this beautiful machine there was great difficulty experienced in giving a plane surface to the harder metals; and, after the finishing stroke of the best workmen, imperfections were apparent, and plane surfaces were avoided as much as possible in old machinery; it will thus be seen that the planing machine is a most important contrivance; it has contributed very greatly towards producing that elegant machinery which may now be seen in our factories executing its appointed work with astonishing rapidity and accuracy.

Fig. 10.

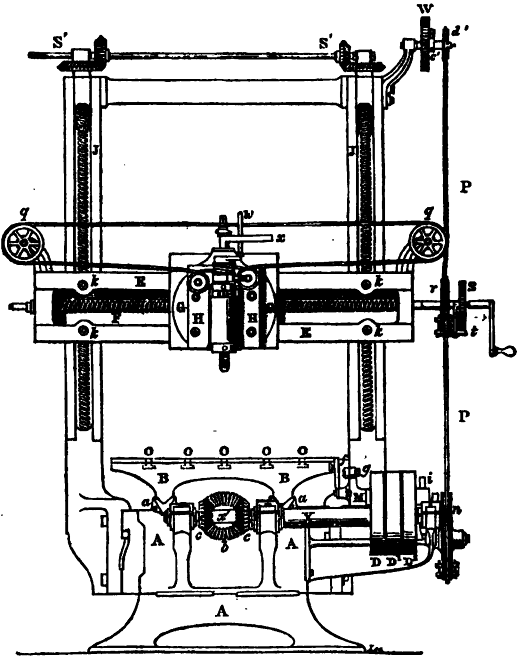


The diagrams 10, 11, and 12 exhibit one of the most improved and ingenious planing machines which have come under our notice. It is self-acting, and intended by the makers* for general work of what engineers call a moderate size; a scale is attached, so that the real size of the machine may be readily conceived.

---

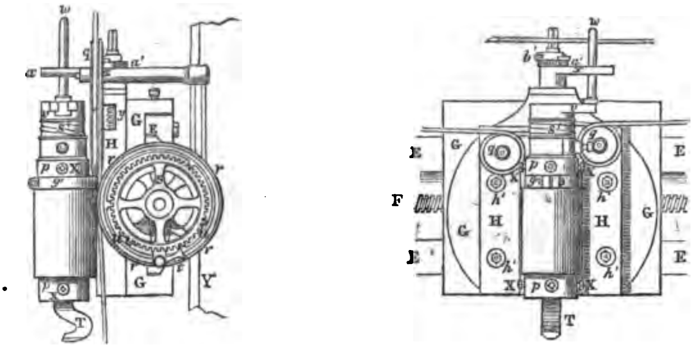
* Messrs. Joseph Whitworth and Co., of Manchester, to whom we are indebted for the above drawing of this, their patent self-acting planing machine.

Fig. 11.



The iron bed A A of the machine is provided with angular bearings *a a* on its upper edges, upon which the table B B slides; these bearings being well lubricated by a self-acting apparatus. The table has a number of longitudinal grooves O O on its upper surface, for fixing the work to it, and is moved with an alternate rectilinear motion by the screw C C, which turns in a patent white metal nut N, fixed to the under side of the table. At the end of the machine where the driving gear is placed (seen in the second figure), the driving screw carries a bevelled wheel *b*, working with the smaller bevelled wheels *c c;* one of these wheels is fixed to the shaft *x'*, which passes through the opposite bevelled wheel and the hollow axis X to the pulley D_1, the other bevelled wheel is fixed to the hollow axis or long boss X, which is attached to the pulley D, the third pulley D_2 being loose. The driving band being pushed from one pulley to another by a bell-crank lever I I and guide *g*. The contrivances for moving this lever and reversing the face of the cutting tool form peculiar features in this machine, and are self-acting by the following arrangement:—On the side of the moveable table B B two stops *ff* are fixed, one of which, when the table arrives at the extremity of its travel, strikes against the upper arm of the bent lever M, upon whose fulcrum or axis is placed the segment wheel *i*, the teeth upon which gear with a pinion on a spindle behind the wheel *l;* the same spindle carries another pinion at the end not seen in the drawing, working with a rack fixed to the horizontal rod R R, which is capable of being moved longitudinally in its bearings *e e*, and, by means of two pins fixed to it underneath, pushes one end of the bell-crank lever I I, and consequently moves the guide, and with it the strap, on to another pulley. This is the machinery for producing the self-reversing motion of the table. On the visible end of the spindle before mentioned, a circular plate *l* is keyed, having a projecting piece or lug carrying a pin. This pin, when the plate is moved round, falls into a slot in the imperfect wheel *m*, moving it and the pulley *n* with the strap P. The lever *o* is provided for the purpose of moving the plate *l l* and pin when required. The movement thus given to the pulley *s* is communicated to the tool socket (for turning it) by an endless strap P, which passes over the pulley *r*, and round the guide pulleys, *q*, *q*, *q*, *q*, attached to the vertical slide E E and horizontal slide G G; they give a proper direction to the strap, and keep it on the tool socket *s'* in all positions of the slides. The strap then passes up to the pulley *d'* attached to one end of a lever L' L, the fulcrum being at L'; the lever carries a click *c'*, which catches the ratchet teeth of a fixed wheel W; this little contrivance is for the purpose of keeping the strap at a proper tension. When this strap is moved by the pulley *n*, it causes the tool socket *s'* to make a semi-revolution, and thus turn the cutting edge of the tool in an opposite direction, to enable it to plane during the motion of the table in either direction. A stop is fixed on the tool socket, which comes in contact with a stop on the vertical slide E, and accurately determines the amount of revolution of the tool socket. The reversing strap P also works the screw F F and moves the plane across the table, by means of a click *t* attached to the axis of the loose pulley *r r;* the click catches the teeth of a wheel S fixed to an axis passing through the hollow axis of the pulley *r*, and turning the screw F; the click thus pushes the wheel S through a small part of a revolution, and the screw, working in a nut at the back of the slide G G, carries the tool, by successive steps, across the table. The distance which the tool moves every time the click catches the wheel is determined by the studs *u u*, which, sliding in the circular grooves or slots, may be fixed at any distance apart. A self-acting vertical motion may be given to the slide H H, when fixed at any angle, by means of the arm *v*, projecting from the socket pulley, the arm carries a pin *w w* passing through a slot in the lever *x*, fitting loosely on the screw *y* of the down slide H H, and having a click *z* working against a small wheel *a'* on the axis of the screw. The lever *x* also bears a pin *b'*, which will act as a clasp round the screw; this is useful when the tool is planing, so as not to allow of its being reversed; it must then be lifted from the work during the motion of the table in one direction; for such a movement the upper part of the tool socket is disconnected from the lower, the motion of the pulley *s* continues as before, but, instead of reversing the tool, it acts by the lever on the screw *y*, and raises the slide H H with the tool.

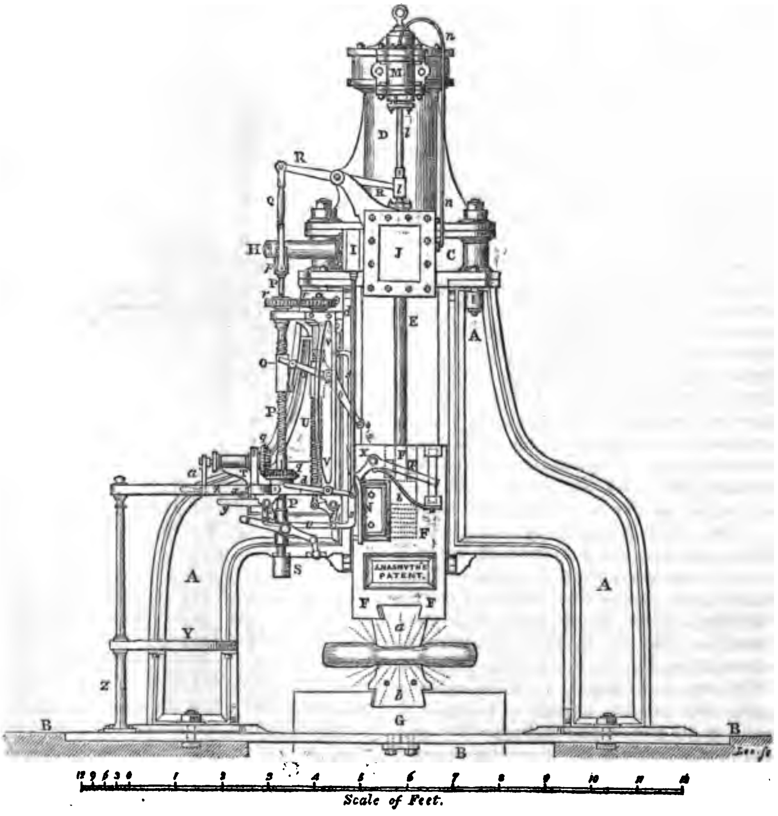
Fig. 12.



The tool is adjusted in its socket by the screws X X, and the socket has a conical bearing, so that any play is prevented by tightening the nut *g'*, which acts as a collar of suspension from the bearing. The bolts *h' h'* are provided to set the tool slide H H at any required angle on the slide G G, which is placed at the necessary height from the work by the vertical screws J J passing through nuts fixed at the back of the frame E E by the screws K K; both screws J J being turned simultaneously by the horizontal shaft S' S', which carries two bevelled wheels gearing with bevelled wheels on the axis of the screws.

III.—*The Forge-Hammer.*—In forging large masses of metal to form them into shafts, anchors, and heavy parts of machinery, the small hammers used by workmen are insufficient to produce the required effect of condensing the material; a ponderous instrument called a forge-hammer has therefore been commonly used, which by its great weight, and a fall of some depth, effects an adequate compression of the heated mass placed upon an anvil beneath it. The general form of this engine is shown in the preceding chapter (*fig.* 33). It may be observed that this hammer will not work so well with larger as with smaller masses; the larger the mass the less space it has to fall through, and consequently the less velocity and momentum is attained. The importance of the engine consists in its capability of giving a powerful blow; as much momentum as possible should be obtained, and the larger the mass the more the momentum should be, which is contrary to the effect produced, as the larger the mass the less will be the momentum, neither is there any means of varying the intensity of the blow; a part of the power obtained is unavailable in the common hammer, for, as we have already remarked, no attention is paid to the laws of bodies rotating about a fixed axis, and the axis or centre about which the hammer oscillates suffers continually from the efforts of the moving mass acting with a certain amount of leverage (the anvil being the fulcrum) to tear away from its axis. It is but very lately that this subject has received any attention, although a very important one, as large bars of wrought iron have frequently been fractured, and disclosed a faulty structure within, the

Fig. 13.



blows of the forge-hammer having penetrated but a little depth below the surface. An engine has lately been brought under public notice by Mr. Jas. Nasmyth, and appears to supply all that is required; its great power, and the facility with which that power can be regulated so as to do its work properly, giving this new hammer a completeness which leaves nothing to be desired. A view of one of these patent steam hammers is given in the following diagrams.

The general structure of the instrument is shown in *fig.* 13; it consists of stout cast-iron standards A A, firmly secured to a broad sole-plate B B, which is imbedded in the strong flooring of the forge. Upon the top of the standards a table C is fixed, to support the steam cylinder D and its valves. The piston-rod E, proceeding from the piston in the cylinder D, carries the hammer block F F, a mass of cast-iron, in which is inserted the hammer-face *a;* it is held fast in the dovetail groove by iron wedges and wooden packing. The anvil-face *b* is similarly secured to the anvil-block G, also of cast-iron, and of a mass sufficient to sustain the blows of the hammer. The hammer block is directed in a vertical motion by guides *g′ g′* on the interior side of the standard, which work in grooves formed in the hammer block. By admitting high-pressure steam into the cylinder through the pipe H, and underneath the piston, the latter is driven upwards, with the hammer; at a proper height the steam escapes, and allows the hammer to fall with great force on the substance placed upon the anvil *b*. The details of the instrument are—the arrangements for admitting and regulating the supply of steam beneath the piston, and the contrivances by which the engine is made self-acting, or by which it opens and shuts the steam-valves, and the means employed for regulating the intensity of the blow.

The steam brought by the pipe H meets in the box I a shut-off valve *c* (*fig.* 14), which is worked by the rod and handle *d d*, under the command of the attendant, who stands upon the platform Y. The steam, when this valve is open as in the figure, passes into the valve-box J, in which a sliding-valve *c* (*fig.* 15) works before a passage *f*, leading to the steam cylinder D, and another *g* leading to the waste pipe K; the steam, when the valve is in the position indicated, enters the cylinder, and drives the piston L upwards; when the valve is moved upwards, so as to open a communication between the passages *f* and *g*, the steam rushes out through the waste pipe K. To prevent the piston from striking the cylinder cover, the holes *h h* are made at a short distance from the top of the cylinder, which communicate with the waste

Fig. 14.

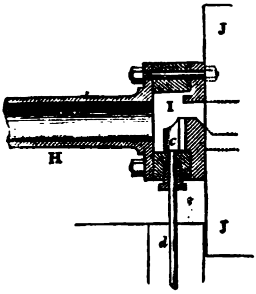
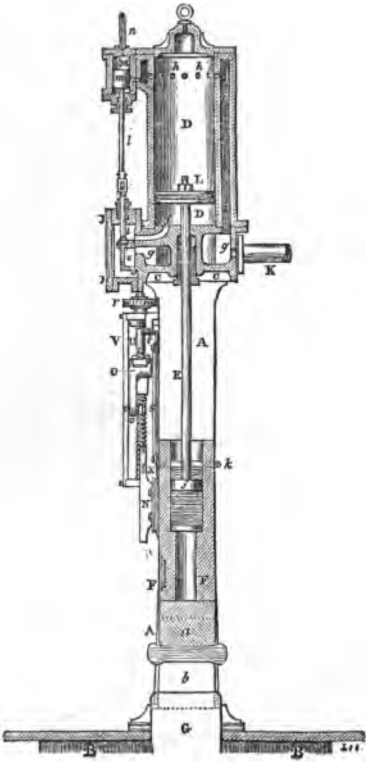


Fig. 15.



pipe by the passage *i;* these holes allow the air and steam in the cylinder above the piston to pass out when performing its upward stroke, until it reaches the holes, when the air above *h h* cannot escape and acts as an air spring upon the piston; the holes also permit the steam underneath the piston to escape when the holes have been passed.

The slide-valve *e* is moved down, in the latter case, by means of a small solid piston *m*, fixed to the upper part of the slide-valve rod *l l*, moving steam-tight in the little cylinder M, which is supplied with steam from the valve-chest J by the copper pipe *n;* the pressure of the steam would thus keep the piston and valve continually in the position shown, and keep the introducing port *f* always open. The upward motion is effected by a lever R R attached at one end to the valve-rod *l*, and at the other to the connecting-rod Q, which conjoins it to a shaft P P, screwed for a considerable part of its length, and is capable of vertical and rotatory motion; to a nut on the screwed shaft is jointed the lever O, carrying at the other end a roller *o*, which moves over the face of a tappet N fixed on the hammer-block; when the hammer rises the tappet strikes the roller *o*, and pushes that end of the lever upwards; the other extremity will of course push the shaft P P downwards and the valve-rod *l l* upwards, by which the steam is let out into the waste-pipe K; the hammer then falls; but in so doing it allows the end *o* of the lever to fall, which raises the screwed rod P, the valve-rod is depressed, and again steam is admitted into the cylinder, causing the hammer to ascend; in that way the hammer is self-acting. The height of its fall it may be seen depends on the height of the lever O; this can be regulated by the attendant, who has the handle *a′* under his command; this handle turns the bevelled wheel *q*, working another on the screwed rod P, which communicates its rotatory motion by means of two spur wheels *r r* to a similarly screwed rod U, carrying the nut in which the fulcrum of the lever O rests. By turning the handle *a′* the nuts are moved up or down as required, which brings the ends of the lever in contact with the tappet N at a greater or lesser height above the anvil.

In the arrangements at present described the steam is allowed to act slightly on the piston before the hammer has reached the substance on the anvil; consequently the whole momentum of the hammer gained by the fall is not allowed to act on the forging, when it is required, the attendant detaches a small handle *y* from a catch, by which the ingenious mechanism lettered *s, t, v, w, x,* X is brought into action. The lever *y* is pressed downwards by a spring *x*, and brings the *trigger w* into contact with a shoulder on the rod P; when the latter descends the trigger catches the top of the shoulder, preventing the return of the rod upwards and the admission of the steam into the cylinder; the admission-valve *e* would thus not be again opened but for the attached arrangement *s, t, v,* X; the handle *y* being pressed down by the spring *x* acts upon the rod *v*, pushing it towards the hammer; this rod is jointed to the bar *s s*, which is attached at the top and bottom to bell-crank levers *t t;* when the bar is pushed towards the hammer it touches one end of a lever X, called the *latch-lever;* the little instrument is heavier on one side than on the other, the heavier arm being pressed upwards by a spring *b′;* which is, however, not so strong as to prevent the arm from descending by its inertia when the hammer is suddenly stopped by the metal on the anvil; in this case the other extremity of the lever is raised, and pushes back the bar *s s*, the rod *v*, and the trigger *w*, allowing the rod P to ascend and reopen the steam-valve. The handle *z* enables the attendant to work the steam-valve manually when necessary.

In the section shown in *fig.* 15 the piston-rod E is seen to be attached to the hammer with a packing, to prevent the sudden jerks which would occur if the rod were merely screwed into the hammer-block; the packing consists of several layers of hard wood, which by their elasticity act as a spring. Above them the keys *k* secure the connection.

The steam-hammer has been applied by the patentee as a pile-driving engine, with the greatest success; some modifications are of course required to adapt the hammer to this purpose, but the arrangements of the self-acting valves are similar. The piston and hammer are supported by a tall standard or guide-pole, down which it is allowed to slide and follow the pile as it sinks in the ground, by means of a strong chain, passing over the standard and round a barrel turned by a small steam-engine. The hammer, its boiler, and the minor details are all placed on a truck or platform, moveable upon a railway, which allows the machine to travel along as the piles are driven. The weight of the hammer used by Mr. Nasmyth is 30 cwt., descending through a space of three feet; it moves so rapidly as to deliver seventy or eighty blows per minute. The new pile-driver is a far superior instrument to the old form, both as regards the effectiveness of a single stroke and rapidity with which the work is done. In the old pile-driver the monkey or hammer is drawn up by a windlass at a very slow rate, and occupies a very long period of time to drive a large pile; the monkey also is carried to a great height to attain as much velocity as possible in the descent, which has the effect of destroying very rapidly the head of the pile, and thus using that energy which should be effectual in forcing the pile into the ground. With the new engine these inconveniences do not result, the hammer being of a great weight, and moving through but a small space, has great momentum with little velocity; this, as is well known in practical mechanics, is the condition desired; for although impact is so much more useful in driving a wedge than pressure, that impact must be of a kind suited to the nature and properties of matter; as a pressure is a misapplication of power on one hand, so a weight very small in comparison with that of the wedge, moving with a very great velocity, is a misuse of power on the other hand. This does not, however, interfere with theory, it is still true according to the law of momentum* that two hammers of very different weights may have the same amount of momentum, as for example in the case of the present engine,

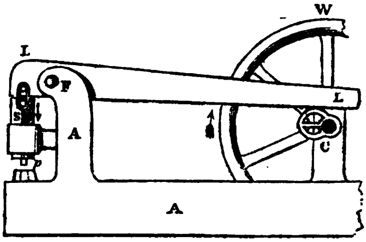
$$30 \text{ cwt.} \times 14 = 420$$
$$14 \text{ cwt.} \times 30 = 420$$

the amounts are equal whether we suppose the hammer of 30 cwt. to fall 3 feet, by which it attains a velocity of 14 feet per second, or the hammer to be 14 cwt. and its velocity 30 feet per second; but the result when the two are applied to useful purposes are soon evident.

We believe the patent steam pile-driver has been used in piling the foundations of the High Level Bridge, at Newcastle-upon-Tyne, at the Viaduct over the river Tweed, near Berwick, and other places, with complete success, and it is considered a most important invention for engineering purposes.

4. *Punching and Plate-cutting Machines.*—In making boilers and other apparatus where iron plates are required to be riveted together, some engine is necessary to form the holes for admitting the rivets; for this purpose the punching machine is used, the desired quality of which is a capability of exercising immense force through a small space—the thickness of the plate to be punched. In a great many contrivances for this purpose, a very heavy cast-iron lever is used, the arm carrying the punch being very short in comparison with the other, the extremity of which a cam, worked by steam or other available power, and carrying on its axis a heavy fly-wheel, lifts during a revolution, and punches the hole. Such an instrument is shown in *fig.* 16. L L is the great lever, the fulcrum being at F, in the strong standard A, the shorter arm carries the punch *p* with its socket *s*, which is kept steady by the box in which it slides. The longer arm is raised by means of the cam or eccentric C, on the same axis with the heavy fly-wheel W; the descent of the longer arm, which consequently raises the punch, is effected by its own great weight. A beautiful machine has been constructed by Messrs. Nasmyth, Gaskell, and Co.†, capable also of cutting iron plates, an operation which was commonly performed by a lever instrument similar to that shown in the

Fig. 16.



---

* See Dynamics, art. 33.

† We are indebted to the kindness of Mr. Nasmyth for the drawings of this machine.

diagram above, the punch being replaced by two cutting edges, acting like enormous shears.

A A are large cast-iron standards, which support the punching and cutting arrangements; the driving apparatus being similar for both parts of this compound machine. We shall first describe the punching instrument, which is seen in *fig.* 17A, *fig.* 18, and the plan *fig.* 19. The first figure is a front elevation, showing the slide B, supported by the standard and between two shoulders *d e*, the former being adjustable by screws; this forms what is called a V slide on account of its shape. It carries at the lower extremity the punch *p*, which makes a hole in the plate R, when the slide is driven downwards. Motion is given to the slide by an eccentric *a*, fixed to one end of a shaft which carries at the other extremity the spur-wheel C; this is connected with a reducing train D E. On the axis of the smaller pinion is a pair of pulleys, one being fast and the other loose, by which means the punching-slide receives motion from the driving shaft of the factory; a heavy fly-wheel G is placed upon the gearing-shaft to equalize the motion. The punch, in making a hole, drives the portion of metal beneath it into the die *h*, through which it falls to the ground. When the object is thus effected the punch rises, being assisted in doing so by two weights *w w;* in this movement the plate very frequently has a tendency to rise also, as the punch fits tightly in the hole it has made; a stop is therefore placed each side of the punch to prevent this action; it is shown in *fig.* 18 by a small

Fig. 17 A.

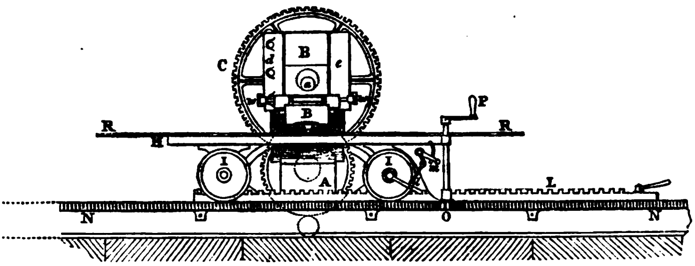


Fig. 17 B.

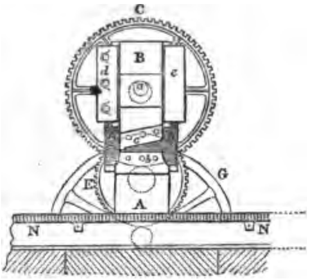
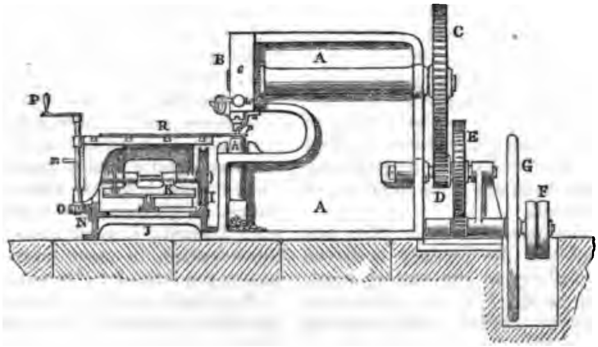


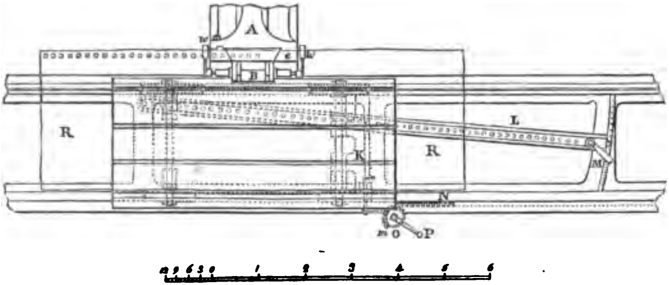
Fig. 18.



curved piece, above the letter *p*, screwed to the standard A. The apparatus on which the plate R to be punched is bolted, is for the purpose of carrying the plate onward as the holes are punched; and this can be done with great nicety.

The plate rests immediately upon a carriage, H I, moving along a railway, the inner rail being a triangular bar, so that the wheels I are unable to run off the rail. A rack N is fixed on the outer side of the railway, gearing with the pinion O, which is fixed on the lower end of a spindle worked by the handle P; the workman in attendance is thus enabled to move the table and plate at pleasure along the railway, by turning the handle P. The distance between the holes, called the "pitch," or the distance moved by the carriage after every punch, is determined and regulated by a highly scientific and elegant contrivance. Underneath the table a pall K is attached to an axis fixed to the carriage, catching the teeth of a rack L; this rack is moveable about an axis at one end, and at the other extremity carries a pointer, *fig.* 19, and tightening screw; it will be readily understood that by this means the most minute variations may be made in the pitch of the holes; in some cases this is necessary, as in the outside and inside plates of circular and cylindrical forms, where any two radii, or the direction in which the rivets are required to lie, will be less distant with the inside than the outside plate. By a scale M the workman can obtain these small differences of pitch with the greatest facility, by turning the handle *m*, it acts upon a link *l* attached to the pall K, and, raising it, permits the carriage to slip over one tooth; of course, by fixing the moveable rack in a more diagonal position, the less will be the pitch of the punched holes. The table has a movement at right angles to the motion of the carriage along the railway, so that a double row of holes may be punched, the distance between them being accurately preserved.

Fig. 19.



Scale of Feet.

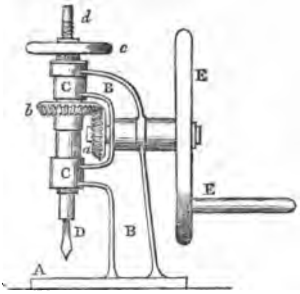
When the punching of a plate has been completed, the carriage is driven along the railway to the cutting headstock seen in *fig.* 17B; its general arrangements are similar to the punching machine; in this case the slide B carries a cutter *c*, and, when the plate is placed upon the edge of the cutter *b*, the former descends, and, acting like a pair of scissors, cuts the iron plate.

By a contrivance in connection with the machine the punch can be instantly arrested, also it can act either in conjunction with the plate-cutter or separately.

5. *Drilling Machine.*—The common method of drilling small holes with the bow, although a very useful because simple mode, is inapplicable where great precision is required or large holes are to be bored in metal. The lathe is frequently made a drilling machine, either by fixing the tool in a chuck and driving it on the end U of the mandril (see *fig.* 1), or putting the blunt end of the drill against the point of the spindle *s* in the moveable headstock, and driving it by the screw and wheel *w* against the work to be drilled, which is then put on the chuck. This manner of drilling is, however, limited; in some cases it would be inconvenient, in many more impossible to use the lathe for this purpose.

For small work the portable drilling machine exhibited in the following diagram is a neat and simple contrivance, and may be driven by the hand. Upon the bed A the standard or vertical post B is firmly fixed, supporting two bearings C C for the drill-spindle, in which the drill D is held. The spindle and drill are turned round by means of two bevelled wheels *a b*, the latter being worked by the fly-wheel and handle E. To produce the requisite vertical motion of the drill there is a small wheel *c* working the screw *d* on the top of the drill-spindle. This pretty little machine is made by Messrs. Nasmyth, Gaskell, and Co.

Fig. 20.



A beautiful self-acting drilling and boring machine* is represented below (*figs.* 21, 22). A strong cast-iron frame A A is firmly bolted to the iron bed B, supporting the drill arrangements at the upper, and the moveable work table at the lower part. The drill is worked when the driving band is on one of the pulleys C, which are arranged as in the lathe (described in page 73), for changing the speed; at one

Fig. 21.

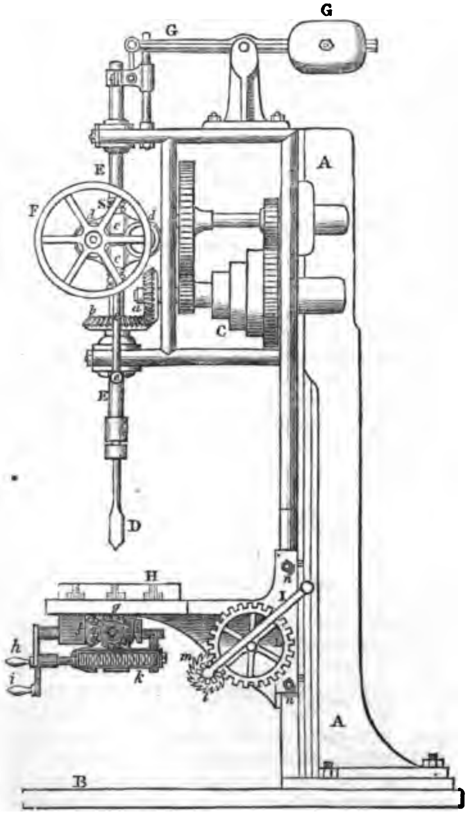
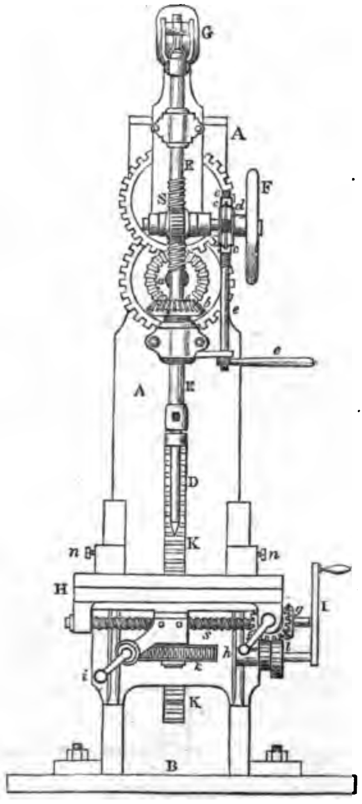


Fig. 22.



---

* By Messrs. J. Whitworth and Co., Manchester.

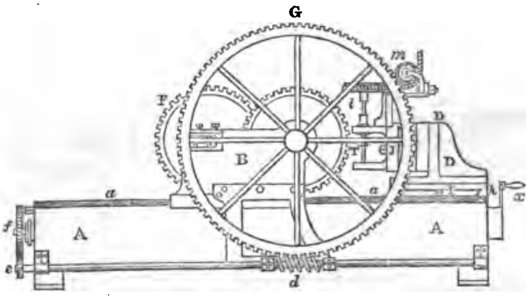
end of the spindle a bevelled wheel *a* is fixed, gearing with another similar wheel *b* on the drill spindle E. The contrivance for producing the self-acting property of this drill, or a downward motion, as it bores the work on the table, consists of a screw S on the drill spindle, which works with a pair of worm wheels, one on each side of the spindle; on the axes of these wheels there are two small pulleys *d d*, carrying on their circumference a friction apparatus composed of two pieces of iron *c c;* these embrace the pulleys *d d*, and when pressed together will of course create considerable friction when the pulleys revolve, the tightness of their hold can be regulated by turning the rod and handle *e e*, at the upper end of which is a screw working in an inside screw cut in the friction pieces *c c;* the thread passing through the upper piece is cut in an opposite direction to that passing through the lower piece, so that in turning the handle the straps are forced together or pushed asunder; the drill D may thus be pressed downwards with any required force, for if the wheels *c* were fixed, the spindle would descend through a space equal to the distance between two contiguous threads, or the pitch of the screw S; if they were loose no motion would be given to the spindle, therefore when the handle *e* is turned and friction produced on the pulleys *d d*, the worm wheels do not revolve freely, but push the spindle slowly downwards. The bevelled wheel *b* does not participate in this vertical motion of the drill spindle, as it would be put out of gear with the wheel *a;* it slides vertically on the spindle, but cannot revolve on it because of a groove running up the spindle, into which a key or feather catches. When the spindle and drill is required to be lifted from the work, the wheel F on the axis of the worm wheel is turned by the hand. The spindle is balanced by a weight and lever G.

The upper part of the table H, supporting the substance to be bored, is capable of a lateral and a rotatory motion; the former is obtained by the screw *s* working through a nut fixed underneath the table, and turned by the bevel wheels *f g*, and the handle *h*. The handle *i* produces the rotatory motion, by turning a worm which acts on the wheel *k*, its axis being connected with the under side of the table. The whole may be moved up or down at pleasure by the handle I, which turns the pinion *l* and the wheel I gearing with it, the spindle of this wheel carrying a pinion underneath the table, working in the rack K; a ratchet wheel *m* and detent is provided to retain the table at any height to which it is raised, until the pinching screws *n n* are tightened, by which it is held in its place.

6. *Wheel-cutting Machine.*—Many mechanical contrivances have been made for the purpose of cutting spaces in the circumference of wooden models and iron wheels, the general arrangements being a support on which the wheel to be cut is fixed, a cutter which can be pressed against the part to be cut away, and some combination of wheels or other elements, by which the wheel under operation can be moved round on its axis a distance equal to the pitch[*] at each movement, to present new work for the cutter.

The wheel-cutting machine[†] repre-

Fig. 23.



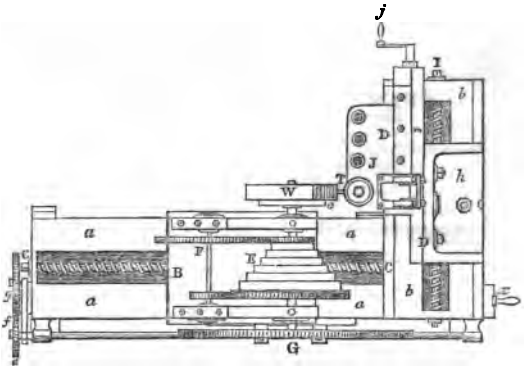
---

[*] See chap. I. p. 48.

[†] By Messrs. Collier and Co., Manchester.

sented in the following figures is so arranged that it might be used as a lathe if required. The bed-frame A A is of cast-iron, and carries on its planed upper surface *a a* the headstock B, which is moved along the bed by a screw C, passing through a nut fixed to the under surface of the headstock, and carries the wheel W in which the teeth are to be cut. At one end of the bed-frame the headstock D, in which the cutter is fixed, slides on the planed surface of the bed *b b* transversely with respect to the motion of the headstock B. The two principal parts in this machine are therefore the headstock and its adjustments for holding the wheel in which the teeth are to be cut, and the headstock for carrying the cutting tool.

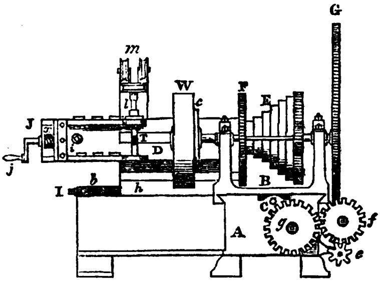
Fig. 24.



The former, B B, is provided with a set of cone pulleys E, and a speed arrangement F similar to that described in the lathe (p. 73). On one end of the cone shaft there is a face plate *c*, on which the wheel W is fixed; the opposite end of the shaft carries a large index-wheel G, working with a worm *d* on a shaft which has at one end the pinion *e*, this pinion gears with a wheel *f*, which communicates its motion to another wheel *g*, the latter and the pinion *e* being changeable at pleasure, according to the number of teeth required in the wheel W. The wheel *g* is on the end of a shaft which reaches the other end of the bed-frame, and is there turned manually by the handle *x* as the teeth are cut in the wheel.

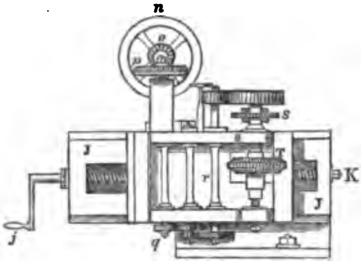
The cutter headstock D is moved along its bed by the screw I, working in a nut fixed underneath the sole-plate or slide *h;* the upper part *i* of the slide is moveable on the surface of *h* horizontally, to allow the cutter slide-rest to be set at a convenient angle for cutting the teeth of bevelled wheels, the angle being measured by an index on the upper surface of the plate *h*. In the position shown in the drawing the cutter is adjusted to cut the teeth for a spur-wheel. A slide-rest J is attached to the upper part of the headstock D, moving horizontally in front of it, or before the wheel; it is worked by the handle *j* and screw *k*, carrying the cutting tool T, which revolves rapidly by the action of a cord passing over a pulley on the top of the spindle *l* in which the tool is fixed, and thence guided by the pulleys *m* to a driving shaft in the workshop. To cut screw-wheels or worm-wheels, the slide-rest J has a vertical motion by which it can be set at any required angle, which is indicated by an index on the headstock.

Fig. 25.



The arrangement of the cutter shown in the above diagrams, exhibits the method adopted for cutting the teeth of wooden wheels or of patterns; when iron wheels are to be worked, the slide-rest is fitted as in *fig*. 26, the cutting tool T being a wheel with a periphery suitable for leaving the teeth of a proper form. The driving band passes over a pulley *n*, on the same axis with the bevelled pinion *o*, which works the bevelled wheel *p;* on its axis, at the lower end, is a pinion *q* which gears with a reducing-speed train of three wheels; the axis *r* of the last of these wheels carries, at the upper end, a pinion working with a wheel on the cutter-spindle, which carries another wheel *s* to work a pinion behind it on a spindle which acts on the screw K, and produces the necessary motion of the slide J instead of using the handle *j;* this part of the machine is therefore self-acting, the cutter T moving along as it cuts the tooth. The other movement—that of the wheel G and consequently the wheel W, the space of one tooth every time the tool has cut one space—might, by a simple combination, be made also self-acting.

Fig. 26.



To use this machine, the wheel in which teeth are to be cut is firmly fixed on the cone-shaft, and change-wheels *e g* of a proper number of teeth, according to a table given, are put on their respective shafts; the cutter is then set to work, and, when a space is cut, the handle *x* is turned in most cases once round, and the cutter again commences its work.

In adapting this machine to the purposes of a lathe, the headstock D is removed and a proper slide-rest adjusted in its place, the remainder is then but a modification of the lathe; it will be seen from this that a lathe also may be readily adapted to cut the teeth of wheels.

7. *The Printing Machine.*— This machine, for letter-press printing, is an improvement on the original manual printing press; it is adapted generally for being worked by steam-power, and is capable of producing many more copies than the common printing press, the number of printed sheets obtained from the latter, with the assistance of two men, being on an average 250 per hour, having an impression on one side only; but, by the machine subsequently described, the mean produce is 1000 sheets per hour, printed on both sides, or eight times the former quantity, the manual assistance being supplied by two boys. The machines as arranged by Messrs. Applegarth and Cowper, are those generally used, with some slight modifications. The principal features consist in the use of endless tapes, between pairs of which the blank paper sheets are carried over large iron cylinders, which, in the course of a revolution, press the sheets on the surface of metal types placed beneath, which have been previously covered with ink. The following illustration represents one of the improved forms now in use*.

The framing of the machine, A A, is of cast-iron, and supports three principal parts,—the table T, on which the form of types to give the impression is laid; the inking apparatus *a, b, c, d;* and the suite of cylinders, B′, L, M, B, round which the sheets to be printed are carried by the tapes, including the feeding table K.

In the figure the form-table T is represented as being at the maximum distance traversed towards the end T of the machine, the nearer extremity being visible immediately above a pulley *r* and below the rollers *a a;* a number of pulleys *r r* form the support on which it travels. The table consists of two parts, the middle being appropriated to the reception of a pair of forms†, and the ends are provided with an even metal surface to form ink tables.

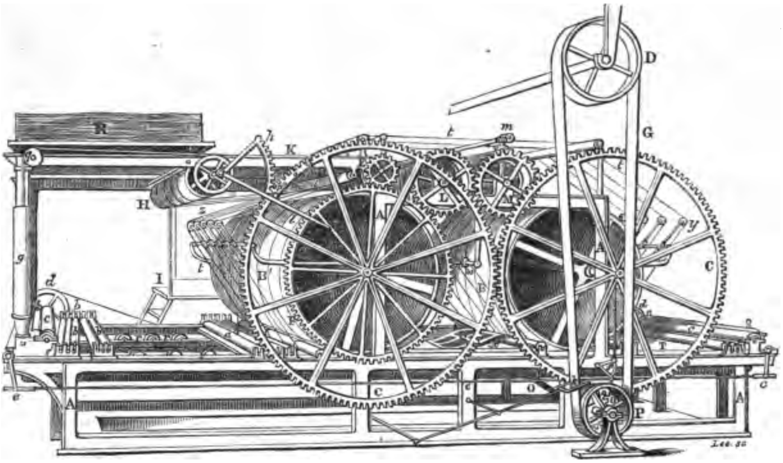
On the end of the shaft O, which passes underneath the machine, is fixed a bevelled wheel which gears with another bevelled wheel on an upright shaft at the top of which there is a pinion, catching in a rack somewhat

---

* Constructed by Messrs. Dryden and Co.

† A *form* is a rectangular iron frame (called a *chase*) in which several pages of types are placed, and made tight or "locked up" by wooden wedges.

Fig. 27.

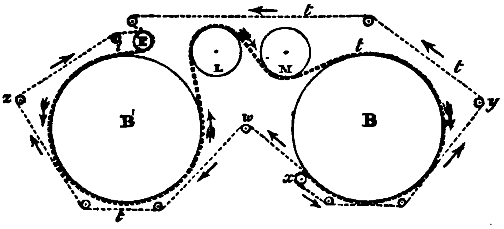


similar to that in *fig.* 105 (Mech., Treat. II. p. 58); this rack is attached to the under side of the form-table T, but is at liberty to move laterally, because the pinion being a fixture as regards its position, when one end of the rack reaches it, a slight motion must of course take place in order to bring the other side of the rack in contact with the pinion. In order to retain the pinion in contact with the rack, its shaft having no bearing at the upper end, a small pulley or wheel is fixed there, which pushes against a flange whose face is opposite to the teeth of the rack; the pinion is thus prevented from leaving the rack; the lateral movement of the rack is effected in a proper manner by a kind of parallel ruler motion, so that both ends move simultaneously, and an equal distance.

The forms are tightened between the sides of the table by pieces of wood called "furniture," and are inked by two sets of rollers, one of which is seen at *a a*, and the other is placed in a similar position near the other large cylinder. These rollers receive the ink at each traverse from the ink-table T, which fetches the ink from an apparatus marked *c*, where a quantity of ink is kept in a box, and is allowed to come in contact with the roller *c*, which is turned slowly by means of a cord passing round the pulley *d*, and finally reaching another pulley on the axis of the drum B, on the opposite side of the machine; as the under side of the box is made so as nearly to touch the circumference of the roller *c*, the latter as it turns can carry away a small portion of the thick pasty ink used in printing; beneath the roller is another, invisible in the drawing, which receives the ink, and, being in contact with *c*, is made to revolve; this roller, however, is in a vertically moveable bearing, connected with the small lever and rod *e e*, the latter being attached at one end to the bell-crank lever *f*, and as the upper arm of this lever is furnished with a small rod joining it, with an eccentric on the axis of the cylinder B, by the nature of an eccentric an up and down motion will be given to the lever *f* when the machine is in motion, and produce an alternate horizontal motion of the rod *e e;* by this means it may be seen that finally the roller in question will be moved up and down as the cylinder B revolves, and it is so arranged as to descend when the ink table T arrives underneath. A quantity of ink is thus deposited on the ink table, but not very evenly distributed on its surface, as it should be; to effect this a series of small rollers *b b* are allowed to roll on the table as it returns, they work loosely in their bearings, being merely let down between the divisions of their carriages, which are seen in the figure; in this state their under surface is somewhat lower than the surface of the ink table, so that the latter lifts them up a little as it meets them; thus the rollers press with their whole weight on the table; but this of itself would be insufficient to distribute the ink: the rollers *b* are therefore not placed directly across the table, but somewhat diagonally, as may be observed in the drawing, which permits them to be pushed across the table while they revolve, as we might infer from the principle of the resolution of forces. In this way the ink is very evenly distributed on the plate, which, as it proceeds inward, transfers the ink to the rollers *a a*, to ink the printing types. The rollers *a a*, *b b*, and the unseen roller below *c*, are made of a soft material, composed of treacle and glue in such proportions as to give the compound the consistence of caoutchouc.

The iron cylinders B′ B are supported by the framing A, and are adjustable at the proper vertical position by screws at the bearings of their spindles; each cylinder carries a large toothed wheel, that seen in the figure on the right hand gearing with a driving pinion *o* immediately behind the fast and loose pulleys P, which connect the machine with a drum D on the driving shaft, by a strap G; thus the whole of the upper part of the machine is driven by the pinion *o*, as the two large wheels are in gear, and on the spindle of the left hand cylinder B a toothed wheel F is fixed, which drives the pinion E and the wheel L, the latter driving its companion M. The tapes *t t* retain the sheet in its place, while the cylinders B′ B carry it downwards to receive an impression from the form of types. Their direction of motion is better shown by the accompanying diagram (*fig.* 28); E is called the entering drum, because the sheets here enter between the tapes, the motion of the drum then carries the tapes and sheets round in the direction of the feathered arrows, over the drum L, underneath M, and over B; when in the act of running up the inner side of the cylinder the tapes separate, and the sheet is thrown out; the one series of tapes turn over a small roller *x*, pass outwards in the direction of the plain arrows, are carried over the small pulleys *y*, and another small roller above the cylinder, to a roller above the entering drum E; the other series of tapes are brought upwards to a number of small pulleys *w*, whence they pass underneath the cylinder B′ up to the pulleys *z*, and thence under* the roller *i* to the entering drum E; these sheet holders pass over those parts of the sheet which form the margin, and are consequently untouched by the inked types. The tapes are adjusted to a sufficient degree of tightness by the pulleys *y*, *z*, their supports being screwed up in nuts resting on a transverse bar. To enable the cylinders to effect a good impression a piece of woollen cloth called "blanket" is attached to that part of the drum which bears the sheet.

Fig. 28.



The sheets of paper must necessarily be taken by the tapes at the entering drum so that they may meet the form at the proper place to receive an impression; and to effect this important matter a number of wide tapes called the web are placed on the table K; they pass round the drum H and a small roller near the entering drum E; on this web the sheet of paper is laid. The spindle of the drum H carries a sector *h*, and wiper or lever *l*, the former gearing with the great wheel C, and the latter, being close to the face of the wheel, is struck by a pin *p* fixed to the wheel, driving it downwards; the drum then turns through a part of a revolution by the action of the wheel on the sector, and the sheet of paper is driven so far as to be caught by the tapes at the entering drum E; the drum is brought to its former position by means of a weight in the cylinder *g*, to which it is attached by a string *q*.

---

* In the figure the tape should pass below the roller *i*.

The drum M is a "registering" apparatus, or a contrivance for making the sheet of paper which has received an impression under the cylinder B′, arrive at the proper time on the cylinder B to receive the impression on the other side. Although the forms of type are placed in a proper position on their table T, yet it is found that, in a sheet printed off without any further adjustment being made, the lines of print on each side do not "back," and it may be observed in printed books where this particular has not been attended to, if we hold a leaf up to the light the leading lines at the top of the pages do not coincide or overlay each other; in such a case the "register" has not been properly effected. The drum is raised or lowered to effect this object by means of a worm *m*, working a pinion on the axis of a screw which holds the bearing of the drum at one end, and by a similar screw at the other end of the drum; the handles of both movements are therefore worked at the side opposite to that seen in the figure. The immediate consequence of any motion of these supporting screws is that the drum is raised or lowered; but it will be evident, on reflection, that this will cause the sheets to travel over a less or greater distance at the particular end affected, and its progress hastened or delayed accordingly. This is a final adjustment.

From the above description we believe the action of this beautiful machine will be understood by every reader. A boy stands on the platform I, where he has the command of the table or web K and the ream of damped paper R; he takes a sheet from the pile and places it on the web so that the edge nearest R touches wooden stops attached to the web, and another edge, nearest himself, touches a stop fixed to the table; as the wheel C turns, the pin *p* strikes the wiper *l*, the sector falls in gear, and the sheet is driven by the stops between the tapes at the entering drum; it is then carried downwards, while the table, moving in unison with the cylinders, brings the form of types first under the inking rollers, and then beneath the cylinder, to meet the sheet at the proper place; the latter, when the impression is made, passes, as before stated, over L and under M, reaches the top of the second cylinder B, and is borne downwards to meet the other form of type; the impression being made on both sides, the sheet S is thrown out on the table and taken away by another attendant.

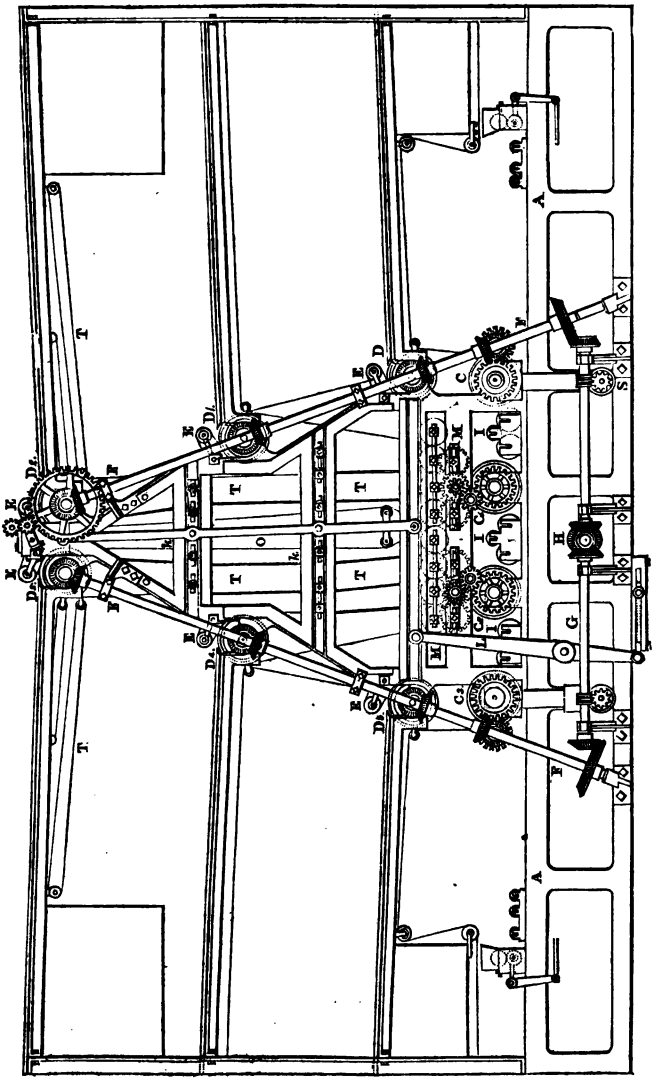
The necessity for rapid printing which arises in working newspapers has led many persons to attempt alterations and arrangements in the machine, by which a much greater number of copies could be completed in the same time; the number of rollers for carrying the sheets was increased to four, and they were reduced to a smaller diameter; the machine thus constructed prints one side only, and gives about 4,500 printed sheets every hour. In 1846, Mr. Little patented a printing machine, which was to produce 10,000 copies per hour. This is the greatest rapidity of production ever attempted publicly; to effect this the patentee uses eight cylinders, but in a manner very different from the usual method; it exhibits great simplicity in construction as well as elegance in action.

Up to the present time[*] the eight cylinder machine has been but once in operation, and that for a short time; one with four cylinders, upon the same principle, having been some time at work, we shall describe the arrangements of that machine, which are only different from the other in being of smaller dimensions. The first figure exhibits a side view, and the second a section with the tapes indicated by dotted lines; A A is the bed or frame, upon rollers attached to the inner side of which the ink and form table B B moves from end to end of the machine; immediately above the table are placed four drums, C, C, C, C, for pressing the sheets of paper which they receive from the rollers above them on the surface of the inked type. The printing drums are supplied with sheets of paper from six different places, D, D_1, D_2, &c., where a revolving bar or roller E descends at the proper time by the action of a cam upon the larger revolving roller D, taking hold of the

---

[*] May, 1848.

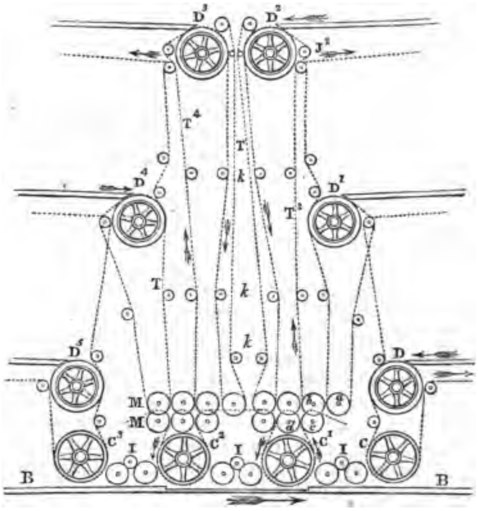
Fig. 29.



edge of a sheet placed there by the attendant, and pulling it between the tapes, which by their motion carry it through the machine.

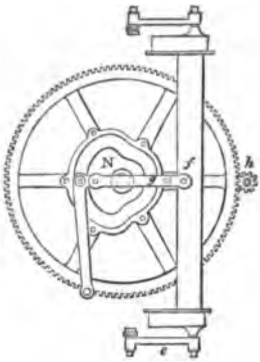
The two extreme printing drums C, C_3, and feeding drums D, are driven by two shafts, F, F, carrying bevelled wheels, the shafts themselves being driven by another smaller shaft G in connection with the shaft H, on which a fast and loose pulley is placed to hold a driving band. The two inner drums C_1 C_2 being driven by a rack on the vibrating table, the inking rollers I I are placed between the printing drums, and receive ink from a part of the moving table B, on which the ink is distributed by the small rollers K K. The supply of ink is effected in a manner similar to other printing machines; the ink is placed in a trough *t*, at the bottom of which a roller communicates a small quantity in its revolution to the roller underneath it, the latter roller being pushed down by an eccentric motion so as to touch the ink table every time it returns to the end of its travel. The alternate motion of the form and ink table is effected by a pinion and sliding apparatus similar to the common printing machine. These form the general features of the machine; its principal peculiarity consisting in the mechanism by which the two interior printing drums, C_1, C_2, are enabled to print a sheet both ways. In the common fast machine two only out of four drums printed each travel of the form table, the remaining two being raised; this was a necessary precaution, for it may be seen by *fig.* 29 that if a sheet entered at D^2, passing between the tapes to the drum C^1, round it, and up to the receiving box at J^2, when the form returns the printing drum is useless, as it can receive a sheet in one direction only; it is therefore raised up by a cam: this difficulty is completely obviated in Mr. Little's machine, with respect to the drums, by the use of two rows of small drums L L. The tapes from the feeding drums pass round the upper row, as is shown in the figure, and another set of tapes connect the lower row and the printing drums; the latter combination is thus independent of the former, and the sheets have to pass from one series of tapes to another before they receive an impression. This allows the upper row of drums to be shifted, so that the tapes T may direct the descending sheet of paper on the proper side of the drums. In the figure the form-table B is supposed to be moving in the direction indicated by the arrow, driving the lower part of the interior drum in the same direction; these drums will be fed with sheets from the drums D^4 and D^2, the printed sheet returning upwards by the tapes T^4 and T^2 respectively; but when the table returns the drums C^1 C^2 will be driven round their axes in the reverse direction, and they could not possibly be fed with sheets, as in the former movement. The peculiar motion by which this difficulty is obviated is a shifting of the upper row of drums M, the width of a drum to the left hand side of the diagram, *a* being brought over *c*, and *b* over *d*, by which motion the feeding drum D^2 is enabled to send a sheet to the printing drum C^1; the same occurs with the other drums. The traverse motion of the upper series of drums is effected by a lever L,

Fig. 30.



in connection with a horizontal cam arrangement underneath the machine; one end of the lever being attached to the slide holding the drums, and the lower end to a small rod *e* (*fig.* 31) connecting it with a bar *f* which carries the rod *g*, a pin on one end of this rod rests against the cam N on the axis of a spur-wheel driven by the pinion *h* in connection with the driving shaft H.

Fig. 31.



The two extreme drums C C^3 do not print both ways, as may be inferred from an inspection of the arrangements. In the motion indicated in *fig.* 30 the drum C^3 is lifted up by a cam worked by the axis *s* while the form passes, the first impression being taken by the drum C^2. Thus three print at each travel of the table. The small rollers *k k* are merely for the purpose of directing the tapes T, and keeping them tightly stretched; the lever N, the fulcrum of which is at the upper part of the machine, and the lower end attached to the slide of drums M, serves to carry the rollers *k k* with a traverse motion to the movement of the drums M. A machine of this description is stated by the inventor to be capable of supplying 6,210 sheets per hour.

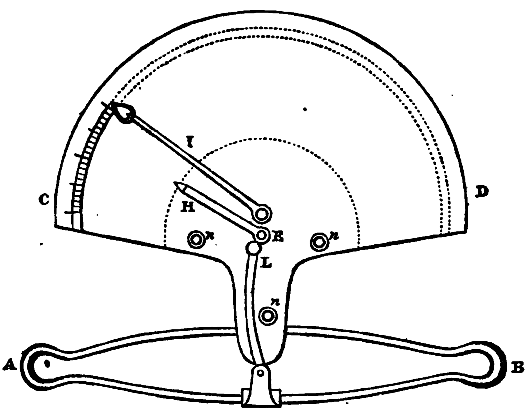
A printing machine has lately been erected in New York, which can supply 12,000 copies per hour, it is called "Hoe's fast press," and works on the old principle of putting the columns of types on a large moving drum, around which several smaller drums are placed for carrying the sheets to be printed. (October, 1848).

CHAPTER III.—*Dynamometers.* — *Regnier's.* — *The Friction Dynamometer.* — *Colladon's Dynamometer.*

A DYNAMOMETER is, generally speaking, an instrument for measuring the relative strength or power of prime movers of machinery, and is therefore a very useful instrument, as it enables us to determine the comparative powers of men, horses, and other animals, steam and water-wheels, as also the resistance of or power required to work, machines We shall here describe the most useful and interesting dynamometers which have come under our observation.

I. *Regnier's Dynamometer.*—M. Regnier, at the request of Buffon and Guéneau, who wished to make some experiments on the relative strength of men of different ages, in different states

Fig. 32.



of health, and of the different parts of the body, constructed a dynamometer very serviceable for such purposes*. It is represented in *fig.* 32, consisting of a steel spring A B, about twelve inches long, and of an elliptic form, to the upper branch of which is firmly fixed the scale C D, carrying a hand H, turning on the centre E, and moving by the action of the lever L; this lever it may be seen is attached at its other extremity to the lower branch of the spring. Above the hand H is an index I, which points to the division on the scale; it moves somewhat stiffly over the face of the scale, so as to prevent its shifting from the point to which it is driven, or in fact to make it self-registering. The index is moved by a pin on the under side of the hand H. When the two sides of the spring are brought towards each other by either pressing them with the hands, or pulling at each end A B, the rod L L will push the hand H round its centre and move the index I, which will indicate on the scale the number of pounds' pressure exerted. This is a very good instrument, although it may be made of a more simple form; from its portability (the original being about two-and-a-half pounds' weight only) it is a very useful dynamometer for measuring the strength of men and animals.

The Friction Dynamometer.—The elegant instrument known by this name, or the brake, is the invention of M. Prony, who, by considering that if the friction necessary to use the whole power of any engine or prime mover could be measured, the power of that engine would be determined, constructed a dynamometer in which friction is made to use up the whole power or available energy of a moving power, and then measured according to the laws of friction. The following diagram exhibits the form of this dynamometer, as used by M. Morin in his researches on the power of water wheels. C C is a collar of iron 30.5 inches internal diameter (which was sufficiently large to allow it to be put on the larger shafts), 1.2 inch thick in the middle C C (*fig.* 33B), and 6.3 inches broad, having a flange (shown in second figure) to prevent the friction band, which moves upon it, from slipping off; the outer surface is truly turned, in order that it may be fixed concentrically with the shaft on which it is placed. This attachment of the collar to the shaft of the engine under experiment is effected by six screws, *b b b*, having a thread cut for about ten inches of their length. When the collar is placed upon the shaft, its concentricity may be ensured by the screws, which take hold of the shaft, great care is taken with this operation, as it is a very important part of the experiment. When the shaft is of iron, and small, the collar must be mounted upon a tube of requisite size. As the screws are liable to be bent on account of their length, the collar is also wedged upon the shaft by pairs of wedges. Upon the collar is placed an articulated strap or band S S S, consisting of eight separate plates of iron, one-fifth of an inch thick and 3.94 inches broad, jointed by pins of from one quarter to one-fifth of an inch in diameter; the curve allowed for this chain is somewhat greater than that of the collar, so that the angles formed at the joints of the plates hold the grease which is supplied to the collar and band, or any extraneous matter which may come between the collar and band. At the termination of this chain two stout half links are joined to the bolts

Fig. 33 A.

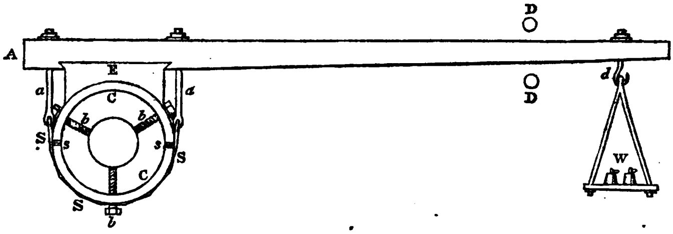
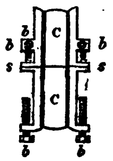


Fig. 33 B.



---

* Journal de l'Ecole Polytechnique.

a a, which are 23.6 inches long and 1.2 in diameter. This description of chain is preferred to a band of sheet iron, as the pressure is more equal on the surface of the collar. The bolts *a a* are screwed perpendicularly through the lever A B, which is a piece of pine 7.9 inches square at the end A, and tapering to 5.2 inches at the end B. Between this lever and the upper part of the collar is placed a cushion E made of hard wood, which presses upon the collar; a small hole being made through the cushion and lever to allow of the collar being lubricated. At the end B of the lever a scale-pan W is attached by the hook *d*, to hold the weights used in experiment to measure the friction.

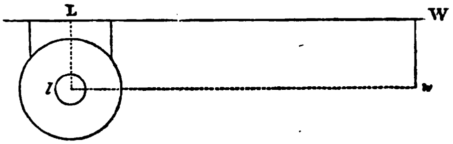
In using this apparatus care must be taken that the machinery to be tried is in proper order, that the revolving parts work evenly. The collar and band are then adjusted as is directed above, the lever being in a horizontal position between two cross-bars D D, which thus prevents it from falling or being carried round when the axle of the machine turns, which it would have a tendency to do; weights are then placed in the scale until an equilibrium is obtained, when the lever will assume a horizontal position, oscillating a little when any small variation occurs in the intensity of the force exerted by the machine. The number of revolutions per minute made by the shaft or spindle (or the collar of the brake) is counted by the experimenter, to find whether the inertia of the masses in motion exerts any influence on the results; if the number of revolutions be constant the motion is uniform, and the power of inertia is nothing*.

The most effective velocity of a machine may be experimentally determined by this instrument, by varying the load upon it from 0 to that which will stop the motion of the machine.

There is a limit to the capability of a dynamometer of the size and proportions we have stated, for M. Morin found that, when well lubricated, there was an abrasion of the surfaces of the rubbing parts when the friction amounted to from 1 to 1.2 tons. Where a power is to be tried which would require more friction than is measured by one ton, the collar must be correspondingly increased in diameter, which would of course give the weight more leverage.

The calculation of the power given by a machine, as obtained by the use of this instrument, is very simple, for suppose the collar were fixed upon the shaft of a steam engine revolving 100 times per minute, the weight required to equilibrate it being 150 lbs., and the length of the lever 10 feet. The weight or force resisted by the machine is the same as would be raised by a wheel having a radius equal in length to the lever or L W (*fig.* 34), so that the number of revolutions of the axis *l*, multiplied by the length L W of the

Fig. 34.



---

* When the lever is in equilibrium, or when the weights cause the lever to be suspended in a horizontal position, the friction of the band and cushion consumes all the available power. If then we call

P, the available power at any distance

R, from the axis;

v = velocity of the circumference of the shaft;

F = friction at the surface of the collar;

r = radius of the surface;

We shall have, the whole being in equilibrium,

$$P R = F r, \text{ or } P v = F \frac{r}{R} v.$$

But in experiment a weight (W) is acting with a leverage l (or the length of the brake) to produce this equilibrating friction; whence

$$W l = F r, \text{ and therefore } P v = W \frac{l}{R} v;$$

and as $\frac{l}{R} v$ expresses the line which would be described by the point at which the weight is suspended if it were given, we obtain the following rule:—

The available power of the machine is found by taking the product of the weight on the lever, and the path which the point of suspension of the weight tends to describe.

lever by the ratio of a circle to its diameter (or 6.2832), and the weight at the end W of the lever will give the working power of the machine in pounds raised one foot per minute. Thus, in the example we have taken, the power is—

$$100 \times 10 \times 6.2832 \times 150 = 1{,}042{,}480 \text{ lbs.}$$

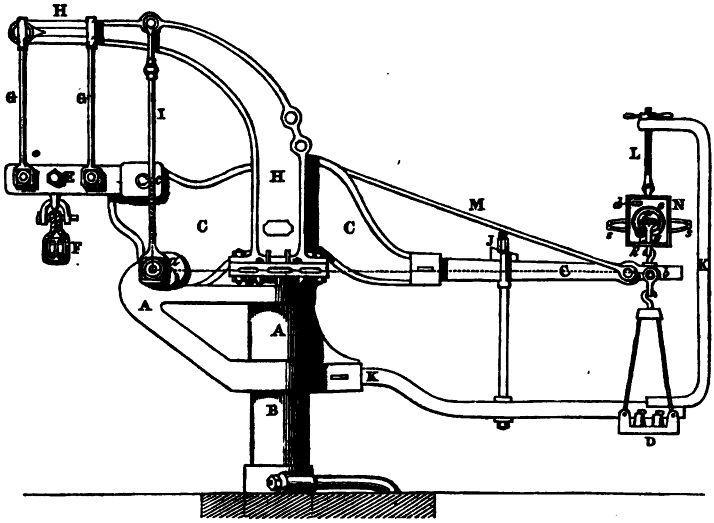
raised one foot high per minute; this may be converted into horses' power by dividing it by 33,000 lbs., which is the common estimate of the power of a horse in raising a weight one foot high in a minute. In this way the power of the steam-engine is about equal to 31½ horses.

This effective and generally applicable dynamometer has been variously modified by different observers; it is, or may be made for the experiments usually required, very portable, inexpensive, and a most useful piece of apparatus in practical mechanics.

The dynamometer of M. Morin weighed 500 lbs., but so great a mass of material is not required. It is said that the whole apparatus (not including the weights used) may be made of not more than 30 lbs. weight for trying engines exerting a working force of not more than twenty horses' power, and at an expense of about twenty-five shillings.

3. *Colladon's Dynamometer.*—The instrument which we have thus named is an application of the lever lately made by M. Colladon, of Geneva, for the purpose of measuring the power of steam-vessels. One of these instruments has been erected at the Royal Dockyard, Woolwich, which is of sufficient strength to experiment with steam-vessels having engines of 1000 horse-power*. It consists of a strong cast-iron frame A A, supported by a foot B, on which it is capable of turning horizontally, and which is firmly fixed to a stone flooring; upon this frame rests, by a knife-edge, the bent lever C C, with its fulcrum at *a;* one end of the lever holds a scale D by a knife-edge *b*, while to the other is supported a pair of strong plates E, carrying between them the pulleys F, to which ropes from the steam-vessel under experiment are attached. This acts on the shorter arm of the lever by a knife-edge *c;* the effective leverage is therefore shown by the dotted line, *c* and *b* being the extremities, and *a* the fulcrum; in the Woolwich instrument *a b* is to *a c* as 5 to 1 nearly, the lengths being respectively 8 feet 11 inches, and 1 foot 9 inches. A parallel motion G G prevents the piece E from moving out of the horizonal position when the ropes are being pulled

Fig. 35.



---

* Through the kindness of the Lords Commissioners of the Admiralty, we are enabled to give a sketch of this dynamometer.

by the steam-vessel. The sustaining frame H is screwed to the frame A, and also helps to support by the rod I the lever C, the knife-edge of which at *a* is terminated by a small oval plate, one end of which may be seen in the diagram; this carries a knife-edge, which rests upon the stirrup of the suspending rod I, so that the two knife-edges are in a vertical line. To prevent too great a play of the scale end of the lever, there is a stirrup and wedge J, by which contrivance the arm is allowed in experimenting to vibrate but a few tenths of an inch. The stirrup-rod is fixed to an arm K, reaching from the frame A to a short distance beyond the arm of the lever, and, turning upwards, terminates in an horizontal position, and carries a screw L to support a separate dynamometer for measuring the extra weight of the long arm of the lever, which is several hundred weight greater than that of the shorter arm, and for obtaining a mean of the oscillations of the lever during a trial. The rod M is for strengthening the long arm of the lever, and preventing the possibility of a flexure.

The additional dynamometer N is composed of two parts, not peculiar to this instrument,—the spring *s s* which, in bending or contracting, turns a small axis carrying the hand which points to the scale *e;* and the disc *f* turned by clockwork, turning by simple friction a small horizontal wheel placed before it, whose axis works in a bearing *g* attached behind the disc to one side of the spring.

The hand which revolves before the scale *e*, as the spring moves, passes over a series of divisions expressing pounds' weight, the scale being marked from 0 to 1200 lbs.

The small horizontal friction wheel, supported by the frame *g*, being in connection with the lever, will of course descend as that bends, from its position in which it touches the centre of the disc *f*, where it is unmoved although the disc may be turning by the action of the clock-work; in proportion as the wheel is drawn downwards, it will be turned more rapidly by friction with the disc*; it is in connection with a small clock-scale *h* for showing how many turns it has made during any time of revolution, and a scale upon its own circumference indicates the parts of a turn; both circles being divided into a hundred parts—a decimal division, which should be always made, where possible, in any measuring instrument. By a spring catch the friction wheel can be raised at pleasure from the disc.

A small portion of a clock-face *d* is placed at one corner of the box, to show how long the disc *f* has been revolving since the commencement of the experiment.

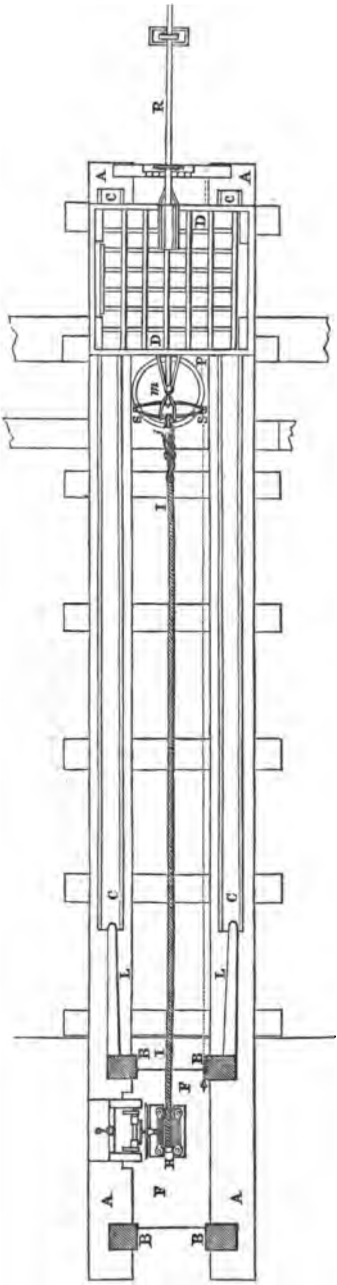
In making an experiment with this machine, the vessel to be tried is connected by strong ropes to the block pulleys F, when the working parts of the vessel are ready; weights are provided in the dynamometer house, and an indicator in the vessel, to show the effective force of the engines during the time of the trial. When the dynamometer ropes are pulled tightly by the vessel, the whole power is put on, and the scale of the dynamometer loaded with weights until the arm of the lever is rather more than balanced; the screw L supporting the box dynamometer is then worked by its handles so as to raise the box, and cause the spring *s s* to help in supporting the lever arm; this operation is continued until the longer arm rises from its bearing on the stirrup J and slightly oscillates, the wedge having been pushed backwards to allow requisite space; the hand indicating the pressure on the spring also oscillates, and the small friction wheel is pulled downwards from the centre of the face wheel which it touches, partaking of the oscillating motion; the clock-work having been set going, an observation is made for two or three minutes of the dynamometer on shore and the indicator in the vessel simultaneously, when the machine is stopped, and the number of divisions through which the little hand on the dial is connected with the friction wheel will show the revolutions made by the wheel during the two or three minutes; as the disc *f* turns once in a minute, we can find the mean distance at which the friction wheel should have been from the centre to revolve the observed number of times; then it can be readily found, by means of the hand pointing on the scale *e*, the pressure required on the spring to pull the friction wheel to the mean distance found; and thus the mean tension on the spring is obtained in pounds' weight.

By this instrument, it must be understood, the power exerted by the paddles with a known expenditure of steam-power is ascertained for a vessel in still water and not in motion; it indicates no inferior or superior steaming qualities in the vessel tried. If the power of two steam-vessels were thus registered and found to be equal, it would not follow that the vessels would be equally available for service with respect to their locomotive power. Again, the power measured by this dynamometer must not be supposed to exhibit the force exerted by the paddles when the vessel is in motion, for the paddles in the experiment work at one spot and create a current, the direction of which is the same as that of the revolving paddles; when one paddle-board strikes the water, it pushes the fluid before it, and would create a space behind it filled with air but for the surrounding water, which rushes after the paddle-board; the descending paddle-board, therefore, strikes the fluid which is moving in the same direction; and the velocity with which the water moves is a quantity to be subtracted from that of the paddle-board, to find the true effect in water at rest. With vessels in motion this does not occur to so great an extent; owing to the inertia of the fluid, the vessel is propelled by the action of a paddle-board, and the next meets the water in advance of the former paddle-board; the water in this case is not obliged to rapidly travel after the paddle-board, and would not do so at all but for the unavoidable *slip* of the paddle.

---

* See page 66 of the present treatise.

Fig. 1.



CHAPTER IV.

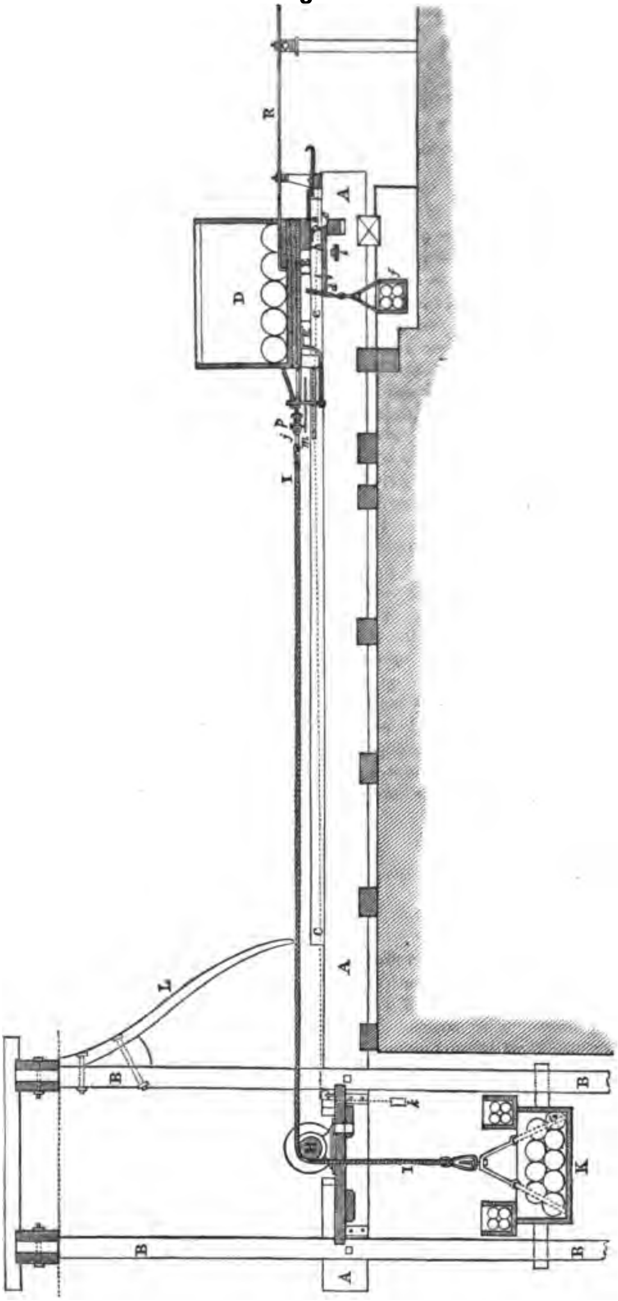
Friction.

In the course of our remarks on motion, we have necessarily had to mention the powerfully retarding influence of friction on a moving body. The subject is one of very great practical importance, as no problem in practical mechanics, relating either to locomotive bodies or the equilibrium of any combination of solid materials, can be solved without taking into account the power of friction. A treatise upon this subject has already appeared*, in which the investigations of Coulomb and Vince are detailed at some length; since the time when these philosophers conducted their experiments, the same inquiries have been entered into by G. Rennie, N. Wood, and Arthur Morin, the last experimenter, with the view

---

* Mechanics, Third Treatise.

Fig. 2.



of confirming the results announced by Coulomb, which had been questioned by several philosophers, especially Professor Vince, who, it will be seen in

a preceding treatise*, obtained results different from those stated by Coulomb, more in consequence of the view which he took of the subject than from the actual discordance of his experiments with those of Coulomb.

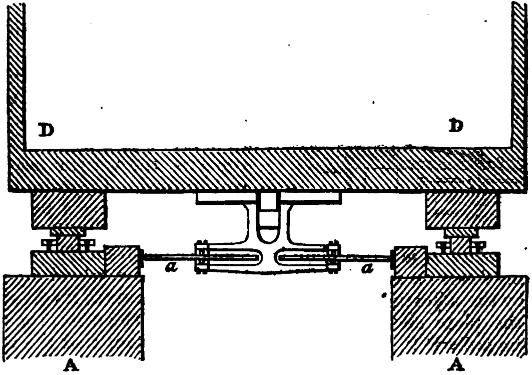
The experiments of M. Morin and Mr. G. Rennie were made with respect to the pressure, relative velocity of motion, and extent of the surfaces in contact, both with and without unguents.

We shall first describe the beautiful arrangements which M. Morin made for his experiments, and then state his results, with those of Mr. G. Rennie.

The apparatus used by M. Morin is one capable of giving very accurate results, and in its general principles most useful for all researches respecting motion and forces; it may be justly termed self-acting, as the results of any trial are registered by itself. The following diagrams will explain the construction, with the subsequent description†.

A A A A, are beds of oak, about a foot square in section, fixed in a truly horizontal position, at a distance from each other of 2½ feet, which forms a bench for the sliding apparatus; upon these beds the pieces C C are temporarily fixed, holding the body whose friction is to be tried, some of which is placed on the under side E E of the sliding box D D. In experimenting, bullets were used as the moving power. A table F F is fixed between the four uprights B B, to support the pully H, over which the cord I passes. This cord is attached at one end to the sliding box, and passing over the pully at the other end to the box K, in which weights are placed to drag the sliding box along the bench. Beneath the sliding box D, and attached to the bench, is placed a small bent lever *d c e*, one end of which presses upon the iron heel *h*, fixed to the under side of the box D, the end *d* carrying a box of weights *f*; this was used as an additional force to give the starting motion to the box D. It acts so long as the end *e* of the lever is in contact with the heel *h*, or until the arm *d c* comes in contact with the stop *i*; while the arm *e* pushes against the heel *h*, the force will be uniform, as that part of the arm which touches the heel is curved, and thus preserves the point of contact at an equal vertical distance from the plane of the fulcrum *c*; also the weights in the box *f* are suspended from an arc of a circle whose radius is equal to *d c*, and by that means act with a uniform force whether the lever is in a horizontal or inclined position. In some of the experiments, this lever was dispensed with, a slight push being sufficient to give an initial velocity to the sliding box. To prevent any lateral motion of this box when an experiment was made, a sort of tail or long rod R (*figs.* 1 and 2), was fixed behind it, which ran between rollers; this, however, was found insufficient when experimenting with unguents; and the superior contrivance shown in *fig.* 3 was adopted: A A are the oak beds seen in section, D being the sliding box, under which were fixed two pairs of wheels *a, a*; these were adjusted so as to allow a play of about an inch between them and the sides of the bed. These wheels, rotating about very small axles, were found to produce no noticeable friction during the experiments.

Fig. 3.

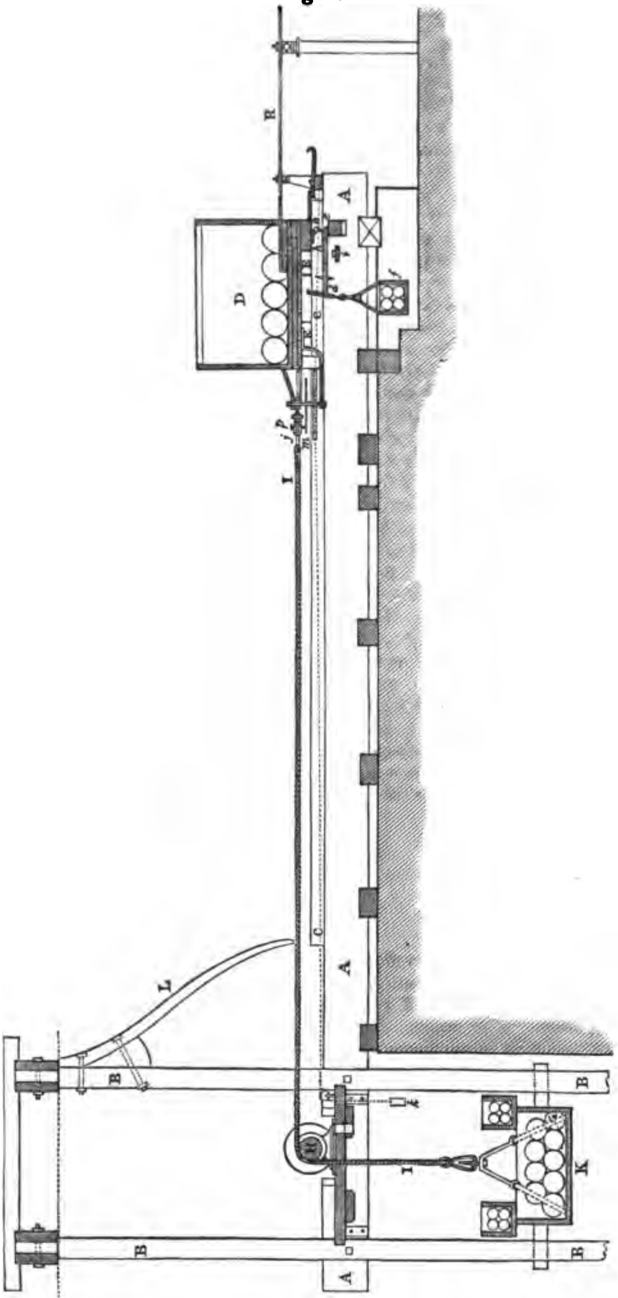


---

* Mechanics, Third Treatise, art. 11, *et seq.*

† Mémoires de l'Institut, 1833.

Fig. 2.



of confirming the results announced by Coulomb, which had been questioned by several philosophers, especially Professor Vince, who, it will be seen in

a preceding treatise*, obtained results different from those stated by Coulomb, more in consequence of the view which he took of the subject than from the actual discordance of his experiments with those of Coulomb.

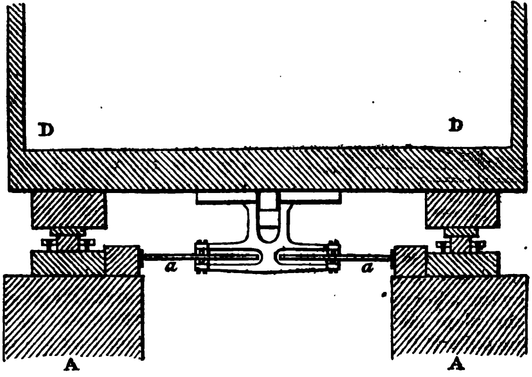
The experiments of M. Morin and Mr. G. Rennie were made with respect to the pressure, relative velocity of motion, and extent of the surfaces in contact, both with and without unguents.

We shall first describe the beautiful arrangements which M. Morin made for his experiments, and then state his results, with those of Mr. G. Rennie.

The apparatus used by M. Morin is one capable of giving very accurate results, and in its general principles most useful for all researches respecting motion and forces; it may be justly termed self-acting, as the results of any trial are registered by itself. The following diagrams will explain the construction, with the subsequent description†.

A A A A, are beds of oak, about a foot square in section, fixed in a truly horizontal position, at a distance from each other of 2½ feet, which forms a bench for the sliding apparatus; upon these beds the pieces C C are temporarily fixed, holding the body whose friction is to be tried, some of which is placed on the under side E E of the sliding box D D. In experimenting, bullets were used as the moving power. A table F F is fixed between the four uprights B B, to support the pully H, over which the cord I passes. This cord is attached at one end to the sliding box, and passing over the pully at the other end to the box K, in which weights are placed to drag the sliding box along the bench. Beneath the sliding box D, and attached to the bench, is placed a small bent lever *d c e*, one end of which presses upon the iron heel *h*, fixed to the under side of the box D, the end *d* carrying a box of weights *f*; this was used as an additional force to give the starting motion to the box D. It acts so long as the end *e* of the lever is in contact with the heel *h*, or until the arm *d c* comes in contact with the stop *i*; while the arm *e* pushes against the heel *h*, the force will be uniform, as that part of the arm which touches the heel is curved, and thus preserves the point of contact at an equal vertical distance from the plane of the fulcrum *c*; also the weights in the box *f* are suspended from an arc of a circle whose radius is equal to *d c*, and by that means act with a uniform force whether the lever is in a horizontal or inclined position. In some of the experiments, this lever was dispensed with, a slight push being sufficient to give an initial velocity to the sliding box. To prevent any lateral motion of this box when an experiment was made, a sort of tail or long rod R (*figs.* 1 and 2), was fixed behind it, which ran between rollers; this, however, was found insufficient when experimenting with unguents; and the superior contrivance shown in *fig.* 3 was adopted: A A are the oak beds seen in section, D being the sliding box, under which were fixed two pairs of wheels *a*, *a*; these were adjusted so as to allow a play of about an inch between them and the sides of the bed. These wheels, rotating about very small axles, were found to produce no noticeable friction during the experiments.

Fig. 3.



---

* Mechanics, Third Treatise, art. 11, *et seq.*

† Mémoires de l'Institut, 1833.

The box is stopped without concussion by two springs L L (*fig.* 2), which are fixed by bolts to the uprights.

These arrangements form the common part of the apparatus; but it was required to measure the effort exercised by the descending weight during the motion, and also to note the spaces run during a certain time of motion. It is this part of the machinery which possesses so much merit on account of its completeness and elegance. The dynamometer, or instrument which measured the force acting on the moving box, will be understood with the aid of *figs.* 4 and 5, in which S S, *s s*, represent a compound spring of cast steel, composed of two branches, that nearest the box D being held by a fork fixed to the box, and the other branch S S by a hook *j* to the cord I; by calculation, the greatest thickness of each piece of steel required in order that it might bend to the extent of three feet with a pressure of 205.5 lb., was found to be nearly two-tenths of an inch. The ring *r*, by which the weights affect the spring, is fixed in the centre of the branch by screws; attached to it is a small tube, holding a pen *p*, the point of which, as may be seen in *figs.* 5, 6, rests upon the plate *m m;* this plate is circular, moving freely on an axis, which is supported by the adjusting screws *c c*. Underneath the plate and on the same axis is fixed the pully P P, with a thread *t* passing once round it; one end of the thread is attached to the right-hand extremity of the beds, and the other end is stretched by the small weight *k* (*fig.* 2): the thread is thus prevented from slipping over the pulley, when the latter is carried with the sliding-box along the bed. *Fig.* 6 exhibits in a more distinct manner the important part of this apparatus. By this contrivance the circular plate *m* is made to revolve. A sheet of paper is fixed upon this plate to allow the pen *p*, which is previously supplied with Chinese ink, to draw a line upon it: if the spring were not stretched at all, the line described would be circular; but as the spring bends the pen will mark a corresponding deviation from the circular line, and the amount of this deviation is the measure of the flexion of the spring, or of the pulling force of the weights on the sliding box; again, as the plate revolves while the box is moving, the variation in the deviation of the line, or curve drawn from the circular, indicates the varying intensity in the action of the force on the spring. The curve thus drawn on the paper was denominated by M. Morin the curve of tensions.

Fig. 4.

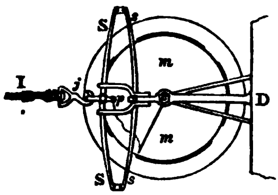


Fig. 5.

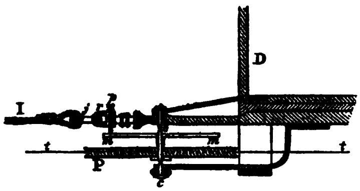
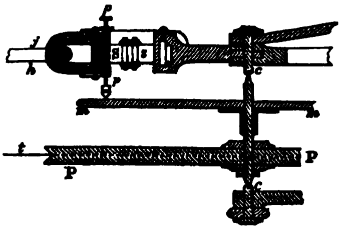
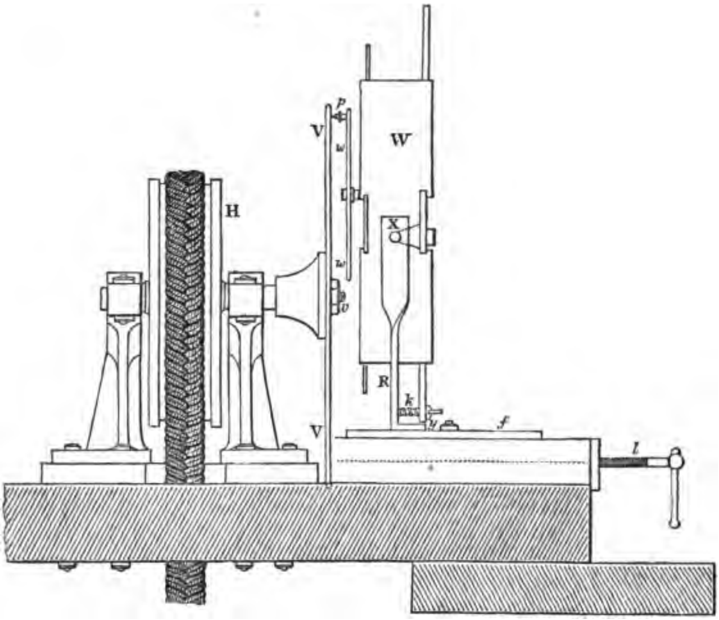


Fig. 6.



The other self-registering apparatus was to determine the spaces moved by the sliding box in regular intervals of time, or the law of movement. It was placed in connection with the pulley H (*fig.* 7). On the same axis with the pulley H is a smooth plate of metal V V, fixed firmly to the axis by a screw *v*, which pressed it tightly against the broad shoulder formed on the end of the axis, the vertical face of which was truly turned. Opposite to this plate the wheel *w*, carrying a pen *p*, turns on an axis which passes into the box W; a simple clock train is contained in the box, by which the wheel and consequently the pen is made to revolve with a uniform motion. In the use of the instrument it is most important that the wheel *w* be perfectly parallel to the plate V V; unless it is so, the pencil will not mark during the

Fig. 7.



whole time of its revolution. The box W is supported by an axis X on two posts R, standing upon their feet *y*, which are fixed to the sliding table *f* by screws; in the foot of one of these posts, the hole through which the tightening screw passes is lengthened into a slot or slit, so as to allow of the foot being pushed backwards or forwards for a horizontal adjustment of the clock case; the case has a vertical rotation upon its axis X, and the wheel *w* is adjusted truly perpendicular by the screw *k*, working through a fixed nut, and in the lower part of the box; a spring is also fixed to the nut, and presses against the box to insure the steady movement of the box when the screw is turned; the parallelism of the pencil wheel W and the plate V V being accurately effected by these arrangements, the table *f*, carrying the box and pencil wheel, can be moved towards the plate V V by the screw *l*, so that the pencil *p*, when moved, may make a mark on the paper.

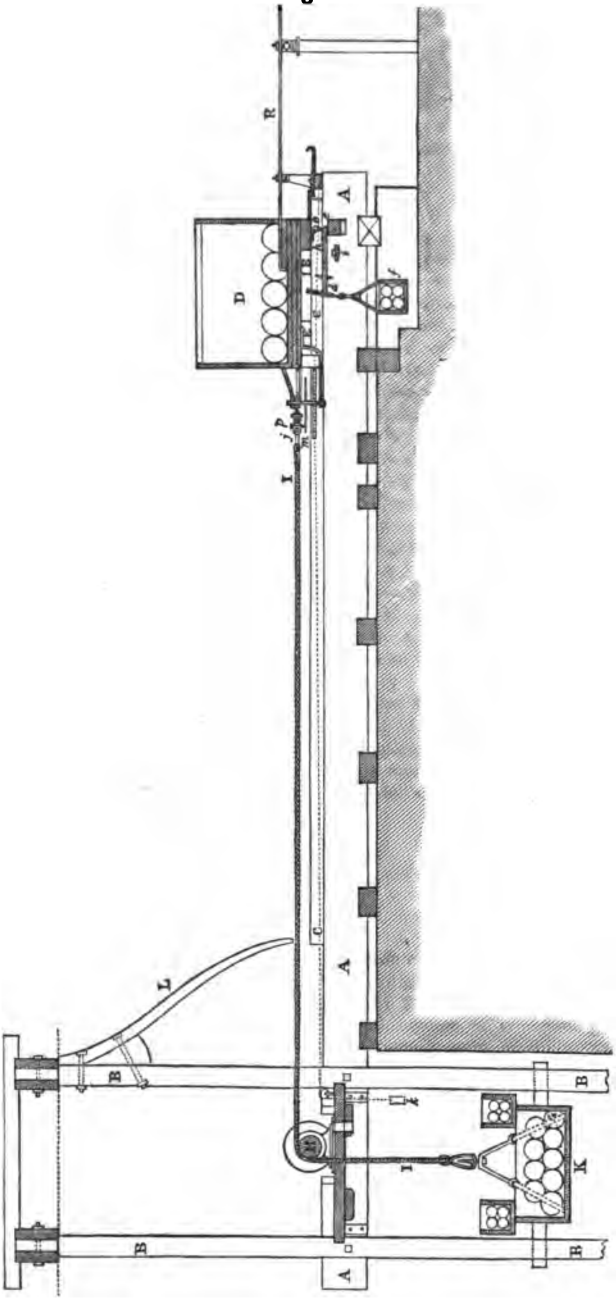
Each of these self-registering instruments draws curves, by which is shown, in the first, the tension of the spring during the motion of the box D; in the second, the time of motion, and the space moved through during each portion of that time,—that is, the rate of motion.

The following diagrams exhibit the manner in which the curves are drawn by the instruments in one of the experiments on the friction of oak upon oak.

Fig. 8 A is the curve of the dynamometer attached to the box D, the lines drawn from the centre of the circle to the curve traced representing geometrically the tension of the spring at those points. In the second *fig.* 8 B, the curve is set off, the base line A B, in geometrical language called the abscissa, representing the space described during the time the curve is drawn, and vertical lines (or ordinates) as C D, the tension of the spring.

In *fig.* 9 A is shown the curve drawn by the registering instrument for determining the rate of motion. The pen *p* in connection with the clock-work in the box W is allowed to describe a circle, which, in the experiment we have mentioned, was done in two seconds; and the circle thus drawn was divided into ten equal parts, and through these dotted circles were drawn, concentric

Fig. 2.



of confirming the results announced by Coulomb, which had been questioned by several philosophers, especially Professor Vince, who, it will be seen in

a preceding treatise*, obtained results different from those stated by Coulomb, more in consequence of the view which he took of the subject than from the actual discordance of his experiments with those of Coulomb.

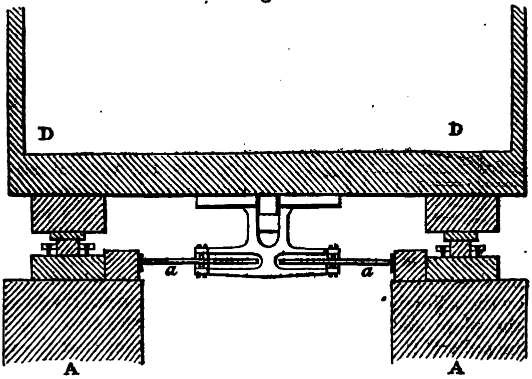
The experiments of M. Morin and Mr. G. Rennie were made with respect to the pressure, relative velocity of motion, and extent of the surfaces in contact, both with and without unguents.

We shall first describe the beautiful arrangements which M. Morin made for his experiments, and then state his results, with those of Mr. G. Rennie.

The apparatus used by M. Morin is one capable of giving very accurate results, and in its general principles most useful for all researches respecting motion and forces; it may be justly termed self-acting, as the results of any trial are registered by itself. The following diagrams will explain the construction, with the subsequent description†.

A A A A, are beds of oak, about a foot square in section, fixed in a truly horizontal position, at a distance from each other of 2½ feet, which forms a bench for the sliding apparatus; upon these beds the pieces C C are temporarily fixed, holding the body whose friction is to be tried, some of which is placed on the under side E E of the sliding box D D. In experimenting, bullets were used as the moving power. A table F F is fixed between the four uprights B B, to support the pully H, over which the cord I passes. This cord is attached at one end to the sliding box, and passing over the pully at the other end to the box K, in which weights are placed to drag the sliding box along the bench. Beneath the sliding box D, and attached to the bench, is placed a small bent lever *d c e*, one end of which presses upon the iron heel *h*, fixed to the under side of the box D, the end *d* carrying a box of weights *f*; this was used as an additional force to give the starting motion to the box D. It acts so long as the end *e* of the lever is in contact with the heel *h*, or until the arm *d c* comes in contact with the stop *i*; while the arm *e* pushes against the heel *h*, the force will be uniform, as that part of the arm which touches the heel is curved, and thus preserves the point of contact at an equal vertical distance from the plane of the fulcrum *c*; also the weights in the box *f* are suspended from an arc of a circle whose radius is equal to *d c*, and by that means act with a uniform force whether the lever is in a horizontal or inclined position. In some of the experiments, this lever was dispensed with, a slight push being sufficient to give an initial velocity to the sliding box. To prevent any lateral motion of this box when an experiment was made, a sort of tail or long rod R (*figs*. 1 and 2), was fixed behind it, which ran between rollers; this, however, was found insufficient when experimenting with unguents; and the superior contrivance shown in *fig*. 3 was adopted: A A are the oak beds seen in section, D being the sliding box, under which were fixed two pairs of wheels *a*, *a*; these were adjusted so as to allow a play of about an inch between them and the sides of the bed. These wheels, rotating about very small axles, were found to produce no noticeable friction during the experiments.

Fig. 3.



---

* Mechanics, Third Treatise, art. 11, *et seq.*

† Mémoires de l'Institut, 1833.

had been drawn through the extremities of the ordinates, a number of tangents were drawn to the curve, all meeting the tangent A T, and, on perpendiculars to them being drawn, it was found that they constantly cut one another in the point F, and demonstrated mechanically that the curve was a parabola.

With this complete and elegant apparatus M. Morin was enabled to experiment upon a great number of substances used in the arts, and deduce several laws relating to the friction of bodies in motion one upon another; in his experiments on the friction of oak moving upon oaks, for example, the results which he obtained from four trials were as follows:—

No. of Exper.	Surface in contact. square feet.	Pressure. lbs. av.	Friction. lbs. av.	Ratio of Friction to Pressure.	Velocity of motion in ft. per sec.
1	2.79	295.	141.1	.477	2.26
2	2.79	970.4	466.4	.480	1.34
3	0.94	120.5	56.9	.472	4.92
4	0.03	604.	72.9	.484	3.47

Comparing the figures thus given by the experiments, we may observe that there is little difference between the amounts in the fifth column, the mean of them being 0.478; this quantity is called the *coefficient* of friction, and, to find the friction of two pieces of oak moving one upon another under any pressure, we multiply the pressure by this coefficient. As this coefficient or ratio appears to have nearly the same value notwithstanding the variation in the pressures—from 120 lbs. to 970 lbs.—we may justly infer that the friction is proportional to the pressure; again, we find that the ratio has not altered although the surface used in the first trial was so much greater than in the last, whence it appears that extent of surface in action does not affect the amount of friction; lastly, on comparing the velocity column with the ratio, it exhibits no connection at all with the small variations in the coefficient of friction, and evidently does not affect it. From very many similar experiments, he determined—

1. That the friction is constant and proportional to the pressure.

2. That it is independent of the extent of the surface in contact.

3. That it is independent of the velocity of motion.

It will be seen that these laws were stated by Coulomb; but they were not entitled to so great confidence as in the present case, on account of the want of uniformity in the results which he obtained.

Other general laws of friction have been also fully established by the experiments of M. Morin and other philosophers. It has been found that the friction of moving bodies, and that of bodies at rest, are subject to the same laws, although the experiments which have been made with the view of determining the friction of quiescence do not so closely agree with the general laws.

Mr. Rennie carried on his experiments with a much greater insistent pressure than that used by M. Morin, indeed so far as to reach the limits of abrasion. In one experiment he tried a weight of 10 tons per inch on hardened steel, which abraded under so great a pressure. From a number of trials with different substances, such as leather, cloth, ice, wood, stones, and metals, he drew the following conclusions:—

1. With fibrous substances (such as cloth,) friction is increased by surface and time, and diminished by pressure and velocity.

2. With harder substances, such as woods, metals, and stones, and within the limits of abrasion, the amount of friction is as the pressure directly, without regard to surface, time, or velocity.

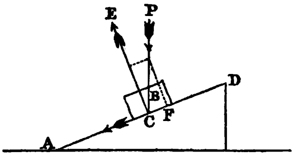
3. With dissimilar substances gliding against each other, the measure of friction will be determined by the limit of abrasion of the softer substance.

4. Friction is greatest with soft, and least with hard substances.

The friction of substances, when the rubbing surfaces are covered with an unguent, as tallow or oil, has been tried, and from the numerous experiments of M. Morin it appears that the friction in this case does not vary in amount according to the substance used, but to the nature of the unguent; it varies also with the quantity of the unguent between the rubbing surfaces; when hog-lard and olive-oil were interposed so as to form a very thin continuous stratum, between the surfaces of wood sliding on wood, metal on metal, and metal on wood, the proportion of the friction to the pressure is between 0.07 and 0.08. With tallow the same value for friction was obtained, except in the case of metals sliding on metals, when the mean friction was found to be 0.10; by which it appears that tallow is not so well adapted as an unguent for metals when sliding on or against metals as the two former substances.

A very useful consideration arises from the principles of friction, namely, the limiting angle of resistance, or the angle at which a body pressing on another will begin to slip. Suppose the body B (*fig.* 10), be placed on the inclined surface A D, it will press by its weight on the direction P C, while the plane resists in the direction C E; resolving the force P into two, one in the direction of the plane's resistance or C E, and the other parallel with the surface of the plane or F C, by the principle of the parallelogram of forces, the lines F C and E C will respectively represent the relative force in each of the directions A D, E C; now the body B will or will not slide on the plane according as the friction produced by the weight B is less or greater than the force F C, acting along the plane; the conditions upon which the equilibrium of the body D depend are therefore two—the magnitude of the angle E C P, and the friction of the substance, and, as the relative friction of the same substance will not vary when the angle E C P varies, it is obvious that there must be some angle at which the friction brought into action by the pressure E C will be less than the moving force F C; and the body will commence sliding; the angle at which the two forces become equal is called the limiting angle of resistance*.

Fig. 10.



The same general laws are found to apply in the friction of axles upon their bearings. This subject, of so much interest in machinery, was also investigated by M. Morin, with a suitable apparatus. From his experiments he concluded that for axles of iron and cast iron in cast-iron or brass bearings, when lubricated with oil, hog-lard, or tallow, the relation of the friction to the pressure is the same as in the following table:—

	$\frac{F}{P}$
When continually lubricated	0.054
Lubricated in the common way	0.070 to 0.080
A little unctuous, dry or wetted with water	0.140 to 0.160

These results are similar to those obtained in the experiments on plane surfaces sliding upon one another. When water was used to moisten axles which had been greased, the relation of the friction to the pressure was found to be more constant for a length of time, the water keeping the lubricant cool, and preventing its expulsion.

The following tables give the most useful results obtained by M. Morin, from his experiments on the friction of plane surfaces,—in the first series, when they have been some time in contact, and in the second series, where the surfaces are in motion one upon the other. In the experiments for the purpose of ascertaining the values given in the first series of tables, the same general laws were found to hold as in the friction of motion, although in some instances there was considerable disagreement.

---

* Let the force P C = P, the angle E C P $=\alpha$, and f the coefficient of friction, then the force in the direction E C is P. cos. α, and in the direction A D, P. sin. α, the limiting angle of resistance is obtained when

$$\text{P. sin. } \alpha = f \text{ P. cos. } \alpha;$$

if we divide this equation by P. cos. α, we reduce it to

$$\tan.\ \alpha = f,$$

or the tangent of the angle of resistance is equal to the coefficient friction of the substance; from this the angle may be readily found. We are indebted to the Rev. H. Moseley for bringing the subject into notice.

I.—Friction of Plane Surfaces, when they have been some time in contact.

Surfaces in contact.	Disposition of fibres.	Lubricant.	Relation of friction to pressure.	Limiting angle of resistance*.
Oak upon oak	Parallel	None	0.62	31° 48′
	Ditto	Rubbed with dry soap	0.44	23 45
	Perpendicular	None	0.54	28 22
	Ditto	Water	0.71	35 23
	Vertical upon horizontal	None	0.43	23 16
Oak upon elm	Parallel	Ditto	0.38	20 49
Elm upon oak	Ditto	Ditto	0.69	34 37
	Ditto	Rubbed with dry soap	0.41	22 18
	Perpendicular	None	0.57	29 41
Ash, fir, beech, and service-tree, upon oak	Parallel	None	0.53	27 56
Leather (tanned) upon oak	Leather flat	Ditto	0.61	31 23
	Sideways	Ditto	0.43	23 16
	Ditto	Wetted with water	0.79	38 19
Black dressed leather (for straps) Upon plane surface of oak	Parallel	None	0.74	36 30
Upon round surface of oak	Perpendicular	Ditto	0.47	25 11
Hemp matting upon oak	Parallel	Ditto	0.50	26 34
	Ditto	Wetted with water	0.87	41 2
Hemp cord upon oak	Ditto	None	0.80	38 40
Iron upon oak	Ditto	Ditto	0.62	31 48
	Ditto	Wetted with water	0.65	33 2
Brass upon oak	Ditto	Ditto	0.65	33 2
Copper upon oak	Ditto	None	0.62	31 48
Ox-hide upon cast iron (as for pistons)	Flat, or sideways	Wetted with water	0.62	31 48
	Ditto	Oil, tallow, or hog-lard	0.12	6 51
Black dressed (or strap) leather upon cast iron pulley	Flat	None	0.28	15 39
	Ditto	Wetted with water	0.38	20 49
Cast iron upon cast iron	...	None†	0.16	9 6
Iron upon cast iron	...	Ditto	0.19	10 46
Oak, elm, yoke elm, iron, cast iron, and brass, sliding in pairs upon each other	...	Tallow	0.10‡	5 43
		Oil or hog-lard	0.15§	8 32
Calcareous stone upon calcareous stone ‖	...	None	0.74	36 30
Hard calcareous stone (muschelkalk) upon calcareous stone	...	Ditto	0.75	36 52
Brick upon calcareous stone	...	Ditto	0.67	33 50
Oak upon ditto	Wood, endways	Ditto	0.63	32 13
Iron upon ditto	...	Ditto	0.49	26 7
Muschelkalk upon muschelkalk	...	Ditto	0.70	35 0
Calcareous stone upon muschelkalk	...	Ditto	0.75	36 52 33 50
Brick upon ditto	...	Ditto	0.67	22 47
Iron upon ditto	...	Ditto	0.42	32 38
Oak upon ditto	...	Ditto	0.64	36 30
Calcareous stone upon calcareous stone ¶.	...	With a layer of mortar (of three parts of fine sand, and one of slacked lime)	0.74	

---

* This column is taken from Mr. Moseley's work, before referred to.

† The surface retaining some unctuosity.

‡ When the contact has not continued long enough for the expression of the tallow.

§ When the contact has lasted a sufficient time for the lubricant to be expressed and bring the surfaces to an unctuous state.

‖ The stone used in these experiments was a soft stone from Jaumont, belonging to the great oolite formation, of a yellow colour, and tolerably homogeneous, weighing 135 lbs. per square foot.

¶ After a contact of from 10 to 15 minutes.

II.—FRICTION OF PLANE SURFACES, moving one upon another.

Surfaces in contact.	Disposition of fibres.	Lubricant.	Relation of friction to pressure.	Limiting angle of resistance.
Oak upon oak	Parallel	None	0.48	25° 39′
	Ditto	Rubbed with dry soap	0.16	9 6
	Perpendicular	None	0.34	18 47
	Ditto	Wetted with water	0.25	14 3
	Vertical upon horizontal	None	0.19	10 64
Elm upon oak	Parallel	Ditto	0.43	23° 17′
	Perpendicular	Ditto	0.45	24 14
	Parallel	Ditto	0.25	14 3
Ash, fir, beech, wild pear-tree, and service-tree, upon oak	Parallel	None	0.36 to 0.40	20° to 22′
Iron upon oak	Ditto	Ditto	0.62	31° 48
	Ditto	Wetted with water	0.26	14 35
	Ditto	Rubbed with dry soap	0.21	11 52
Cast iron upon oak	Ditto	None	0.49	26 7
	Ditto	Wetted with water	0.22	12 25
	Ditto	Rubbed with dry soap	0.19	10 46
Copper upon oak	Ditto	None	0.62	31 48
Iron upon elm	Ditto	Ditto	0.25	14 3
Cast iron upon elm	Ditto	Ditto	0.20	11 19
Black dressed leather upon oak	Ditto	Ditto	0.27	15 7
Tanned leather upon oak	Flat or sideways	Ditto	0.30 to 0.35	17° to 19°
	Ditto	Wetted with water	0.29	16° 11′
Tanned leather upon cast iron and brass	Ditto	None	0.56	29 15
	Ditto	Wetted with water	0.36	19 48
	Ditto	Unctuous, and wetted with water	0.23	12 58
	Ditto	Oil	0.15	8 32
Hemp in threads or in cord, upon oak	Parallel	None	0.52	27 29
	Perpendicular	Wetted with water	0.33	18 16
Oak and elm upon cast iron	Parallel	None	0.38	20 49
Wild pear-tree upon cast iron	Ditto	Ditto	0.44	23 45
Iron upon iron	Ditto	Ditto	0.44*	23 45
Iron upon cast iron and brass	Ditto	Ditto	0.18	10 13
Cast iron upon cast iron and brass	...	Ditto	0.15†	8 32
Brass upon brass	...	Ditto	0.20	11 19
upon cast iron	...	Ditto	0.22	12 25
upon iron	...	Ditto	0.16†	9 6

---

* The surfaces wear when there is no lubricant.

† The surfaces retaining a little unctuosity.

Surfaces in contact.	Disposition of fibres.	Lubricant.	Relation of friction to pressure.	Limiting angle of resistance.
Oak, elm, yoke elm, wild pear-tree, cast iron, iron, steel, and brass, sliding in pairs upon one another, or themselves	Ditto . .	In the common way, with tallow, hog-lard, oil, cart-grease, &c.	0.07 to 0.08*	4° to 4° 35′
		Slightly unctuous to the touch	0.15	8° 32′
Calcareous (oolitic) stone upon calcareous stone	...	None	0.64	32 37
Muschelkalk upon calcareous stone	...	Ditto	0.67	33 50
Common bricks upon calcareous stone	Ditto . .	None	0.65	33 2
Oak upon calcareous stone . .	Wood, endways	Ditto	0.38	20 49
Wrought iron upon calcareous stone	Parallel . .	Ditto	0.69	34 37
Muschelkalk upon muschelkalk	Ditto . .	Ditto	0.38	20 49
Calcareous stone upon muschelkalk	Ditto . .	Ditto	0.65	33 2
Common brick upon muschelkalk	Ditto . .	Ditto	0.60	30 58
Oak upon muschelkalk . . .	Wood, endways	Ditto	0.38	20 49
Iron upon muschelkalk . . .	Parallel . .	Ditto	0.24	13 20
	Ditto . .	Wetted with water	0.30	16 42

III.—Friction of Axles (or Gudgeons) in motion upon their bearings.

Nature of the surfaces in contact.	State of the surfaces.	Relation of friction to pressure when the lubricant is applied	
		In the usual way.	Continually.
Cast-iron axles upon cast-iron bearings	With olive-oil, hog-lard, or tallow	0.07 to 0.08	0.054
	With the same, and water . .	0.079	
	With asphaltum	0.054	
	Unctuous	0.137	
	Ditto, and with water. . . .	0.137	
	Very unctuous	0.073	
	Ditto, and with water . . .	0.073	
Cast-iron axles upon brass bearings	With olive-oil, hog-lard or tallow	0.07 to 0.08	0.054
	Soft cart-grease	0.065	
	Unctuous	0.166	
Cast-iron axles upon lignum-vitæ bearings	Without lubricant	0.185	
	With oil or tallow	...	0.092
	Unctuous (with oil)	0.100	
Iron axles upon cast-iron bearings	With olive-oil, hog-lard or tallow	0.07 to 0.08	0.054
Iron axles upon brass bearings	Ditto	0.07 to 0.08	0.054
	With cart-grease	0.090	
	Unctuous, and with water . .	0.189	
Iron axles upon lignum-vitæ bearings	With oil	0.114	
	With hog-lard	0.135	
Brass axles upon brass bearings	With oil	0.101	
	With tallow	0.093	
Brass axles upon cast-iron bearings	With oil	...	0.052
	With tallow	...	0.045

---

* When the lubricant is unceasingly renewed, and evenly distributed, this value can be reduced to 0.05.

THE "TIMES" PRINTING MACHINE.

Since the chapter of illustrations of machinery was in the press, we have had an opportunity of inspecting a printing machine of a new form, recently constructed for "The Times" newspaper, in which the pages of type are held in chases or iron beds of a polygonal form, which admits of their being placed on the periphery or circumference of a large iron cylinder, supported by framework in a vertical position, and surrounded by eight smaller iron cylinders, each furnished with woollen coverings in the usual manner, so that the type, in revolving with the cylinder, comes in contact with each of the impressing cylinders successively, and thus eight sheets of paper are printed during every revolution.

This vertical cylinder machine is the invention of Mr. Augustus Applegath, and was designed by him especially for the use of "The Times," the circulation of which had outstripped the powers of the four-cylinder machine, which he invented for the same journal in 1827. In page 91 of this treatise, Mr. Little's patent printing machine has been described, the drawing of which illustrates the general features of the old four-cylinder machine, in which two of the printing cylinders pressed on the type when it moved in one direction, and two when it returned, so that, although the type moved *twice* under the *four* cylinders, yet only *four* sheets received an impression, and it may be said that half of the motion was lost. In the vertical machine, the inventor has attained his object of making available the whole motion of the type, by exchanging the reciprocating horizontal for a continuous circular motion *.

In the following figures we have given an elevation, and plan of the machine, with an end view.

The type cylinder A is seen in the plan, *fig.* 1, with several of the printing cylinders, B, at the circumference, to each of which is attached the machinery for carrying the paper to the cylinder. In the elevation, *fig.* 2, the type cylinder is shown with two pages of type, which would be printing a sheet on the drum B′; the forms or beds are fixed by screws to the rings marked A A of the hollow type cylinder; beyond which is seen the great axis or spindle C of the cylinder.

This vertical position of the type involved, however, the necessity of an horizontal motion of the paper while in a vertical position. This is elegantly effected by the feeding apparatus, an end view of which is given in *fig.* 3. The sheet of paper, being laid on the feed-board, D, after the usual manner in news machines, it is caught by the tapes moving round *a*, at the proper time, and conveyed over the entering drum E, and downwards between the rollers *d d*; when it arrives at the proper place it is suddenly stopped by two pairs of flat pieces of wood, F F, *fig.* 1, called stoppers, the tapes round *d d* opening at the same time, so as to prevent them from rubbing against the sheet of paper, which is supported in its vertical position by the stoppers and two small rollers, *e*, *fig.* 3, on delicate springs. The stoppers instantly open, leaving the sheet suspended by the rollers only; the vertical revolving rollers, G G, then collapse, and give motion to the sheet between them, S, *fig.* 1, pushing it between two sets of horizontal tapes, H H, which carry it towards the impressing cylinders, B′, where it meets the inked type, and the printing is effected; leaving the impressing cylinder, the sheet is carried out by tapes in the direction of the dotted line, by two other sets of horizontal tapes, I I, as far as the pair of rollers, K, thence it is carried on by a single pair of tapes at the top of the sheet until it stops, and remains suspended between two pairs of small conical rollers, L L, until the "taker-off" lays it on his board. The direction of motion is indicated by the arrows.

The combination of mechanical movements to effect these objects is highly interesting. The shaft M, which drives the drum E, is in gear with the wheel N, on the shaft O, which carries the series of cam or eccentric wheels, *f*, *g*, *h*.

---

* This practice is more conformable to theory, for in a *continuous* (which must be curvilineal) motion the inertia of the masses in motion does not interfere prejudicially, but, on the contrary, preserves regularity in the movement; in the reciprocating motion, a great weight, moving at the average rate of 45 inches per second, had to be stopped suddenly when at the extremity of its traverse, producing a fearful strain on the parts in action, and a considerable loss of time and the power applied.

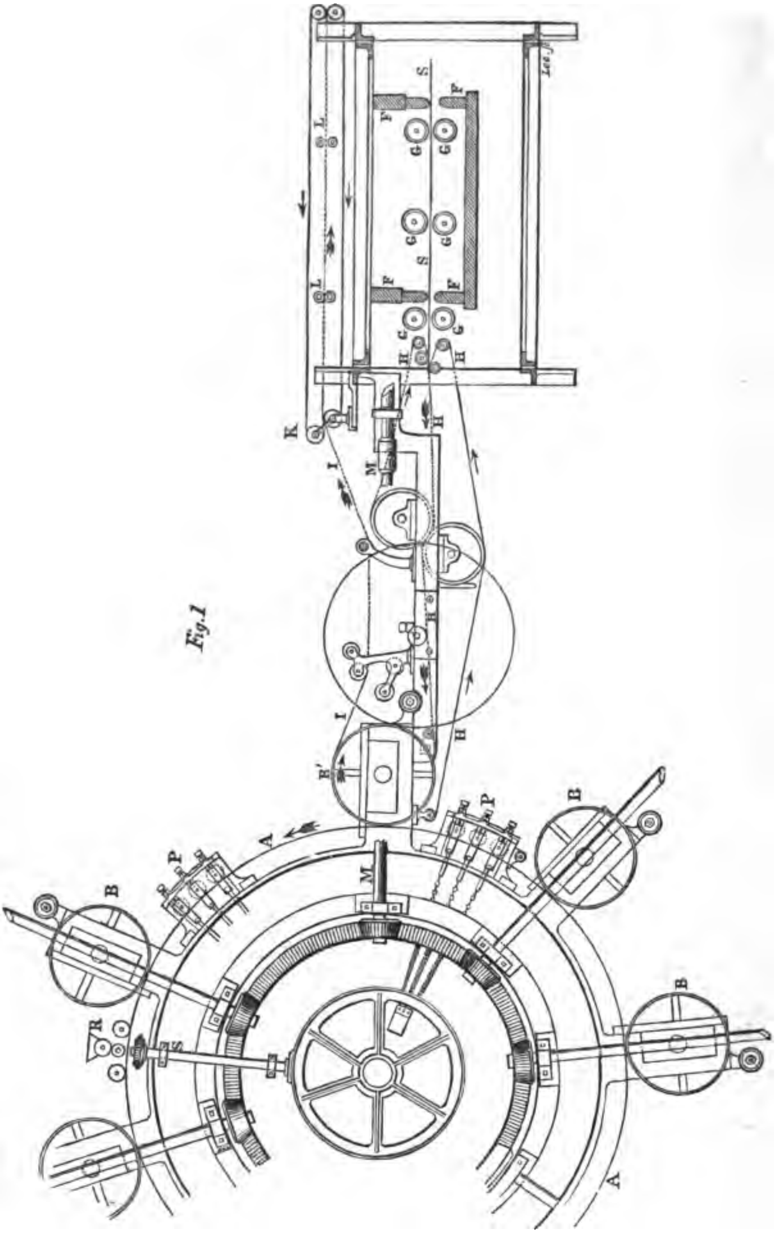


Fig. 1

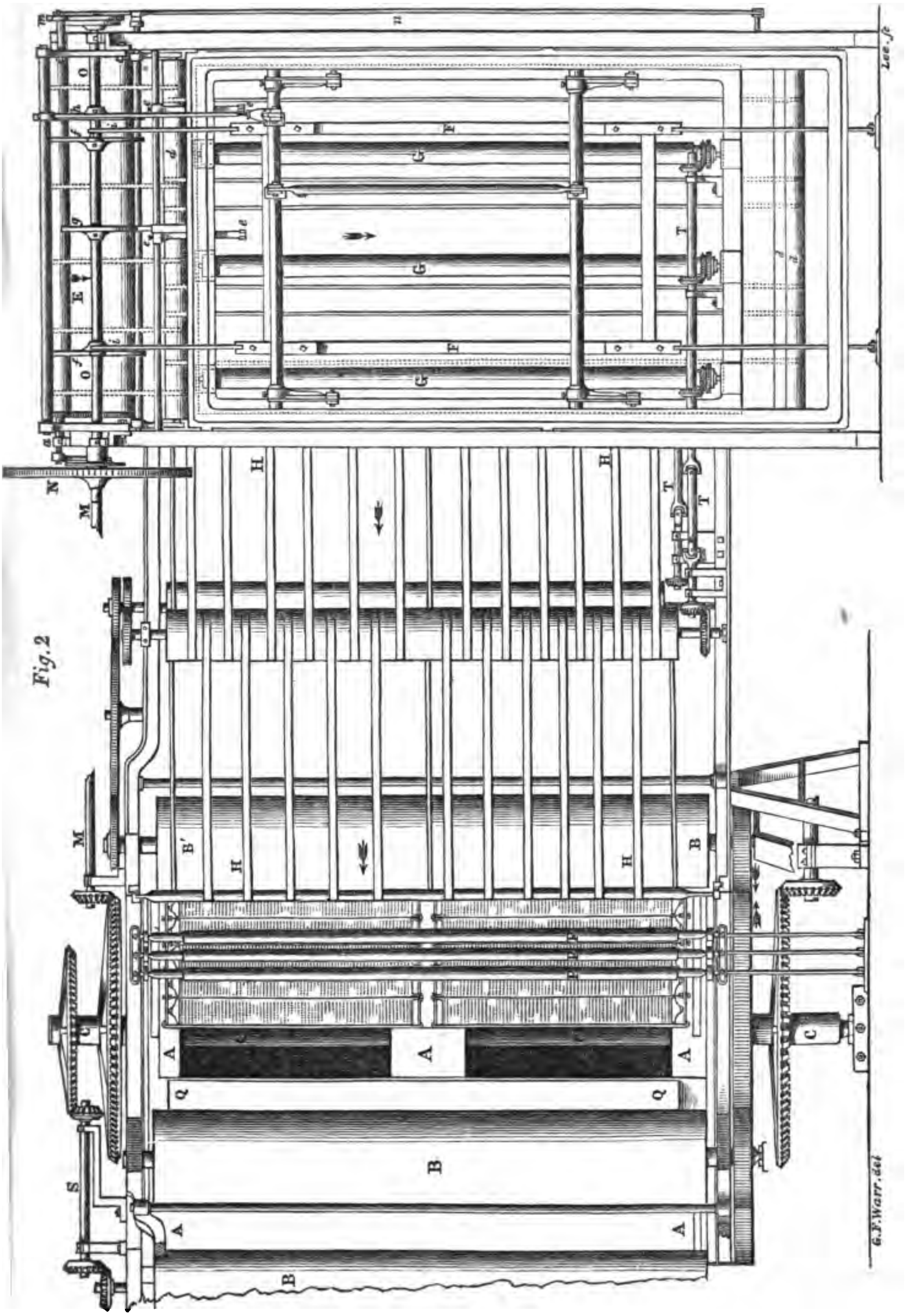
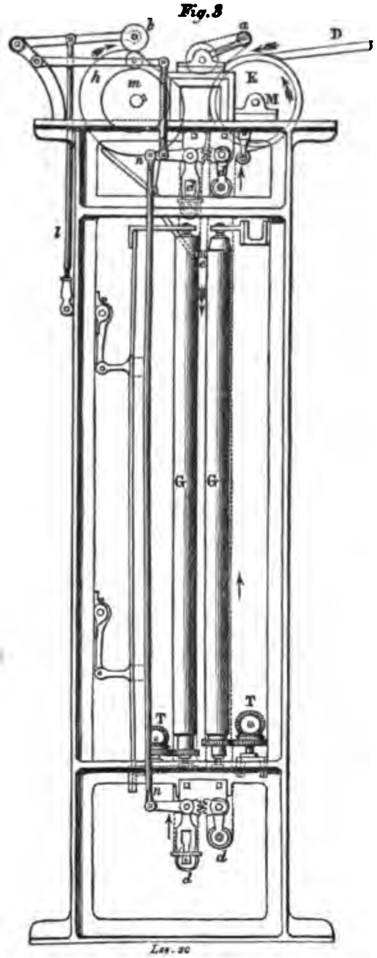


Fig. 2

Fig. 3

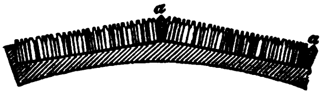


The first of these, *ff*, bear against rollers on the rods *i i*, and, in the course of a revolution, allow them to fall inward, carrying the stoppers F against the paper. The cam *g* acts on the spindle carrying the finger rollers *e*, and, in revolving, pushes them against the sheet of paper. The cam *h*, (seen also in *fig*. 3,) regulates the action of the vertical rollers, G G, at the proper time allowing the superincumbent pulley *b* to fall, pushing the rod *l* downwards, and by the bent levers forcing the left-hand rollers, G, inwards *. The cam *m* serves to raise the rod *n*, which, by bent levers, separates the rollers *d d*.

The rate of motion of the type cylinder is about 60 inches per second, so that an error in the arrival of the sheet of paper to the impressing cylinder of the sixtieth part of a second would cause an error of one inch in the margin, and damage the sheet; yet the simple means we have described perform these delicate operations with such accuracy, that the spoilage of stamped sheets is considerably less with this machine than the old form.

For inking the type there are three rollers, P, between each impressing cylinder; in the common machines the inking rollers bear upon the type and distributing tables by their own weight, but, as they must be in a vertical position in this machine, long coil springs are used (see *fig*. 1) to keep them in contact with the type and distributing table alternately. To distribute the ink on these rollers, a wood table (part of which is seen in *fig*. 2) is attached to the type cylinder, and receives an up and down motion from an undulating railway at the lower part of the cylinder, which affects the distribution of the ink, without the use of the diagonal rollers, in the usual form of printing machines (p. 89). The ink is supplied from a copper vessel at the upper part of the machine, it gradually moves down the box R, *fig*. 1, into one corner of which a roller intrudes; this roller is driven round by a shaft and bevel, S, and carries with it as much ink as the edge of the box will allow; this is communicated to a smaller roller, playing between the ink roller and the distributing table, as in the common machines. The vertical position of the inking rollers proves superior to the ordinary or horizontal arrangement, as the surface only of the letter is touched by the rollers, and the form of type is as clean after printing 38,000 copies as it was at the beginning.

The type cylinder is 64 inches in diameter, and the chases or forms have a bottom of a polylateral shape, so that a form of type is a portion of a polygon; and as the type is of the ordinary shape, the column rules, *a a*, are of a wedge



---

* These rollers are worked by bevel wheels on the spindles T.

form, fixed at one end of the chase by a pin, and drawn tightly by a screw at the other extremity; each column of the type also is individually secured by set screws, and the form thus prepared is found to be completely uninfluenced by the centrifugal force created in working the machine. It might be supposed, as a column of type is not a part of the circle, but a chord, that the impression at the sides of a column would be very strong, but in the middle very faint; but it will be found on calculation, that, owing to the great diameter of the cylinder, the difference between the chord and the arc in this particular, with a column of 2¾ inches in breadth, is $\frac{1}{48}$th of an inch, a quantity of little consequence in such a case; this is compensated for by four overlays of cartridge paper of various widths fixed on the impressing cylinders.

The produce of this machine, at the present time, is 9120 per hour, or 152 sheets every minute, while the reciprocating machines, when at their greatest speed, would not afford more than from 5000 to 6000 impressions per hour; the present machine is not driven at its maximum speed, the chief difficulty being the inability of the "taker-off" to arrange the sheets as rapidly as they are supplied to him by the machine.

March, 3d

X

EQUILIBRIUM OF STRUCTURES.

CHAPTER I.—*Action of Forces.—Relations of contending forces with respect to a body on which they act.—Stability of the Centre of Gravity.*

(1.) STATICS, considered with reference to artificial structures, is a subject of very deep inquiry, and, in order to develope fully the scientific principles regulating the stability of a combination of material parts in certain forms, a familiarity with the more general views of statical science is indispensable. It is our purpose in the following chapters to give an outline of the theory of equilibrium as applied to engineering and architecture, comprehending the operations of framing, roofing, the erection of domes, bridges, centres, and arches generally; other structures, as walls, piers, columns, and lighthouses, are also the subjects of statical investigation, which points out the conditions necessary to be observed in the combination for the maintenance of its form and position, while subjected to the action of forces in directions, and with powers which are indicated by the circumstances of place, material, and form; or, where the probable forces are first considered, the science of equilibrium demonstrates the form of the structure best adapted to sustain the given forces. The practical calculations resulting from such theoretical investigations introduce the consideration of the strength of the materials used, of wood, stone, or metals; the proper form of joints for advantageously fitting the parts together; of cements, and other means of uniting joints; and the artificial methods of strengthening the materials.

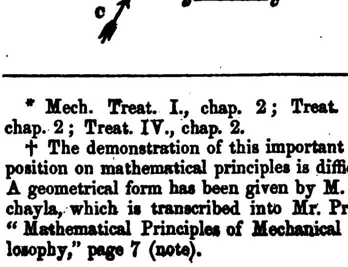
(2.) The forces which may act on the parts of a structure have a tendency to give one or more of them a motion either of translation or rotation. The result of forces acting in one plane, but not in the same nor the opposite direction, have been determined in former parts of these treatises *, by reference to a simple experiment. It has thus been shown that two forces may be sustained by a third force—a resultant—all acting in the same plane, the forces being represented by two sides and the diagonal of a parallelogram, drawn so as to give the relative direction and quantity of the forces †; also, that where a number of unequilibrating forces act on a point in one plane, the lines representing them in magnitude and direction may be formed into a polygon with one side wanting, and that a force applied to the point in the direction of this side, and proportional to it, will produce equilibrium, and the point remain at rest.

(3.) The former of these propositions may be more conveniently stated thus:—

When a body or system of bodies acted on by three forces is in equilibrium, the forces are proportional to the sides of a triangle having their directions.

Thus, in the case of the three forces, A, B, C (*fig.* 1), we may draw a line

Fig. 1.



---

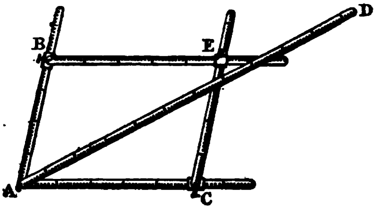
* Mech. Treat. I., chap. 2; Treat. II., chap. 2; Treat. IV., chap. 2.

† The demonstration of this important proposition on mathematical principles is difficult. A geometrical form has been given by M. Duchayla, which is transcribed into Mr. Pratt's "Mathematical Principles of Mechanical Philosophy," page 7 (note).

b c, in the direction of the force B, and proportional to it, also the line *c a* in a similar manner to express the force C, the line *a b*, in the direction of the force A, will be found to be proportional to the force A, if this system of forces produce equilibrium; this line completes the triangle, the three sides of which *a b*, *b c*, *c a*, express the forces in relative magnitude and direction. This and the following, as well as the propositions referred to, may be verified by very simple experiments, using weights for the forces, and cords to transfer their action to the required point.

Parallelograms have been made of wood, the corners being moveable about the pins which join them; a rod intended to represent the diagonal is attached by a joint, and, with the other rods, is divided into equal parts. The following diagram exhibits its construction.

Fig. 2.

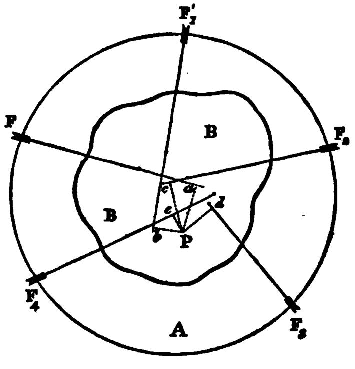


The parallelogram A B C E has two, sometimes all of its sides divided into inches and parts, two, A B, A C, moving easily round a fixed pin at A, which also retains one end of the diagonal A D, divided in a similar manner. The connections at the corners B, C, and E are moveable; the two former consisting of clamps capable of sliding along A B or A C, unless when fixed by their screws; they carry the pins which hold the ends B, C, of the opposite sides B E, C E of the parallelogram; at the remaining corner E there is a double clamp holding the ends of the rods, but may be freely moved along each rod, in order to permit the adjustment of the parallelogram. When this instrument is used, as, for instance, to determine the amount and direction of a force A (*fig.* 1, last page), to counterbalance two forces which meet or would meet, if their directions were produced, in the point *b*, the number of pounds or other units expressing the force B (*fig.* 1) may be measured by so many parts on the rod A B C, reckoned from A, and the clamp with its rod B E slid to the proper mark and screwed up; the same operation may be effected on the rod A B, to show the force C. The point A (*fig.* 2) must be placed on the point *b* (*fig.* 1), and the measured sides brought parallel with the directions B *b*, C *b*; when, on observing the mark on the loose diagonal rod A D, which comes immediately over the axis at E, the number of parts reckoned from the point A will show the number of pounds' force which must be exerted to sustain the two known forces, and the direction of A D will be the direction in which the new force must act.

(4.) It appears also, from the above figure, that, if three forces be in equilibrium, their directions if produced will meet in a point.

(5.) The effect of a number of forces upon any particular point is calculated by their moment or relative power, on the relative amount of leverage with which the forces act; the moment of a force is therefore the product of the force, and the least distance of its direction from the given point, or a straight line drawn from the given point perpendicular to the direction of the force. We can in this way determine the effect of a number of forces acting in the same plane on any body; it may be proved by a simple experiment, as shown in the figure, where A represents the surface of a table upon which a board B of any shape rests through the medium of a few balls, in order to allow of the freer horizontal movement of the board. Weights, F, may then be attached

Fig. 3.

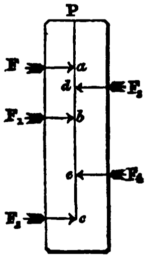


to any points on the board by strings which pass over pulleys fixed to the edge of the table; after a few movements the board will take up some position in which it will remain at rest, or the weights will have found some position in which there is an equilibration. If, then, any point P be taken on the board, and the lines *a* P, *b* P, *b c*, be drawn perpendicularly to the directions of the forces F, it will be found that the sum of the weights multiplied by these respective perpendiculars, tending to turn the point P in one direction, will be equal to the sum of the weights multiplied by their perpendiculars, tending to turn the point in the opposite direction; that is—

$$F_1 \times bP + F_2 \times cP + F_3 \times dP = F \times aP + F_4 \times eP.$$

(6.) Should the forces acting on a body tending to turn it about some point be parallel and opposite, a line drawn from that point will be perpendicular to the directions of all the forces F, F_1, F_2, F_3, F_4, and their moments may be determined as before. If the point of application and magnitude of the first four forces indicated in *fig.* 4

Fig. 4.



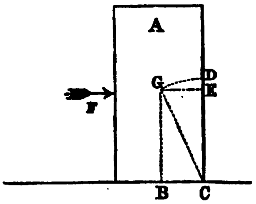
be known, and a fifth required to produce equilibrium, and the value of this additional force given, the distance from P, at which it must be applied, will be found by considering that the moment required must be equal to the difference of the moments of the first three and the fourth forces. Suppose F = 2 lbs., F_1 = 4 lbs., F_2 = 3 lbs., F_3 = 5 lbs.; and the distance P *a*, P *b*, P *c*, and P *d* respectively equal to 4, 10, 18, and 6 inches; then—

$$(2 \times 4 + 4 \times 10 + 3 \times 18) - (5 \times 6) = 102 - 30 = 72,$$

which is the moment required to produce equilibrium in the system; if the force F_4 be equal to 6 lbs., the point *e* of its application must be 12 inches from P, for 12 × 6 = 72.

(7.) In the erection of structures generally, one force has to be constantly taken into calculation—the force of gravity, as the stability of any erection depends on the stability of its centre of gravity, or the common centre of gravity of all its parts. The centre of gravity of any body is said to be stable, when that centre must rise before the whole can be overturned, and unstable when this condition of motion is not necessary. The practical inquiry is, what force must be exerted on the body or structure tending to disturb its centre of gravity, before the stability will be destroyed, or the structure be free to fall? In the following diagram

Fig. 5.



the stability of the body A depends on the fact of C G being greater than C E, for, in order that A may overturn, the point G, or the centre of gravity, must turn upon the edge C as an axis, and, to do so, it will have to rise through E D; this quantity is the difference of C G and C E (or B G), and may always be found when the height of the centre of gravity and the thickness of the body are known, then it is obvious that

$$ED = \sqrt{BG^2 + BC^2} - BG.$$

We have here the space through which the centre of gravity must move; but, to obtain the value of its stability, or the amount of force (F) required to overturn the body, the space E D must be multiplied by the weight (W) of the body, and the force

$$F = (\sqrt{BG^2 + BC^2} - BG)\,W.$$

In the above calculation, the force F is supposed to act on the body at a point on a level with its centre of gravity; its positive efficacy is turning A about its edge C, varying with its position with respect to the point G.

(8.) The finding of the stability of any structure or combination of parts may be said to consist in taking the resultant of the pressures upon each of the surfaces in contact of the system, and determining the point of application of this resultant, and finding the direction of the resultant line of pressure*. Practically we have to calculate with the given strength of the materials, and the manner and means of uniting the parts together.

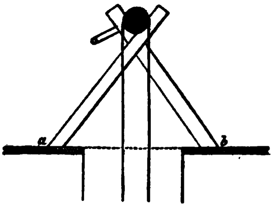
In our remarks on this important part of mechanical science, we shall endeavour to explain the principles which should guide in the arrangement of any structure, and illustrate them by examples of acknowledged good practice.

Chapter II.—*Framework.*

(9.) All combinations or rods, however numerous, may be said to consist of so many frames, which, engaging and counterbalancing the various forces which are exerted on them, sustain the whole structure. A frame may consist of two or more beams or rods according to the purpose designed, and, practically speaking, the size of the frame required; the pressures acting on them generally tend either to compress or stretch; frames are therefore considered with reference to forces acting in these opposite directions. To do so, it is necessary as a first step to find the resultant of the pressures exerted on their respective parts, producing what is called a *racking* of the frame, or an alteration of its form, which of course it is necessary to preserve.

(10.) In a simple support of a windlass (*fig.* 6), such as is frequently used, if the two beams at each end of the axle be inclined to one another very much, or be brought towards the horizontal position, they may give the weight which is raised so great an advantage as to enable it to pull their joints asunder, or force them to the ground; at the same time, they may be placed so near to the vertical position, that the men at the handle, who push horizontally during part of a revolution of the axle, would exert sufficient force to turn the whole arrangement over one of the feet *a*, *b*, the force required to effect this movement being calculated for any given position of the legs *a*, *b*, similarly to the case of the stability of the centre of gravity in the preceding chapter.

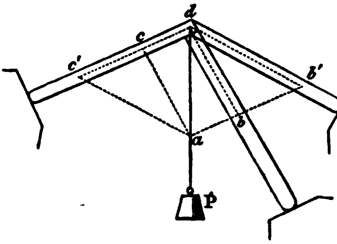
Fig. 6.



(11.) The following figures give a general view of the effect of pressure on the angles of framework, the force, for the sake of experiment, being supposed to be a weight.

The first diagram represents the force as acting upon the point of junction of a pair of beams or rafters. The lines *d c*, *d b*, express the proportional pressure exerted on the beams by the single force measured by *d a*. In such an arrangement of bars, the thrust is very much increased when the angle between the beams is increased; this consequence is shown when the rafter is raised to the position *d b′*, the crushing pressure on each being, in this case, proportional to *d b′* and *d c′*, the latter being twice the amount it was before, and the former somewhat increased, although the force exerted, *d a*, is no more.

Fig. 7.



This figure illustrates the evil effects of allowing a great pressure to be sustained by the joints of a frame where the angle between the parts is very open or obtuse.

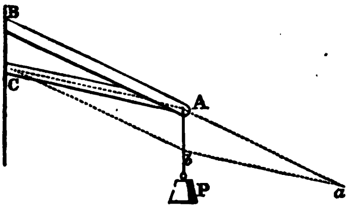
The succeeding diagrams exhibit a simple combination placed under circumstances differing from the preceding case,—in the former, the effect of the force was a compression of the material;

---

* Mr. Moseley, "Mechanics of Engineering," p. 407.

in the following instances, it is observable that the action of the force P on one of the rafters or pieces is stretching; the parts in *fig.* 8 act with great disadvantage owing to the direction in which they resist the vertical force P; this direction is parallel with their length; their disability in this respect will increase most rapidly as their direction is brought nearer to the position at right angles to the force, or horizontal, in the above case. The dotted lines A C, *a* A, exhibit the pressure which must be sustained by the beams A C and A B, respectively, if we imagine equilibrium to exist, in order that the comparatively small force A *b* (the measure of P) may be supported.

Fig. 8.



In *figs.* 8, 9, the combination is better calculated to meet the thrust and strain on its parts. In both examples the force appears to exert about an equal amount of effect; but in practice the latter form is to be preferred, as the greater component of the analysed force is A *c*, in the direction of the piece A C, and this piece receives a thrust from the action of P; in the former case, A *a* is the greater component, which expresses a force tending to tear asunder; and, by the principles of the strength of materials, a thrust is always to be preferred to a strain.

Fig. 9.

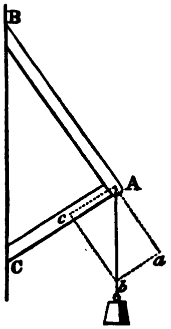
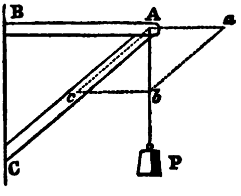


Fig. 10.



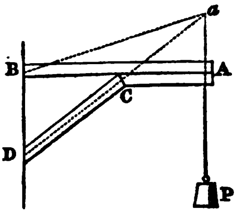
(11.) The last figure forms a support of considerable strength, and is very much used, not merely in its simple form, but also as an important part of large structures in wood, such as centres for bridges and arches, roofs, and wooden bridges. A common and useful modification of this bracket-form is shown in *fig.* 11. The conditions of equilibrium in such a figure may be determined by producing the direction of the strut C D (or piece bearing the thrust) until it meets the direction of the force P, produced; then we may draw a line from B to meet in the point of concourse *a*, as the directions of the forces produced must meet in a point (chap. I., art. 4.) Thus, B D, *a* B, D *a*, will express the quantities of the three pressures producing equilibrium, because three pressures in equilibrium are proportional to the sides of a triangle formed by their directions (*see* last chap.); these lines also express either the force exerted on the pieces of the frame or by them. It appears from this simple geometrical construction that the forces affecting B and D, arising from B D or P, are as follows:—the force in B *a* is

$$P \times \frac{B\,a}{B\,D}.$$

And the crushing force in *a* D, upon the strut, is

$$P \times \frac{a\,D}{B\,D}.$$

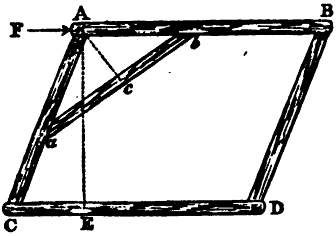
Fig. 11.



If the beam A B be horizontal, as in the figure, the thrust on the strut C D may be determined in terms of the actual parts of the frame, by reference to a very simple geometrical property*, whence we find that the pressure is equal to the force P, multiplied by the product of the length of the beam A B, and strut D C, divided by the product of that part of the length of the beam B C, included between the wall and the point where the strut is applied, and the depth of wall B D, between the beam and abutment on which the strut rests.

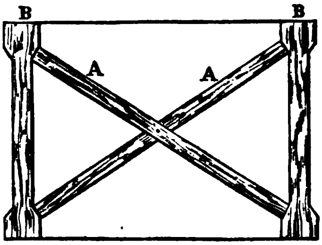
(12.) In constructing frames, the scientific workman pays great attention to its form; a very simple frame is shown in the figure, composed of four rods, A, B, D, C, A, but it is not calculated to resist pressure on its angles, as may be demonstrated by supposing some force, F, to act against the corner A, the lower side being fixed or incapable of translation; the frame will readily rack about the angles C D, as the force F has a moment of F × A E (chap. I., art. 5.), the line A E being the perpendicular to the direction of the force, and the arm by which it acts on the angle C. The angles may be strengthened by braces or ties, as *a b*, which should be so strong, and at such a distance A *e* from the angle A, that the product S of the strength S of the tie and a perpendicular A *e*, shall be equal to the moment of the force F, or as nearly as possible equal to it, or that S × A *e* = F × A E.

Fig. 12.



It may be inferred from the above example that the parallelogram is not a strong form of frame, and that the triangle A *a b* is an improvement; it is well known that the strength of a frame consists in the well-arranged triangles of which it is composed: the triangle is a figure which cannot change its shape without tearing asunder or crushing the constituent materials, whereas in *fig.* 11 the strain is thrown altogether on the joints, and, in order that the frame may rack, it is merely requisite that the adhesion of the bolts be overcome. The following diagram represents a strong frame which may be frequently seen in roofs, partitions, and wooden bridges.

Fig. 13.



The struts A A are attached to the posts B B, at projecting parts called joggles, and bolted together in the centre. A number of these frames, fixed side by side, forms a strong arch, with but a very slight rise in the middle. The celebrated architect Palladio constructed a bridge in this manner.

In a large swing gate a diagonal bar is fixed so as to embrace all the horizontal bars and form the gate into two triangles. In scaffolding the upright poles are strongly bound together by other smaller poles, lashed diagonally; thus making a triangular framework.

Chapter III.—*Roofs.*

The erection of roofs, if the space be considerable, is an undertaking in which all the skill of the carpenter is brought into practice, and, with the arrangement of partitions, constitutes the most difficult part of carpentry.

(13.) The most simple form of covering or roof is that of a flat surface of some convenient material, supported by a number of beams or girders stretching from one wall of the building to the other: this is applicable, and practised

---

* The force on the strut is

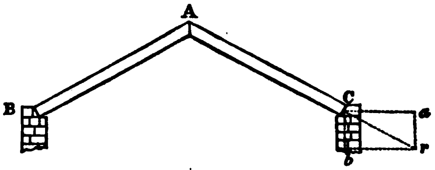
$$P\frac{a\,D}{B\,D} = P\frac{a\,D}{B\,C}\times\frac{D\,C}{B\,D} = P\frac{A\,B}{B\,C}\times\frac{D\,C}{B\,D}.$$

Similarly it may be found that the force acting on the horizontal beam A B, in terms of its real parts, is

$$P\frac{\sqrt{B\,A^2 + B\,D - (B\,A \times B\,D)}}{B\,D}.$$

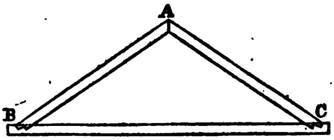
where one wall is higher than another, as in the case of out-houses and sheds of moderate dimensions; but, in most cases of ordinary roof-making, the strength of an unassisted beam would be unequal to the task, and with a flat roof the rain and snow would not be precipitated. To obviate these difficulties, we might suppose the form shown in the figure to serve the purpose of supporting a covering; but it may be shown, from our previous remarks, that the horizontal thrust on the walls B C would act very injuriously, indeed it would be unsafe to attempt such a construction with ordinary walls for support, if the roof were required to be as low as it is represented in the figure. In this position the thrust of the rafter A C will be in the direction C *r*, but by resolving this force into two we may find what amount of horizontal force ensues from this thrust; this will be equal to C *a*, while the vertical pressure C *b* will tend to make the wall more firm; but the latter is very much less than C *a*, and will continue to decrease as the rafters B A C approach the horizontal position. The wall, in consequence of this unsustained thrust, will be pushed outwards, unless it should happen that buttresses might be built against that part of the wall on which the rafter abuts, as in the case of ecclesiastical buildings, or that the wall, instead of being, as is usual, equal in thickness from the top to the base, could be sloping externally, the necessary slope being calculated from the horizontal thrust which it would have to sustain. The strength of the wood rafters has to be considered also; for, if the beams A B, A C, be of very great length in comparison with their depth or thickness, they will bend even by their own weight, and be quite incapable of supporting the superincumbent load of tiles or slates; this difficulty may be obviated to some extent by increasing the elevation of the rafters, where that is admissible; but the weight of the roof would greatly increase, and, of course, the expense. This is not all; the almost vertical roof is, in most cases, a disagreeable object to the eye of taste, as may be observed in many old roofs, such as the Tuileries at Paris; this is a slated roof, with so great an elevation that it appears to the spectator very much like a slated house built upon one of brick.

Fig. 14.



(14.) Considering these circumstances, it is advisable to adopt the low roof, and by some means relieve the walls of the horizontal thrust of the rafters. This object is usually attained by the use of a horizontal beam stretching from wall to wall, and called a *tie-beam*, as is indicated by B C (*fig.* 15). The walls, in this instance, bear only the weight which acts in a vertical direction, while the tie-beam is pushed outwards by the abutting rafters with a force which can be determined as in the preceding case. The beam must be of sufficient strength to resist this strain, but not so heavy as to *sag* or bend in the middle; this it is very liable to do if of a great length, so that it is unadvisable to use this form of support for a roof where the span or distance between the walls is greater than 12 feet.

Fig. 15.

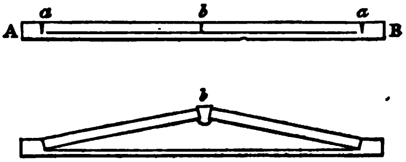


(15.) A description of roof similar to that last-mentioned has been proposed*; the rafters and tie-beam are formed out of a large beam or girder as follows:—The beam A B is cut transversely at *a*, *a*, near the extremities, and between those cuts it is divided longitudinally, (as indicated by the horizontal line,) with the exception of a small space, so that the upper part is not quite separated from the remaining portion of the beam; this upper portion is then cut through at the middle *b*, and the arms *b a* raised to admit a key or wedge between them, as is seen in the lower figure. The inventor calls this contrivance the bow and string rafter.

---

* Transactions of the Society of Arts, vol. 37.

Fig. 16.

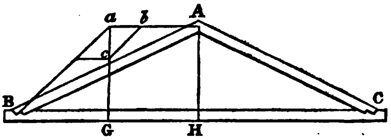


(16.) In estimating the thrust of the rafters on the tie-beam in the above simple form of roof-support, the weight of the roof is frequently considered as placed at A (*fig.* 17), but the effect produced by a mass uniformly distributed is not the same as though the mass were collected at A; but we may consider the load as posited at the centre of gravity of the rafter. If, then, the lines A *a*, B *a*, G *a*, be drawn so as to meet in one point at *a*, the horizontal pressure of the rafter A B at the point A will be represented by *a* A, the pressure at B by *a* B, and the vertical effort of the centre of gravity of the rafter by *a* G (arts. 13, 14); if, then, the line *b c* be drawn parallel to *a* B, the triangle *a b c* will be formed, the sides of which express the proportion and direction of the forces supposed to be sustained by the rafter A C, and the end B of the tie-beam. From this simple construction the horizontal pressure on the tie-beam at B is very easily found; for the ratio of the horizontal pressure (which we will call *h*), and the weight of the rafter (*w*), including whatever may be superimposed, or $\frac{h}{w}$, is evidently $= \frac{\mathrm{B\,G}}{2\,\mathrm{G}\,c}$; then the horizontal pressure or thrust

$$h = \tfrac{1}{2}\, w\, \frac{\mathrm{B\,G}}{\mathrm{G}\,c}.\text{*}$$

That is, the horizontal thrust is found by dividing the half-span by the height of the roof, and multiplying the quotient by half the weight of the rafter and its load.

Fig. 17.



As an example, suppose the rafters of a roof to be 6 inches by 3 in section, and of yellow deal, weighing 33 lbs. per cubic foot, the span 30 feet, the angle of elevation, A B C = 26° 33′, or the height A H = 7½ feet, and the rafters or trusses 2 feet apart, the roofing being of slate, a square foot of which weighs 8 lbs. From these data the weight of the rafter and its load may be determined. The length A B of the rafter will be found by calculating with the two known sides of the right-angled triangle A H B, in the usual manner, thus, $\mathrm{A\,B} = \sqrt{\mathrm{B\,H}^2 + \mathrm{A\,H}^2}$; or
$\sqrt{15^2 + 7\frac{1}{2}^2} = \sqrt{225 + 56\frac{1}{4}} = 16\frac{3}{4}$ feet.
Then the rafter will contain

$$\tfrac{1}{2} \times \tfrac{1}{4} \times 16\tfrac{3}{4} = 2\tfrac{1}{10} \text{ cub. feet,}$$

the weight of which will be

$$2\tfrac{1}{10} \times 33 = 69\tfrac{1}{3} \text{ lbs. nearly.}$$

The load on any one rafter will, of course, be equal to the weight of a surface of roofing found by multiplying the length of the rafter by the space between the rafters or trusses, which, in this instance, is supposed to be 2 feet; therefore $16\frac{3}{4} \times 2 = 33\frac{1}{2}$ square feet of tiling rest on a rafter; or

$$33\tfrac{1}{2} \times 8 = 268 \text{ lbs.}$$

The whole weight, then, of the rafter and its load is $= 69\frac{1}{3} + 268 = 337\frac{1}{4}$ lbs. With this quantity we can find, by the rule before given, the horizontal thrust of this rafter; it will be

$$168\tfrac{1}{2} \times \frac{15}{7\frac{1}{2}} = 337 \text{ lbs.}$$

---

* Or thus, trigonometrically, the ∠ A B C $= \alpha$:—
$\frac{h}{w} = \frac{\mathrm{B\,G}}{2\,\mathrm{G}\,c} = \frac{1}{2}\cot.\,\alpha, \quad \therefore h = \frac{1}{2} w \cot.\,\alpha.$
(In the figure the line G *c* should have been equal to half G *a*.)

It is obvious from the nature of the sloping roof that there must be some inclination of the rafters at which the pressure on the tie-beam will be less than at any other inclination; and, by calculation, it is found that the pressure is a minimum when the angle A B C is 35° 16′, or the tangent $A\,B\,C = \frac{1}{\sqrt{2}}$.

(17.) The simplest addition made to the triangular frame for increasing its strength is the addition of a tie to prevent the tie-beam from bending, and a strut to each rafter, for a similar purpose. These supports are shown in the following figure; it forms a very common arrangement, and is one of considerable strength. The vertical support A D is called a *king-post*, and its function is the sustentation of the tie-beam, which it does effectually; for, supposing that the beam had a tendency to sag or bend, it would pull the king-post downwards, but as the king-post rests upon the rafters by inclined sides or joggles, it must push the ends B C of the rafters outwards before it can descend, so that when the tie-beam pulls the king-post downwards, the rafters are made to stretch the tie-beam more than before, and prevent it from giving way. The king-post, therefore, acts as a tie, and must be attached to the beam in a different manner to that in which the rafters are attached to the upper part of the post; the ends of the latter are mortised into the joggle of the king-post, but it would not be proper to unite the post and tie-beam in that manner, as in the latter there is a stretching, while in the former a thrusting force is exerted; the rafters can thus assist in sustaining a thrust, when the joint is well made, by the shoulder on each side of the tenon; but the king-post would be held to the tie-beam merely by the strength of the small portion of its tenon immediately underneath the pins. Under these circumstances, in the best kind of roofs, an iron strap is used, disposed as shown in the sectional figure.

Fig. 18.

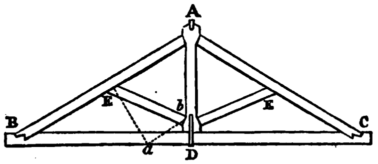
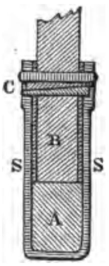


Fig. 19.



The iron strap S, passes under the tie-beam A, and extends for some distance upwards on each side of the king-post B. The upper parts of the strap have eyes at C, through which, and a hole through the king-post, wedges C are driven, in order to tighten the strap. The king-post is therefore suspended from the tie-beam.

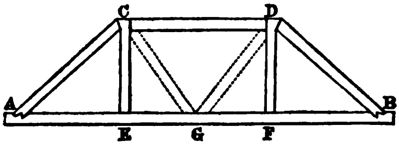
Again, the inclined supports which meet the rafters at E, and abut on the joggles of the king-post at *b*, are called *struts*, and serve to prevent the superincumbent load from bending the rafters. The struts are therefore of great service in the frame, their value in sustaining the rafters depending on their inclination with respect to them; they also prevent the joints B C, at the tie-beam, receiving the injurious pressure which would result from a bending of the rafters. We may thus calculate the effect of the pressure on the strut E. In bending, the rafter and its load would move in the direction E *a*, so that it would not be the whole weight of the roof, but about two-thirds of it in this example, which would press in the direction E *a*. The brace, however, does not support in that direction, it is in E *b*; then, drawing *a b* perpendicular to E *a*, the relative effect of the force will be as E *a* to E *b*. This increased strain on the struts occurs on each side of the king-post, and, when the weight of the rafters and roofing is given, the strain on the king-post may be found in pounds weight. To find the vertical strain on the king-post, arising from both struts, the direction of each strut must be produced until they meet (above D), then measuring upwards on the struts, from a scale of equal parts, any number of units representing the pounds' pressure which the struts are exerting, and completing the parallelogram, its diagonal, which will be in the direction of the king-post D A, represents the whole vertical strain, and, by applying the scale, the amount in pounds weight will be found.

Such investigations enable the practitioner to determine the "scantling" or sectional size of timber which he must use; in doing this, however, he is also guided by the principles of the strength of materials.

Roofs sustained by such an arrangement are very common, as it is generally considered a very proper support where the span does not exceed 30 feet; it is, however, used in greater spans: a roof in St. Paul's Cathedral, of 42 feet span, is supported by trusses of this description.

(18.) When the roof is required of greater extent, and the supports can rest only on the walls at each end of the tie-beam, additional struts and ties must be added to prevent the bending of the timbers. Thus we have new arrangements and a great variety of forms introduced, in many cases, according to the fancy merely of the builder. In the subsequent figure a partially flat-topped roof is represented; in this example the upper parts of the rafters (beyond C and D) may be considered as cut away, and an horizontal piece C D, placed between the upright posts to receive the thrust of the rafters A C, B D. The recommendation of this form of roof is the space obtained for economic purposes, although it is evident that the strength of the frame C E F D, is unequal to the previous arrangement, and also, as C D is supported at each end only, it will be liable to deflect, if of considerable length, owing to the thrust exerted on it by the rafters.

Fig. 20.



A great improvement in this frame is that of bringing the ends E F, of the posts together, as indicated by the dotted lines; the whole is then composed of three triangles: in this form it was used by Mr. Watt for the beams of some of his earlier steam-engines. The action of the different parts are, however, altered in this use of the arrangement; for the point of support, instead of being at A and B, will be at the middle point G, the piston-rod and crank-rod being connected with the ends A B. Thus, A B will be compressed—it will be a strut instead of a tie; A C, C D, and D B will act as ties, producing a thrust on the posts C G and D G.

(19.) The *curb* or *mansarde* roof is another form in which there is a double slope; this modification is shown in *fig.* 21. As in the last instance, the curb roof is adopted for convenience, as it allows of chambers in the roof. The scientific principles of this structure are comprehended in the theory of the polygon. The form requisite to produce equilibrium among its parts may, however, be determined by a simple experiment. Attach four pieces, A, B, C, D, representing the rafters of the curb roof, to a flat board, by the two upper ends of A and D, all the pieces having freedom to move about their joints. When the board is held in a vertical position, and the system of rods left to itself, it will arrange into a polygon similar to that shown in the figure, if the pieces be of equal length and weight throughout; when the whole has taken up the pro-

Fig. 21.

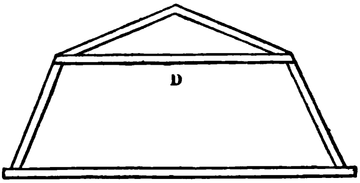
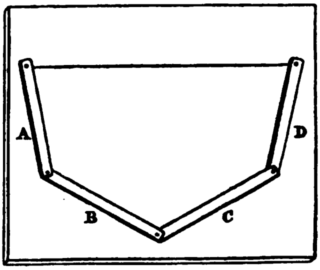


Fig. 22.



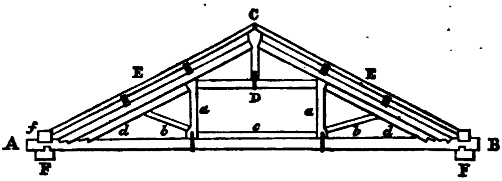
per position there will, of course, be equilibrium, and if the board be carefully inverted, the form may be maintained, although the slightest shock will destroy its equilibrium, which is, in this case, unstable. This polygon will be the correct form for the curb roof; but, owing to its instability, a strengthening collar, D (*fig.* 21), must be supplied. If the rafters are to be loaded unequally, the proportional weights must be suspended from the centres of gravity of the pieces in the experiment, when a different form will result.

This property is of some utility in the principles of arch-building, in considering which we shall make further use of the polygon of pressures.

(20.) Beyond these simple forms of roof-supports many varied arrangements are made, and are necessary, on account of the weakness of the material, when the beams are of great length in proportion to their thickness and depth; by acquainting himself with their relative strength in different positions, as well as the general principles of equilibrium, the builder is enabled to dispose beams of wood or girders and rods of iron of a proper size in proper positions.

The arrangement of a roof-support, similar to that shown in the accompanying figure, is frequently used for large roofs, as the tie-beam is not only more efficiently sustained, but a considerable amount of room is obtained in the middle for household or other purposes*. In this case, the forms explained in arts. 17, 18 are combined; the tie-beam has two points of support, by the two queen-posts *a a*, and the strut *b* gives a support to the rafters at the point where the weight of the purlins acts upon them. The thrust thus produced at the lower ends of the struts, or on the queen-posts, is sustained by the straining-sill *c*.

Fig. 23.



Sometimes the straining-beam D is omitted, and the king-post extended the whole depth of the truss (as before); giving three points of support to the tie-beam; the intermediate spaces are then supplied with struts; the whole thus forming a series of triangles.

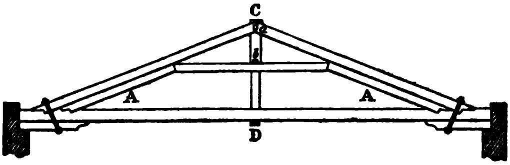
(21.) As examples of roofs, in constructing which the architect has been guided by scientific principles, the roofs of churches and other public works are, in many instances, remarkable, and very instructive.

In the church of St. Paul at Rome there is a striking example of a simple form of roof-truss combined with great span; it is shown in the following figure. Beyond the tie-beam and rafters it possesses only the two false rafters A A, and a straining-beam between their upper ends. The king-post supports the tie-beam by a wood key driven through the extremity of the post at D; another wood key at *b* rests upon the straining-beam, and thus affords some support for the

---

[*] This and the preceding diagrams exhibit what is called a *truss* of the roof; the roof is supported by a number of these trusses, which rest on the walls at the ends A B; they are connected with each other by *purlins* or beams of wood, of which two are seen in the figure, their ends appearing between the rafters below E. The common names of the different parts in the above general kind of roof are as follows: A B is the tie-beam, resting upon the *wall-plates* or beams F F, which latter lie on the material of the wall; the rafters which rest immediately on the tie-beam, and abut against the joggle of the king-post C D, are called *principal rafters;* the upper beams E are the common rafters, and bear the roofing material; they rest upon the top of the king-post C D, and abut against the *pole-plate f*, which is a beam lying in the same direction with the wall-plate F; the *purlins* are laid between the common and principal rafters; the king-post, C D, is short, being tied to a *straining-piece* D, the ends of which abut against the heads of the *queen-posts a a;* these are likewise tied to the tie-beam at their lower extremity; *d d* are *additional* or *false rafters*. *c* is a *straining-sill*.

Fig. 24.

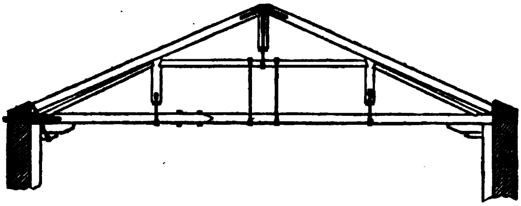


king-post, and a key of iron, *a*, at the junction of the two rafters, completes the suspension of the king-post. The tie-beam in this truss is about 86 feet between the walls, and is but one piece of fir. The following scantling or sectional measures of the different parts may be interesting:—

	Inches.
Tie-beam . . .	23 × 15⅓
Rafters . . .	16½ × 15⅓
Rafters A A . .	10 × 10
Straining-beam . .	12 × 11
King-post . . .	13 × 8¾
Do. above straining-beam	11 × 13
Do. at rafters . .	13 × 13

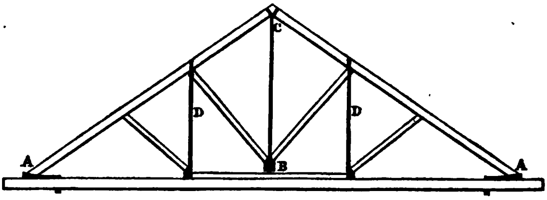
This roof is a very ancient work, having been rebuilt in the year 816*. It exhibits an important feature in these structures, which should always be considered, namely, the use of as small a quantity of timber as possible.

Fig. 25.



In the trusses of the Teatro d'Argentino at Rome, we find the parts which appear in the roof of the church of St. Paul, but with additional ties, which very much strengthen the truss. In this instance there are two queen-posts, which do not reach the tie-beam; the king-post is similarly conditioned, long iron straps or ties being attached, which reach the respective horizontal beams; two iron straps are placed on each side of the king-post strap as an additional support for the tie-beam from the straining-beam. The tie-beam is thus well supported; it is in three pieces scarfed together, and in addition to its own weight and that of the roof, it has supported the machinery during the exhibitions, and the ceilings and other paintings used on such occasions. The span is about 88 feet, and the inclination of the rafters 24°. Examples of English practice seldom exhibit the ties used in this manner, although they are common on the continent; the character as ties of the parts supplied by the iron strap is very conspicuous in these constructions. Mr. Nicholson proposed some time since the employment of iron rods in the place of wood king and queen-posts. According to his method, a roof would appear as in the following

Fig. 26.



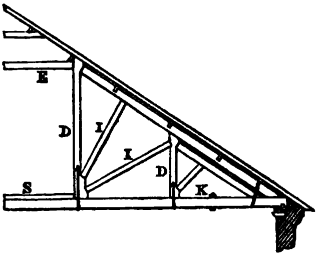
---

* Rondelet, L'Art de Bâtir, tome iii., p. 116.

figure. The king-*rod* B C embraces the heads of the rafters and presents a socket on each side at the foot for the struts; the queen-*rods* D are similarly provided, having a straining-sill between them to sustain the thrust of the struts; the rafters are not mortised into the tie-beam, but fit into iron sockets A A, which are made fast to the tie-beam. This roof is admirably disposed, and accurate in principle; there appears, however, to be an objection to the use of iron on the part of some writers on scientific carpentry, possibly because it has not been much used before time, which is a very weighty reason with many, especially practical persons.

Three roofs are described by Robison, in the *Encyclopædia Britannica*, which exhibit excellent methods of combining the timbers. One is that of the Birmingham Theatre, of 80 feet span. The principal rafters are not carried to the ridge, they abut against the queen-posts D, and are supported by the straining-beam E, it is therefore analogous to the form mentioned in art. 18, with the addition of strengthening ties and struts; the principal queen-posts support two struts I, one abutting against a smaller queen-post, and the other giving support to an open part of the principal rafter; a straining-sill S is very appropriately added to sustain the thrust of the struts I I; to guard against the effect of a tie-beam giving way, if such an accident should occur, a beam K connecting the several trusses was bolted to the tie-beams. This roof is not merely a convenient form (allowing 19½ feet space between the queen-posts for the workmen), but is also a very light structure, as will be seen by the following scantling, or width and breadth of its timbers.

Fig. 27.

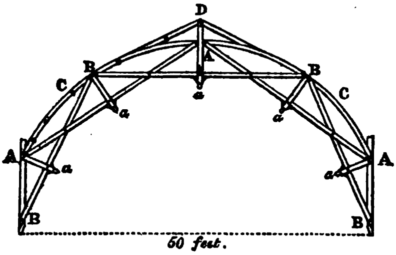


	Inches.
Tie-beam . . .	15 × 15
Straining-beam E . .	12 × 9
Queen-posts D . .	9 × 9
Ditto . . .	7 × 9
Principal rafters . .	9 × 9
Common do. . .	4 × 2½
Principal struts .	9 and 6 × 9
Common do. . .	6 × 9
Straining-sill . . .	5½ × 9

The roof of Drury Lane Theatre was a double truss, one above another, by which arrangement a part of the weight of roofing is taken off the principal or lower truss, although there was a greater weight of roofing in consequence; the strains were, however, prevented from acting injuriously on the walls to a great extent. Dr. Robison considers that this roof was unequalled "in the world for lightness, stiffness, and strength," and that the main truss would bear a load of 300 tons; but it required the walls to be continued much higher than would have been requisite with a roof of the usual outline. A space of 32 feet was allowed for a store-room in this roof.

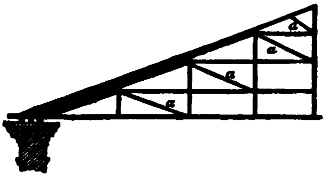
The roof truss exhibited in *fig.* 28 is both strong, light, and elegant; the combination may be decomposed into two roof-trusses; one is formed by the rafters A A, and the other a mansarde form, by the rafters B, B, B; which, abutting on the wall below A, take off a considerable portion of the horizontal thrust which would arise from the solitary use of the rafters A A; a curved rib C C is added, and gives much additional strength to the roof. These rafters are pinned where they cross, and are also secured by five ties *a*; the whole is thus formed into a series of triangles (Art. 12). This roof was constructed by M. Larnier, and it has a span of 50 feet.

Fig. 28.



One of the largest roofs ever constructed is that built over the riding saloon at Moscow; it was erected in 1818, of 150 feet span. It may be described as consisting of a number of flat-topped roofs (see art. 19) placed one upon another, as is indicated in the skeleton diagram; braces *a a* were added to assist the rafters; these, as M. Rondelet very properly considers, should have been placed so as to direct the pressure towards the walls, instead of the middle portion of the truss, which of course is more affected than the extremities by any downward pressure. An accident occurred to this roof soon after its erection in consequence of very warm weather, which caused the beams to shrink;—one of the tie-beams was torn in sunder close to a queen-post, at the point where it was scarfed; the parts separated about three-quarters of an inch, bending the bolts of the joint. It appeared, on examination, that the rupture occurred through a large knot latent in the beam, and not through any fault in construction*. The roof mentioned in several works on carpentry as having been built over this riding-house by M. Krafft, and injured by an accident, was only proposed, and has never been erected.

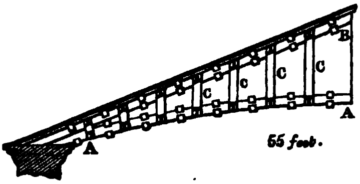
Fig. 29.



(22.) Curved ribs of timber have been successfully applied to the purpose of supporting roofs and other structures, and are strong, economical, and elegant; at the same time affording considerable space in the roof for domestic purposes. The simplest form is that of a solid beam or rib, bent into the required curve, and the fibres of the wood thus curved offer a great resistance to any force tending to bend them in a direction contrary to their curvature. The curved rib is also very useful in bridge-building.

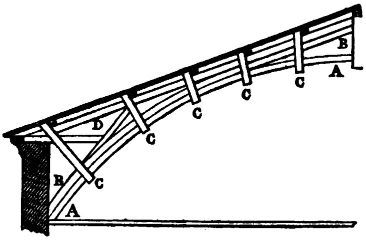
An example of this form of curved rib is given by Philibert Delorme. The rib A acts in this instance as a tie-beam, and is connected with the rafters B by braces C; the span of this elegant roof is 55¾ feet (17 métres).

Fig. 30.



In the roof of the Magazin de la Mâture, at Toulon, we have an instructive instance of the use of curved ribs. A section is shown in *fig.* 31.

Fig. 31.



The lower rib A is curved throughout, while the superior rib B appears as an additional rafter towards the upper part, and abuts against the central post. The struts C are used at intervals to support the rafters D from the ribs, which abut on the walls at a point lower than the rafters, by which, and the small inclination from the vertical at which the ribs meet the walls, the greater part of the usual horizontal thrust is prevented. The span of this strong roof is about 92 feet (28 métres), and affords a great space in the upper story for store-rooms.

Upon this principle a bridge was projected by M. Migneron, a French engineer, in place of the Pont de la Cité, which had fallen to pieces; it was to consist of a number of ribs, upon which the roadway rested at the centre, and was supported at other parts by a number of struts; for some reason it was not adopted, but it would have formed an elegant arch of about 216½ feet (66 métres) span, and 19⅔ feet (6 métres) rise or height*.

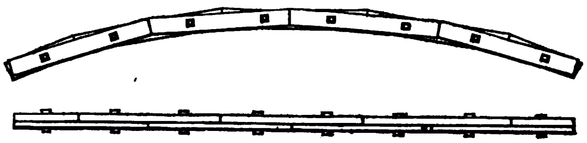
---

* The presence of a knot in an apparently sound beam might be detected by observing the effect of pressure applied at the middle of the beam, while the ends are properly supported. If the beam be of equal width and thickness throughout, and there be no knots, the flexure of the beam will be regular, but if there be a knot, the flexure will be irregular, as the knotty part will not bend so evenly as the sound wood. (See chapter on the *Strength of Materials*.)

In Krafft's proposed roof for the riding-house at Moscow there was an enormous rib, to be nearly 310 feet span.

(23.) The use of parabolic curves of wood or iron has been recommended† to avoid the derangements resulting from the shrinking of the king and queen-posts; these ribs are to be made up of a number of short lengths of wood, the pairs being bolted together, as indicated in *fig.* 32. This has considerable strength, but curved beams are to be preferred. The elasticity of a wooden rod, the thickness of which is not more than $\frac{1}{120}$th of its length, is found to be uninjured when the rod is bent so that the rise or elevation is about the eighth of the span. If two such rods are laid together, bent by twisting ropes, and bolted, they will not alter much when the ropes are relaxed; this rib need not be of a parabolic curve, a circular arc, the height being equal to half the height of the roof, will be sufficient for the purpose.

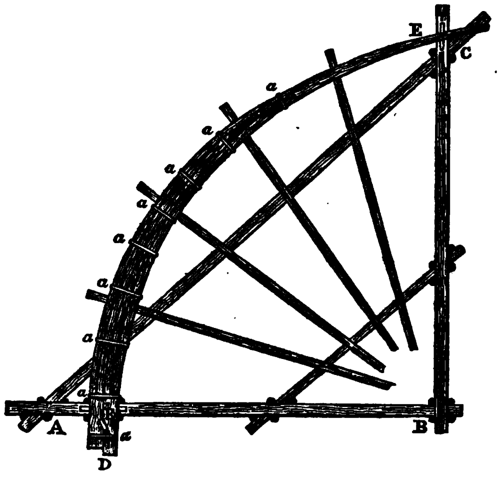
Fig. 32.



Curved timbers or ribs rising from the walls of a building, forming two separate arcs, as rests for the rafters, have been proposed by Mr. Holdsworth; the ribs secured at their lower extremities by a tie-beam, as in the common form of roof-truss. This arrangement is not nearly so elegant as the ribbed roofs we have mentioned, and it appears to require more timber.

In Prussia, bent (*kerfed*) timber is frequently used for sheds, stores, riding-houses, and all buildings in which a great span of roof is required; by this mode of support they construct roofs in

Fig. 33.



---

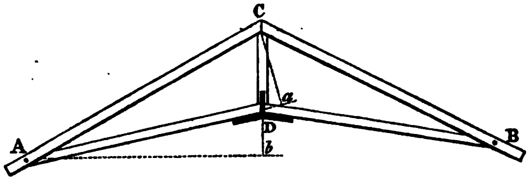
* J. Ch. Krafft, "Traité de l'Art de Charpente," Supplement, 1840.

† Tredgold, "Elements of Practical Carpentry."

which great taste is sometimes exhibited, while the expense is very moderate. Their method of bending the timber is as follows:—The prepared tree or beam of timber is sawn from the larger extremity for about three-fourths or four-fifths of its length, along the middle; the saw is then passed on each side of this central cut, so as to divide the beam into four planks; the latter cuts are, however, not carried on quite to the larger extremity of the beam. The timber is then laid on the horizontal frame A B C, to which the butt or larger end D of the beam is firmly fixed, and the smaller end drawn down gradually into the position E; a number of straps, *a a*, are then made fast to various parts of the beam when the proper curvature is obtained, and a fastening at E made tight. A mortice F is then made to receive a tongue of wood harder than the material of the beam; and, lastly, hoops of iron at every interval of about two feet. The beam may now be taken from the frame, and will preserve the form thus given.

This method of forming wood arches has been applied to bridge building. A bridge of 102 feet span has been described as being supported by *kerfed* beams *.

Fig. 34.



(24.) In many cases of roof-making it is desirable to have higher ceilings than is practicable when tie-beams are used, as mentioned in the preceding examples. The question resulting from further demand on the carpenter's skill is, in what manner can the tie-beam be taken away, and its place supplied by some combination, which shall be effective in sustaining the form of the roof. If from the capabilities of the walls, it be absolutely necessary to prevent the horizontal thrust from playing on them, the readiest alteration which suggests itself is that exhibited in the figure, or the raising of the tie-beam, and use of a short king-post, C D. The rafters will still receive their support, but the strain on the beam A D B will be much increased in proportion as the tie is raised. The strain in this instance may be determined by the method of calculating the moments of the forces acting upon the frame, shown in art. 5. The frame A B C will in this case tend to rack about the joint C, in consequence of the horizontal thrust at A and B, occasioned by the weight (w) of the roof; this thrust we have found to be $= \frac{1}{2} w \times \frac{Ab}{Cb}$ (Art. 16), which acts on the wall at A, in the direction bA, whence its moment will be

$$\tfrac{1}{2} w \times \frac{Ab \times Cb}{Cb},$$

that is the moment tending to turn the rafter A C about the point C, which of course is to be resisted by the raised tie-beam; the force exerted on the latter will be inversely as the arms with which the tie-beam and horizontal thrust act on the racking point, or $Cb : Ca :: $ horizontal thrust : strain on the tie, which is therefore equal to

$$\tfrac{1}{2} w \times \frac{Ab}{Cb} \times \frac{Cb}{Ca}\dagger;$$

thus, as the tie is brought nearer to the rafters, Ca is decreasing, and the quantity $\frac{Cb}{Ca}$ becomes greater, because Cb is constant, and the force on the ties is greater by the latter fraction than the horizontal thrust. In practice iron straps are used to connect the ties at D, they are important particulars in the frame, as upon them depends the stability of the whole truss. The king-post is also subjected by this modification to a much greater strain.

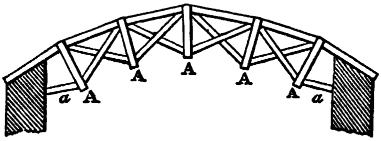
---

* Papers of the Royal Engineers, vol. iii.

† The strain on the ties increases inversely as the cosine of elevation of the tie, or cos. aAb.

(25.) In the old Norman buildings, admirable combinations of struts and ties may be found, veiled by the characteristic ornamental work; in many instances strong roof supports were arranged without tie-beams, allowing a lofty interior. In the truss figured below, posts and struts only are used, but the whole is by principle a firm construction. If the material of the roofing presses equally on the parts of this truss tending to push it inwards, or to collapsion of the frames A A, the diagonal struts meet and neutralize this force; the walls sustain, however, some amount of pressure through the pieces *a a*. In this manner a very large roof might be erected with comparatively short beams; bridges also have been constructed with similar framework.

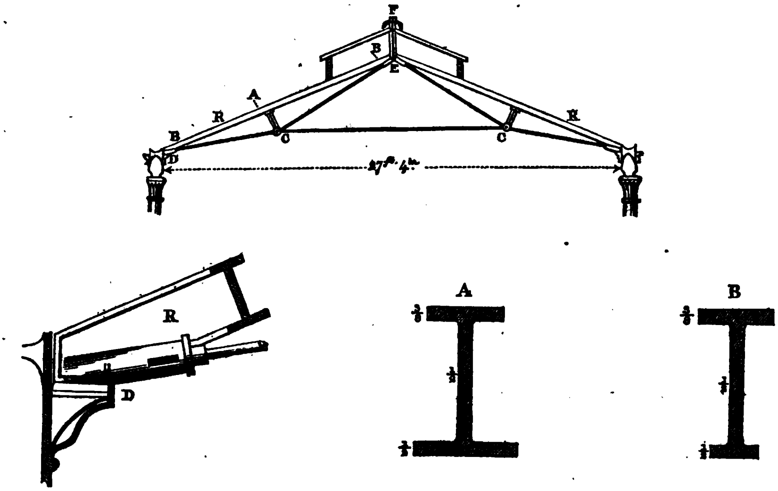
Fig. 35.



In ecclesiastical buildings, the horizontal thrust is a matter of less consequence as the ornamental addition of buttresses to the walls sustain any side effort of the roof; in such positions the tie-beam is frequently omitted to increase the internal elevation of the roof, and collars or little ties supplied near the conjunction of the rafters.

(26.) Iron has now become a favourite material for roof-building and many other purposes to which wood was universally applied. It has been shown from theoretical principles and by practice, that iron is not only more durable but cheaper and stronger than wood; a roof-framing of iron is also lighter than one of wood, and it can be erected at a less cost. The chief applications of iron-work in structures relating to our present subject, are in the roofs of workshops, warehouses, slips for ship-building at dockyards, and passenger and other sheds on railways. Some of the roofs at railway stations present an elegant appearance, from the great apparent lightness of the structure—the whole framing being visible. The following figure will illustrate our description of iron truss, erected at the Birmingham terminus of the Birmingham and Derby Junction Railway. The rafters R R are of cast iron, the sectional form being similar to a double T, as is shown in *fig.* 36; the tie-beam is supplied by the rods D C C D, eminently assisted by the rods C E, or the rods D C E, with the struts C, may be considered as forming a truss for the rafter, and C C a tie supporting the two trussed

Fig. 36.

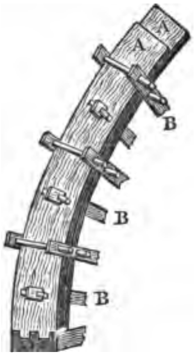


rafters. The heel or lower extremity of the rafter R rests on a bracket D, extending from the top of the column, and carries a cylindrical socket into which the ends of the rods are keyed. The tables of the rafters vary in width: at A or the middle of the rafter, the lower table is 3½ inches wide, and the upper table 2½ inches, at the ends B the same parts are 1½ and 2½ inches respectively; the thickness of the upper table is ⅜ths of an inch, and of the other parts ½ an inch; the depth of the rafters is 4¾ inches. The rods D C are 1 inch, and C C and C E ¾ of an inch in diameter. Surmounting the whole is a ventilating arrangement F composed of oblique shutters. From these quantities it will be seen, that a comparatively small quantity and weight of material is used in this roof, with considerable elegance in the construction *.

Chapter IV.—*Domes and Centres.*

(27.) Roofs of the cupola or dome kind have been frequently constructed of wood and sometimes of iron. The consideration of their equilibrium is somewhat different from that of the ordinary description of covering, being, in fact, much more simple, and the roof much easier to construct. The conditions of the stability of cupolas are more evident in the brick and stone structures than those of wood, but their great simplicity arises from the fact that the parts have a tendency in common to fall inwards, which constitutes them as wedges, preventing the movement; the only precaution necessary being a band or ring at the base or nearly so of the dome to counteract any inclination of the respective rafters to slip outwards. Where a dome is required, which bears its own weight only, it is plain that a very simple combination of pieces of wood are sufficient. Philibert Delorme proposed the erecting of domes with small planks bolted together, forming ribs. According to his method, two or more planks of short length are fixed together, and cut to the curve required, successive ribs being attached to each other, until the proper elevation is reached. The annexed diagram will indicate the appearance of the ribs A A; ties B B are placed at different heights, passing from rib to rib, stiffening the whole framework. The following measures of the pieces or planks will at once show the small amount of timber required to erect one of these roofs.

Fig. 37.



	Ft. in diameter.	In. deep.	In. thick.
For domes	26	9	1
,,	39	11¼	1¾
,,	65	14½	2¼
,,	118	14½	3¼

These measures, however, are greater than those used in some instances. The church of the Madonna della Salute, at Venice, is surmounted by a dome composed of 96 ribs, each of 4 thicknesses, so that each rib is 8½ inches by 5½ inches, while the span or diameter of the dome is 70 feet; in this instance the whole is strengthened by an iron hoop 4½ inches wide and ½ inch thick. Over the Halle aux Blès, or corn-market, at Paris, a wooden-rib dome was erected by M. Moulineau, which was considered a magnificent object; the diameter was about 200 feet, and the curved ribs which formed it were made of three planks, each 13 inches broad, and 3 inches thick; these planks were so put together, that one of 3 feet in length was placed between two others, of 6 and 9 feet in length; thus every superadded portion of a rib fitted into and could be firmly bolted to that below; the ribs were connected at different heights by purlins and iron straps; at one-third the height of the dome every third rib was discontinued, and at two-thirds of the height every second rib; the remaining ribs finally fitting in a ring of timber, above which was placed an umbrella to provide for the necessary ventilation in a convenient manner; this noble dome was destroyed by fire, and replaced by a smaller cupola of iron.

---

* A patent circular malleable iron roof has been patented by Messrs. Neill and Crawford, in which flat bars of iron are used for ribs, bent on their edge into the curved form; these support a roofing of cast-iron plates used as slates, hooking in one another, and rails or purlins placed across the ribs. The comparative weight of this roof is said to be one-third the weight of a wood-trussed roof; it is much cheaper, and affords more room in the interior.

Where the dome is required to support a lantern or other weight in addition to that of its own materials, trussing is necessary to ensure safety, unless the lantern or superimposed weight does not exceed that of the materials which would occupy the circular hole left in the top of the dome; otherwise the upper part will be thrust in, or the lower parts will be thrust out; the former case must be met by proper trussing, and the latter by powerful bands or hoops; even where there is no extra weight a binder is necessary, generally at an elevation of from 50°, or where the tangent to the curve is inclined about 40° to the horizon, or when the span is about $\frac{11}{14}$ths of the diameter, these measures apply where the dome is of equal thickness; the extra support must be applied at a lower part, if the thickness of the dome or the weight of the material decreases as the elevation increases. In using truss work, the strains of a superimposed load are propagated either to some part of the dome above the base, or at the base; wherever it occurs, chains or hoops of proper strength must be supplied; if a tie-beam can be admitted, great assistance will be given to the whole structure. The two sections in *figs*. 38, 39, of half the domes of the Edinburgh Register Office, and that formerly existing over the Pantheon in London, exhibit instances of trussing. In the former it is important that the truss should be well tied in at *a* if there be considerable extra weight to support, otherwise it would be weak; the span of the dome is 50 feet and its thickness 4½ feet; in the latter example, the piece *a*, which will be a strut or tie according as there is little or much weight to be borne, is an important piece; the triangular arrangement in this truss is light and strong.

Fig. 38.

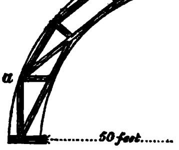
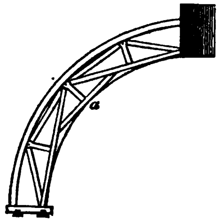


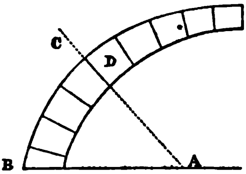
Fig. 39.



(28.) The construction of *centres* or scaffolding for sustaining the parts of an arch until the key-stone is laid, has called forth a considerable amount of mechanical skill. In building a bridge over a stream where it is unimportant that the navigation be prevented, the erection of a centre to build upon is a comparatively easy matter, but in many cases, for physical as well as commercial reasons, it is necessary to make the centre a temporary bridge; the centre must then be built so as to sustain not only its own weight, but that of the materials used in the construction of the arch, which it must do by abutting on the piers laid for the intended arch. A centre so arranged consists of a number of trusses from 5 to 10 feet apart, connected together by bridging joists, upon these boards are laid, forming the bed for the courses of arch-stones; where an arch is large there is a bridging joist under each course, being overlaid with planks, which can be cut away when required.

Although the centre is not pressed with the whole weight of the superincumbent arch-stones, yet it has to support an enormous load if the arch be of a considerable size. The centre is relieved of a portion of the total weight by the friction and pressure thrown by the arch-stones upon one another, indeed, until a particular elevation is reached, the whole weight of a stone is incumbent on the part of the arch beneath it; and we may estimate the pressure exerted on the sustaining centre from the known proportion that the whole weight of a stone is to the pressure it throws on the centre as 1 : sine of the angle of elevation of the joint (which would be sin. C A B in the accompanying diagram, if the stone D were under consideration).

Fig. 40.



This estimation is exclusive of the friction, which will also assist in relieving the centre of some portion of the pressure, and the effect of interposing cement; with a block of dry freestone for instance, the friction prevents the stone from sliding against the centre, until the angle C A B is about 35°, with a block of granite the angle was found to be 33° to 34°*.

The pressure is again diminished in consequence of the tangential thrust exerted more or less by the arch-stones on those in the same course below them, the tendency being to push outwards some parts of the arch. According to the opinion of Mr. Couplet, none of the arch-stones below 30° of elevation press upon the centre, supposing the arch to be semicircular, and of uniform thickness; he also concludes that an arch under these conditions throws upon the centre not more than four-ninths or not quite one-half of its total weight. Dr. Robison considers, for reasons which he does not state, that the pressure is as much as two-thirds of the whole weight.

It results from these views and facts, that it is advisable to exercise some discretion in building up a centre, otherwise it is dangerous to attempt a display of skill; in some cases, centres have completely given way, as in the erection of an arch on the river Derwent, when, as the masons were about to lay the last stone which should complete the structure, the centering broke asunder and fell into the river; as a substitute for mechanical skill, some have added so much useless timber, as to endanger the centre by reason of its own weight alone. A centre, like all other structures, is strongest when composed of the fewest possible parts, so judiciously put together that the thrusts and strains may be fully met.

(29.) It is evident that, as the lower courses of stones are laid first, the possibility of the transference of any pressure exerted at the sides, to the crown or top of the centre, should be carefully guarded against, and that as the principal weight when the arch is nearly finished must be on the crown, that part of the centre should be well supported. In elliptic or flat arches, the pressure commences earlier in the building than in the semicircular arch; when the limit of the angle of resistance † is reached, the stones slide upon each other down on the centre, and the pressure varies with the elevation of the joint, so will the friction vary with the elevation ‡. If we suppose that the angle at which the stones begin to slide is 33° (with fresh mortar interposed), then the coefficient of friction being 0.66, we may calculate by the formula given in the note, that as the joints attain the following angles of elevation, so will the proportion given of their weight be thrown on the centre :—

Angle of elevation.	Fraction of the weight.
33°	0.000
35	0.033
40	0.137
45	0.240
50	0.341
55	0.440
60	0.536

It appears from the calculation, that when the stones lie inclined at an angle of 45° the pressure on the centering is about one-quarter, and at 60° above half the weight of the arch-stone.

---

* Rennie, "Phil. Trans." 1829. This is said of the arch-stones of the present London Bridge; with fresh finely-ground mortar the stones slipped at 25°.

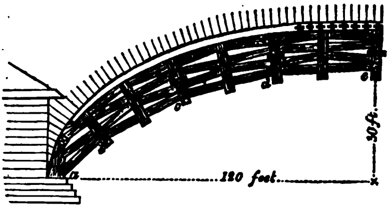
† See *Construction of Machinery*, chap. IV., page 107.

‡ The following equation arises from the figure given above, a = angle of elevation = C A B, f = the coefficient of friction, and P = the pressure on the centre, and W the whole weight of the stones in question, then

$$P = (\sin. a - f \cos. a)\ W.$$

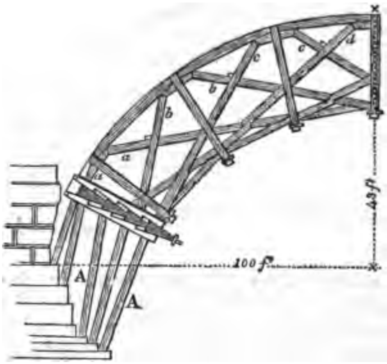
Several interesting examples of centering are afforded by the French engineers. M. Perronet used the following centre for a bridge built at Neuilly, of 120 feet span and 30 feet rise. There is evidently but a small abutment or resting place allowed for this centre on the pier; the whole is a very simple arrangement, consisting of a number of struts, *a b*, *b c*, *c d*, in courses one above another, disposed triangularly; but it will be evident that the strength arising from this figure in composition does not apply here, the triangles are not complete, and the angles are too obtuse, so that any force exerted on the angles must act with prodigious effect on the frame. (See art. 11 of the present Treatise.) The struts were bound by a number of king-posts, and the frames, which were 6 feet apart, were bound together by horizontal bridles *. This disposition of the timbers, in addition to the oblique footing of the lower beams or feet, deprived the centre of that stiffness so necessary to support the uncompleted arch; the history of this erection sufficiently shows the deficiencies of the centre :—after the laying of the masonry had commenced, the crown of the centre alternately rose and sunk; when twenty courses of stones were laid on each side, the centre was found to have sunk one inch; with forty-six courses resting on it, and the crown loaded with fifty tons to prevent it rising at that part, the whole frame sunk half an inch more; and, by the time the keystone was set, the centre had sunk 13½ inches, and risen at the haunches or lower parts.

Fig. 41.



The centre used by Mr. Mylne for building Blackfriars Bridge is a contrast with that of Perronet in point of stiffness, and generally in the skilful disposing of the strut-beams; half of a frame is shown in *fig.* 42; the feet A have a good support in a position nearly vertical, which gives them a much greater effect. By a brief observation of the figure it will be seen that the framing is very simple, being made up of seven pairs, *a a*, *b b*, of rafters, abutting at their upper ends against straining beams, and held by king-posts; or it may be said to be a combination of simple roofs; the direction given to the strains is such as to convey them all to those parts which can best support them,—little of the pressure at the haunches can reach the crown; in the case of the truss *a a*, one leg rests directly over the legs A, and the other passes across the framing to the opposite side of the arch, which in building would be pressed down, and thus oppose a useful force to the leg *a*; the other rafters act in the same way. The progress of the work proved the accuracy of the design, the centre not sinking at all, although closely observed; it took more timber by about a third than Perronet's centre, and offered more obstruction to navigation; it is much stronger, but account should be taken of the flatness of the Neuilly Bridge; in the latter case, the ratio of the height to the span is 1 : 4, while in Blackfriars Bridge we have 1 : 2½, the actual dimensions of the latter being 100 feet span by 43 feet rise.

Fig. 42.

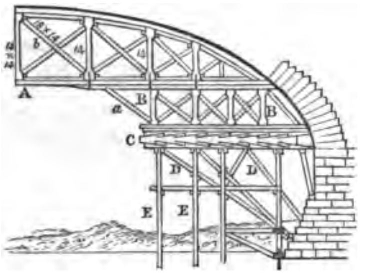


The centering used for the present London Bridge is a beautiful piece of framework, although it is not entirely supported from the piers. *Fig.* 43 represents half of this centre as used for the middle arch, the span being 152 feet and the rise 37 feet 10 inches; this is a composition of diagonal struts, which, as is remarked in the present treatise, chap. I. p. 122, is the strongest disposition of framing-pieces; all the upper framework rests upon the great beam A, 131¾ feet long, which lies on three diagonal frames B; below these is the arrangement of wedges and their receivers or *striking-plates* C, supported by the struts or feet D D, and the piles E; these prevent the possibility of any accident arising from the great obliquity of the feet; the oak-strut *a* also is of great service to the long beam A, meeting it just where a great downward thrust is exerted by the beam *b*. There were ten of these frames or ribs to form each centre, containing altogether 800 tons of timber; a centre was, however, erected in ten days.

---

* The scantling of the timbers was as follows :—strut-beams, 17 by 14 inches; king-posts, 15 by 9 in.; horizontal bridles, 15 by 9 in. each half; and eight other horizontal ties, 9 by 9 in.

Fig. 43.



(30.) It is necessary, in forming the centre of an arch, to make some arrangement for steadily removing it when no longer required. This, which might appear a simple operation, is one calling for great care, otherwise some parts of the yet unsettled structure will sink and thrust others out of their place, to the imminent danger of the whole. There is, however, a sinking or settlement of the arch when the centre is removed, sometimes to an alarming extent; this is generally allowed for in laying the arch stones. M. Perronet used small blocks beneath the arch stones, which, soon after the bridge was keyed in, he began to remove, commencing at the lowest part on each side; as he proceeded, the arch stones slid after the centre, and the joints opened so as to create some alarm, but by the timely assistance of mechanical skill the decentering was safely effected, not, however, without considerable alteration in the form of the arch, which is a dangerous circumstance. In the bridge of Neuilly, the sinking during the progress of the building was very nearly 14½ inches, but when the centering was cleared away, it was depressed 10⅓ inches more, making the total sinking nearly 25 inches (22 pouces, 9 lignes). In another case—the bridge at Mantes—the sinking on removing the centre was 9¼ inches, which, with the previous alteration while on the centre, made a total fall of above 22¾ inches*. This result on the clearing away of the support may be fairly attributed to bad mason's work, but Perronet removed his centres very soon after the completion of the arches, which also were very flat at the crown. The elegant granite arch built over the Dora Riparia, near Turin, spanning 147½ feet and rising but 18½ feet, only descended 6 inches in the five days during which the centre was being removed; this, though it contrasts very favourably with the large quantities before stated, is much greater than we find or allow for in arches erected by our engineers. Dublin Bridge, of 105 feet span and 22 feet rise, through the stiffness of the centering, sank but one inch during the erection, and 1¾ inch on striking the centre; Grosvenor Bridge, over the Dee, at Chester, of 200 feet span and 42 feet rise, exhibited no open joints during the operation, and it finally sank but 2½ to 2⅝ inches; Blackfriars Bridge moved but 1¼ inch in settling, the arches being, however, 43 feet rise for 100 feet span; Bow Bridge, a single arch of 66 feet span and 13¾ feet in height, descended one inch in building, and only half an inch on easing the centering, which was half an inch less than the amount allowed when the work was set out; and Waterloo Bridge, in which centres were used after the model of Blackfriars, on decentering, sank but 1½ inch in any arch, the span being 120 feet to a rise of 35 feet. The method of striking or lowering the centre in the British works now mentioned is different from that of M. Perronet and other continental engineers; the contrivance adopted in Blackfriars and many other bridge-centres is shown in *fig.* 42, and at C, *fig.* 43; the lower ends of the trusses rest on a bed, the under side of which is of a zigzag form, to receive the wedge placed between it and another similarly cut piece, resting on the feet A and D, their surfaces being covered with copper. When the arch is completed, the operation of lowering the centre is performed by driving the wedges inwards; this is effected either by common mauls, or, as at Blackfriars, by means of a heavy beam suspended from some part of the centre, and driven, like a battering ram, against the wedge.

---

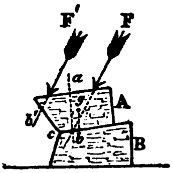
* Mémoires de l'Académie Royale des Sciences for the year 1773, p. 33.

Chapter V.—*General Equilibrium of Structures in Stone, &c.—Walls of Buildings, and Revetments.*

(31.) In ordinary buildings of brick, stone, and analogous materials, the contemplation of their stability or resistance to pressure is different and far more intricate than in the structures we have hitherto considered; where beams of wood are compressed they are generally placed in the direction of the strain or pressure to be supported, and we have then an outline figure or frame in which the directions of all the forces can be traced, but in built-up solid forms, such as walls, piers, arches, and bridges, the inquiry into the effects of forces calls for closer observation, and, after all that can be done, the results are less certain.

(32.) When a single mass is acted on by any forces it will be compressed, or crushed, or its parts torn asunder, according as the forces exceed the force of cohesion of the atoms composing it, but when two solids, as A, B, are resting together, not being connected by cement, and some force or resulting force F act on the upper solid, their mutual stability depends on the inclination of the joint or faces in contact of A and B, and also of the inclination or direction of pressure with respect to the joint, or the angle Fba. In the former case friction is evidently an important element in the calculation, being the only force that prevents the body A from slipping on the upper face of B; it appears from the last column in the tables (p. 108), that if the angle Fba, which the force makes with the vertical ab to the surface of the solid B, be not greater than the limit of the angle of resistance of the substance, then A will not slip; if greater, stability cannot exist unless some other force be in action to oppose F. Again, the second condition necessary to the equilibrium of A upon B is that the direction of the force F should pass through the surfaces in contact, otherwise the upper body will be liable to turn over upon one of its edges, should the force be sufficiently great to overcome the small resistances which are offered to it; thus, it is dangerous if the force be in a position such as $F'b'$, for rotation is likely to occur round the edge c, unless the moment (art. 5, p. 116) of the centre of gravity g, added to the cohesion of the surface in contact, be equal to the moment of the force F'. It must be recollected that the vertical gb through the centre of gravity requires the same conditions as the foreign force F.

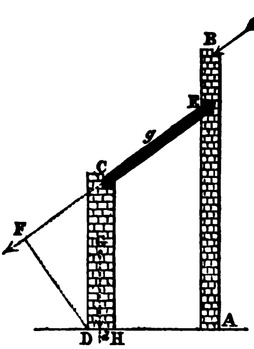
Fig. 44.



(33.) Walls, when simply used as a means of separating portions of land, have but their own weight to sustain, and require no particular consideration in a theoretical point of view; but when they are used for supporting other structures, the operation of the pressures upon them demands investigation. The walls of ordinary houses, for the most part, have the pressures upon them in a vertical direction, some horizontal thrust, however, is to be supposed; they should decrease in thickness upwards, owing to the load varying in quantity, and also because they are less liable to be disturbed by any forces which may accidentally act against them, when the centre of gravity is nearer to the ground. A great portion of the stability of houses at present constructed is obtained from the girders and joists of the floors, which tie the walls together; and it has frequently occurred, when these important beams have decayed, that one or more of the walls has fallen.

(34.) In churches and some other buildings the walls have to support a great thrust from the roof; this is shown in the accompanying figure, where two walls are represented as sustaining a high and low roof.

Fig. 45.



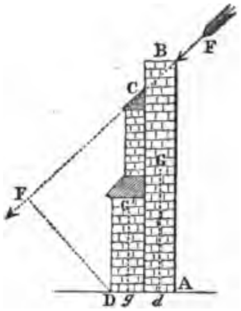
A B is supposed to bear the thrust of the upper roof, as indicated by the arrow P; this thrust is carried by the aisle roof E C in the direction E F, which, with the thrust of the roof itself, is tending to turn over the wall C D about the edge D; then the moment or power of the force F to upset C D will be found by drawing the line F D, D being the edge about which motion would take place. To resist this effort we have the weight of the wall D C, the moment of which is obtained by multiplying the weight by the distance D *d* of the vertical line through its centre of gravity G, from the edge D; this distance, if the wall be homogeneous, would be equal to one-half the thickness of the wall; collecting these amounts we obtain the following equation for equilibrium:—

$$F \times FD = \text{weight of the wall } DC \times Dd.$$

Here F stands for the thrust from the wall B A and that of the roof C E, combined. In addition to the resistance arising from the weight of the wall, there is another of importance in practice, namely, the cohesion of the parts of the wall; if the forces F were so great as to produce rupture anywhere between C and D, the cohesion of the parts must have been overcome.

(35.) In consequence of this overturning thrust, which occurs in many, especially Gothic, ecclesiastical buildings, buttresses are applied to the wall, at regular distances, in order that additional strength may be given to it; at the same time they are, in many cases, so admirably disposed as to appear like ornamental appendages to the structure, yet, if removed, the destruction of the whole edifice might be the immediate consequence. In the accompanying figure a wall A B is assisted by the buttress D C.

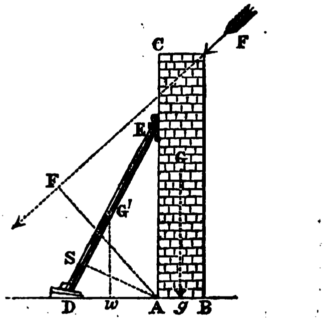
Fig. 46.



Then, to find the conditions of equilibrium for this figure, we must exchange the edge of the wall for the external edge D of the buttress, and the arm of the force F will be F D, while the opposing forces will be the weight of the wall and its leverage *d* D, and the weight of the buttress and its leverage *g* D. When a pinnacle is superimposed on the buttress it adds its weight to that of the buttress; but the stability of a body is injured by increasing height without thickness (art. 7, p. 119). Buttresses are to be preferred before adding uniformly to the weight of the wall; some buttresses frequently forming a portion of an arch are called flying buttresses, and perform their office of supporting, as the rafters in *fig.* 45 support the high wall A B, by transferring the pressure to some part able to sustain it.

(36.) For temporary purposes shores are

Fig. 47.



used to sustain sinking walls, earthworks, and other structures. The effect of a shore may be considered by taking its moment acting contrary to the oversetting force. Thus, if A B C be the body supported, because pressed down by a force F, the disturbing force is F × F A, and to oppose it we have the strength of the strut or shore D E multiplied by S A, added to the product of the weight of A B C and the perpendicular to its direction A g. In addition to this there is also a resistance to the rotation by reason of the weight of the shore pressing against the wall; this is not equal to the whole weight of the beam, but is equal to the weight $\times \frac{D w}{D E}$, $G' w$ being the vertical from the centre of gravity.

(37.) In all these cases equilibrium only has been considered; the forces tending to injure the edifice have been supposed to be equal in their effect to the conservative resistances, neither the former nor the latter preponderating; but it will be evident, from a moment's reflection, that it is impossible that a building could stand under such circumstances,—the most gentle wind would throw it down; it can have no stability; the resistances must be greater, and, generally, are very much greater, than the destructive pressures, to allow for various extra-theoretical considerations, such as, the action of the weather, storms, sinking of foundations, inequality in the strength of the material, badness of the cement, and imperfection of workmanship*. All these things are against the builder.

(38.) In engineering a most important subject is the best method of supporting earth by brick and stone walls, called *revetments*, which occur in forming docks, quays, reservoirs, bridges, and fortifications; yet, although of so much importance, both on account of the circumstance in which they are required, and the number of indispensable works in which they are principal parts, it might be thought that their principles and practice are well known; they are, of nearly all other subjects, least understood. The action of the earth upon them, with the character of the resistance they offer to the load, is a mystery. Much calculation has been made respecting them, but no satisfactory results have been deduced; for there is so much disagreement between the results of different calculators, each of which has found that his own must be the truth, that the engineer finds little of useful direction from mathematicians.

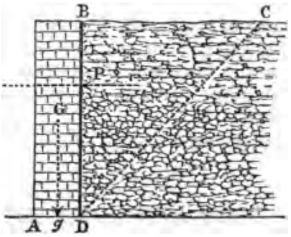
(39.) When earth has to be supported, it is necessary to observe the natural slope or angle of repose of the particular kind of earth in question, that is, the angle at which it will begin to slip. The following table shows the natural inclination of several substances:—

Fine sand (dry) . .	21°	0′
Do. do. . . .	34	39
Do. do. . . .	39	0
Loose shingle (dry) .	39	0
Common earth (dry) .	46	50
Do. do. (damp) .	54	0
Compact earth . .	55	0

It appears from this table, that when sand is heaped up behind a wall, a much greater quantity and weight presses against the wall, than when the substances below it are in similar circumstances. The angle of slipping must be found by actual experiment, in every case.

(40.) If a wall is to be built so as to bear up a mass of earth, it may be a vertical rectangular wall, or a leaning wall, or of a trapezoidal form having one sloping side. Between these forms a comparison may be instituted by taking the moments of the weight of wall, and the pressure of earth. Thus, if the wall A B (*fig.* 48) support the earth B C D, and C D be the natural slope or the line of repose, all the earth contained in the angle B D C will be pressing against the wall with a horizontal force P, tending to push it either forward on its base, or over the edge A. The former motion will occur if the friction of the wall on its base, added to the strength of the cement at the failing

Fig. 48.



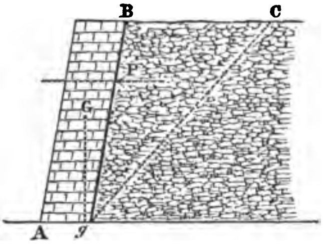
---

* The effect of *time* is not so much thought of in building in the present age.

part, is not equal to the force P. To find the friction of the wall it is necessary to know its dimensions and the weight of a cubic foot of its material, as the friction is the product of the whole weight and the coefficient of friction of the substance (*see* page 106). The possibility of the wall being turned over upon its edge depends on the product of its weight and the distance of the edge A from the perpendicular A*g* to the vertical from its centre of gravity G, being less than the product of the pressure P of the earth, and the perpendicular to its direction from the edge A. This it will be seen, supposes the parts of the wall to be so firmly attached to each other as to form a solid mass—a supposition which cannot be depended on in any case, as practice abundantly proves.

(41.) When the wall is inclined, as in *fig*. 49, it is called a leaning revetment. It is certain that a much greater force will be required to overturn it, as its weight has so much more leverage. It is advisable, however, that the inclination of the sides should not be so great as to throw the perpendicular G*g* without the base of the wall, in case of any failing in the earth, and also because the earth would have more power to push out the lower parts, if the masonry be at all unsound.

Fig. 49.



(42.) A sloping revetment, *fig*. 50, is built with a base greater than the summit or top, the slope or *batter* being external; this is a much stronger disposition of the materials for the arm A*g*; and, in consequence, the moment of the weight of the wall is greater; the centre of gravity also is nearer to the base; it is certain that at the top of the wall there is no pressure of earth, but the weight increases as the depth increases; this will appear by drawing a section, *fig*. 51, of the triangle of earth A B C, B C being the natural slope. If A *a*, equal to one-third of the height be taken, the triangle A *a c* of earth pushes against it, and no more; if A *a* be removed, that portion will slip away; considering the second portion *a b* of the wall, there are three triangles in the section *a b d c*, which would also slip away if the part *a b* of the wall were removed; finally, the lowest portion *b* B supports the five triangles of earth *b* B, C *d*. We should naturally conclude that little or no thickness of masonry is required at the top of the wall, and the stability of the whole wall is much increased by lowering the centre of gravity (*see* art. 7). These remarks receive an illustration in the fact that revetment walls in failing have frequently given way first by bulging outwards at about one-third of the height of the wall. Generally, the foundation being well laid, the sliding motion of the whole wall is less in danger than its oversetting.

Fig. 50.

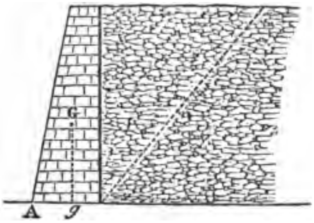
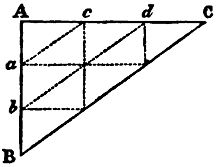


Fig. 51.



(43.) Some experiments were tried by the direction of Col. Pasley* to determine the actual powers of different revetments. He used models of revetments of different kinds; and first tried what weight would overset the model by itself; this weight he calls its stability; then heaping up shingle behind it, the weight required to overturn it (if any) measured its stability under the pressure of the earth: with a rectangular wall, it was found that its stability by itself being 47 lbs., it required with the shingle behind it but 30 lbs.; showing a loss of 17 lbs. of stability; with a wall sloping towards the shingle (*fig.* 50 reversed), the stability alone being 51 lbs.; with the shingle it required 80 lbs. to pull it over; whence it would appear that the weight of the earth on the sloping side gave an additional stability of 29 lbs.; but a sloping wall of the form shown in *fig.* 49 lost by the application of the earth, its stability alone being 85 lbs., and with the shingle 77 lbs. Using a leaning wall, it had a stability of itself of 86 lbs., which was increased to 110 lbs. by the backing of the earth. The sloping sides in all these cases was one-fifth of the height of the models; in some cases they were prevented from sliding forwards. One objection is to be made to these experiments; the models were not built up, but consisted of wooden cases filled with earth, and could not therefore properly represent a structure of brick or stone.

---

* Course of Military Instruction, vol. 3. p. 566.

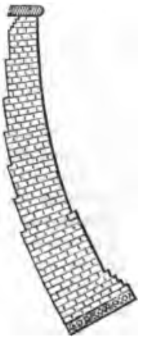
(44.) M. Mayniel* published, some time since, an experimental and analytical treatise on this subject, in which he gives, as the result of experiment and theory combined, the requisite thickness of retaining walls for the following different substances. The thickness given is supposed to be for a height equal to unity:—

Material of Wall.	Weight of a Cubic Foot of Wall.	Common Earth.	Coarse Gravelly Earth.	Sand.	Rubbish.	Clay.	Semi-Fluid.	Wet Earth.
Cut stone .	204 lbs.	0·13	0·16	0·26	0·17	0·41	0·44	0·24
Brick . .	132	0·16	0·19	0·33	0·24	0·17	0·54	0·34
Rough stone	163	0·15	0·18	0·30	0·22	0·16	0·49	0·29
Rolled flints	178	0·14	0·17	0·29	0·21	0·15	0·47	0·27

Thus, if a brick wall of 30 feet in height (if the material were according to the above specific weight) had to sustain gravelly earth, it must be 5·7 feet in thickness, or for sand 9·9 feet, or for wet earth 10·2 feet thick.

(45.) Some walls are built with a curved slope or batter, as occurs in some railway cuttings. This form, however, tends to throw the centre of gravity forward, and the line of pressure through the mass of the wall nearer to its external slope. These are injurious consequences. A wall of this description, built up against a clay earth on the Euston incline of the London and Birmingham Railway, about 26 feet high, and having a thickness of from $1\frac{1}{2}$ to $5\frac{1}{2}$ bricks, as appears from the diagram, gave way on one occasion, when some buildings were commenced near the cutting; the wall was dangerously driven inwards, and a number of iron girders were placed between the walls to give mutual support.

Fig. 52.



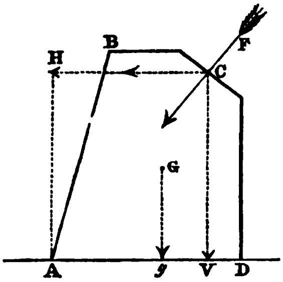
The previous consideration of the equilibrium and stability of walls, indicate sufficiently the great importance of gaining knowledge of this subject; the question how to build a wall, is the great question to be solved, when any structure is undertaken; proceeding without duly weighing the requirements of the walls, how many buildings, how many public works have failed, after the builder's natural enemies, which were mentioned in a preceding paragraph, had exerted their quality-testing powers for some time! What has been said above relates to particular kinds of walls; before, however, concluding the chapter, it will be useful to take a view of a wall in a more general manner, applicable to such cases as piers and abutments.

(46.) If a wall, abutment, or pier A B C D (*fig.* 53), have to sustain a force F in the direction F C, it is required to find what effect will ensue, and what is necessary to sustain equilibrium. Resolve the force F into two, H and V, one in a horizontal direction C H; and the other in a vertical direction C V; the latter force, consequently, tends neither to push the structure out of its place A D, nor over on the edge A; the component H endeavours to do both. The resistance to horizontal motion will be the vertical forces producing friction, if the wall be supposed to rest on its base without cement, or the resistance is $f \times (W + V)$, *W* standing for the weight of the wall, and *f*, as before, for the specific friction of the substances between which the friction would occur. If the structure be separated, the part above ground sliding on the lower, the first resistance to be overcome would then be cohesion of the cement or the substance of the wall itself, whichever possessed the less tenacity. Then the overturning force or moment of H, if the wall withstood sliding, would be $H \times AH$, which, if there be equilibrium, must be equal to the opposing forces together, or $W \times Ag + V \times AV$. If the wall be rectangular, as A H C V, then the mass of it will be equal to the height, multiplied by the thickness and the density of the materials, or $AH \times AV \times$ density, so that a strain occurring horizontally is met by this quantity, producing friction, which expresses the opposition which the wall can afford; the moment of inertia or resistance offered to an overturning motion is the mass multiplied by the distance of the vertical through the centre of gravity, which would be one-half AV, and the product is $\frac{1}{2} AV^2 \times AH \times$ density. From this statement we learn that the resistance to a horizontal disturbance increases (with respect to thickness) as the thickness increases, but the opposition offered to rotation increases as the square of AV, or the thickness.

---

* Quoted by Borgnis, Traité Elémentaire de Construction appliquée à l'Architecture Civile. Paris, 1823.

Fig. 53.

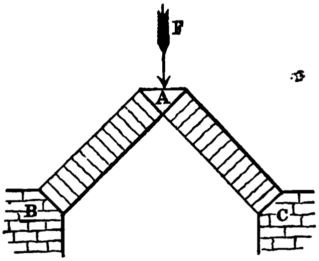


CHAPTER VI.—*The Arch.—Flat Arch. —Equilibrium of different Forms.— Coulomb's Views.—Rupture.*

(47.) THE highly scientific and beautiful character of the arch, as well as its social utility, has enlisted many able mathematicians in the work of developing its rigid principles of equilibrium, and closely-observing practical men in establishing the proper precautions to be taken in order to give stability and elegance to arched structures. However, the common theories of the arch are founded upon suppositions which practice does not require, neither can obtain.

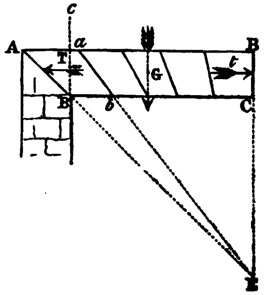
An arch may be said to be a structure in which a line, drawn from the vertical to the centre of gravity, does not pass through the joints of the structures; they are of an angular, flat, or generally of a curvilineal form. An angular arch combines the most simple with the strongest form, as the thrust exerted can only labour at one object—the crushing of the materials. It has proved useful in many cases. It is obvious that any force F (*fig.* 54), acting on the key-stone A, is met by two resisting pressures in the direction of the sides AB, A C, and becomes a question of the equilibrium of three forces. However, practically considered, the force is not generally at F alone; the sides are loaded, and sometimes the top is not.

Fig. 54.



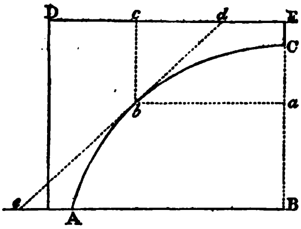
(48.) The flat arch or plate-bande is a useful and very common form of arch; it is composed of a number of bricks or stones with inclined faces AB, *ab* (*fig.* 55), the extremities abutting on walls or piers. By calculation it may be shown that the joints should be so inclined that the faces produced would meet in E, or the directions of the joints must meet in a common centre. The possibility of the parts slipping, or of turning over upon the edges A, C, B, is determined, by considering the weight of the half A B C D, acting through the centre of gravity G, and the thrust T, occasioned at the abutment A B, and also at the opposite part of the arch (at B C), by that weight; whence it is easily shown that there is a limit to the span of this kind of arch, when the depth of the stones and the inclination A B *c* from the vertical of the last joint A B are given; the span, when the last joint is inclined at 45° can be but five times the depth; if A B *c* be 30°, it can be extended to 7½ times the depth*. It also appears that the horizontal thrust at A B is not according to the depth B C of the arch, but varies as the square of the span. It would follow from this, that by adding to the weight of the arch its stability is increased, as it would be equivalent to adding weight to the piers supporting.

Fig. 55.



(49.) In the curvilineal and most general form of arch there are many varieties of outline, and, of course, requiring different calculations for their equilibrium. By the common theory of the arch the arch-stones are supposed to be uncemented and unaffected by friction, but to form a number of wedges, which by their mutual thrust, are supported in their hanging form. With these conditions mathematicians have undertaken to determine what must be the curve A *b* C (or *intrados*) (*fig.* 56), when the exterior profile D E (or *extrados*) is a straight line; or if the intrados A *b* C be given, what must be the form of the extrados D E. Also, the arch-stones being equal in size, what should be the form of the curve.

Fig. 56.



(50.) In the first case, calculation points out a curve, A *b* C, differing from a circle, inasmuch as the radius A B is greater than C B; this result is obtained on the supposition that the pressures on the arch-stones are vertical, and the material equal in specific weight in all parts; neither of which conditions is obtained in practice as required. This figure, however, is useful: when practical circumstances are introduced, friction and cements added, we find its susceptibility to change of form by reason of irregular pressures, no longer a matter of fear to the builder. The centre arch of Blackfriars Bridge is 40 feet rise (B C), and 100 feet span (twice A B), and by comparison with the following table, calculated by Dr. Hutton, for an arch of similar height and span, with an horizontal roadway, proves to be nearly the same in form :—

E *c*	*b c*	E *c*	*b c*
0	6·000	25	12·489
2	6·035	30	15·980
4	6·144	35	20·066
10	6·914	40	26·894
15	8·120	45	35·135
20	9·934	50	46·000

The above figure is drawn according to these measures.

(51.) In the second case, if a portion of a circular arc be required for the interior curve or intrados of the arch, it may be found that the stones and loading should vary as the cube of the secant of the inclination of the tangent to the horizon; in the figure above, *d e* is the tangent to the curve A C at the point *b*, and its inclination to the horizon is the angle *d c* B; then, if the cube of the secant (*e d*) of this angle be found for different points, as C, *b*, and A, of

---

* We have the following equation to determine the angle A B *c* (= *d*), or the length of the arch (= 2 *l*), or the depth B C (= *d*) :—

$$\tan. a = \sqrt{3\,\frac{(l^2 - d^2)}{a^2} - \frac{6\,l}{d}}.$$

the curve, we shall be able to draw the following figure, remembering that the number found expresses the length or depth of the line *c b*. The extrados here obtained sinks from the crown E (*fig.* 57) of the arch until it reaches F at about 35° from the vertical B E; it then takes a sudden turn and rises rapidly, so that the arch stones or loading should vary as the line *b c*, or between F and D it should rapidly increase to infinity. It is thus evident, if this be true, that no disposition whatever of the weight could keep up a semicircular arch; such a figure could not be equilibrated; the lower parts A would be driven outwards; neither could an arc greater than about 20° on each side the vertical E B, be used with an horizontal exterior, or roadway; arches, however, have been constructed with level, or nearly level, extrados, of a circular form, even to the extent of a semicircle, and stood a very long time. Most of the bridges and other arch-works, still remaining to record the wealth and science of ages passed away, are of a semicircular form.

Fig. 57.

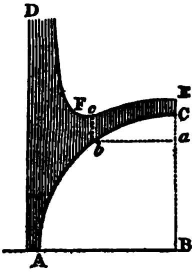
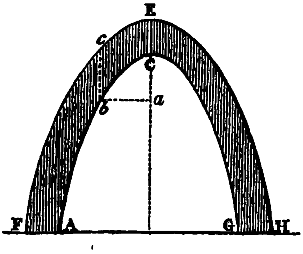
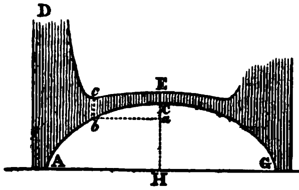


Fig. 58.



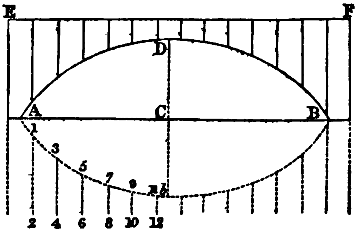
(52.) According to theory, if the curve of intrados A C G be given a parabola (*fig.* 58), the extrados F E H must be a parabola also. If the curve is to be an ellipse A C G, *b c* must vary somewhat less than the circle; still it will finally run up to an infinite height at D. With this figure it may be seen that about 45° may be allowed with a nearly horizontal roadway, which is much greater than in a circular curve*. The elliptical arch is, however, frequently used, on account of its elegance, and flatness at the crown, in a manner quite inconsistent with theory. Waterloo and London Bridges are fine examples of elliptical arches.

Fig. 59.



(53.) It may be also proved mathematically, that when the arch-stones are of equal weight, the figure of the arch should be that of a chain, when suspended loosely between two points, a curve called, in consequence, the *catenary*†; if, however, the pressure be not equal, an arch built of this form would be very unstable. Dr. Robison adopted a practical modification of it thus:—Supposing the span of the arch to be A B (*fig.* 60), the required height or rise about C D, and the roadway E F to be horizontal, let the figure be inverted, and A *b* B a chain of uniform weight forming a curve which at first will be a true catenary; then take pieces of chain 1 2,

Fig. 60.



---

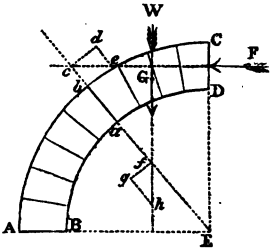
* The depth *b c* ($= y$) may be found for any point by the following formula: $a =$ semi-axis major (A H), $b =$ semiaxis minor (C H), and $x =$ C E. Then $y = \frac{a\,b^3}{(b-a)^3}$.

† *Catena*, Lat., a chain.

3 4, to represent the weight purposed to be put on the arch, at equal distances, 1, 3, 5; these weights will alter the curve, pulling downwards the chain between the middle *b* and the points of support, and allowing the middle *b* to rise. If the ends be trimmed to the straight line (care being taken to keep the weights unaltered), the curve A *b* B will be formed, and give the true form for the arch. It is said to have been adopted, on one occasion, for an arch of 60 feet span and 21 feet rise.

(54.) Coulomb made an excellent investigation respecting the principles governing the arch, independently of any given shape of intrados or extrados. In this inquiry he considered the possibility of an arch failing, either by some parts opening and the stones turning on their edges, or the direct pushing up or down of any part of the arch, as the effect of any force generally liable to act upon it. The following figure will indicate the very natural conditions which he supposes. A portion or half of an arch A B D C, standing on A B, is kept up by a horizontal force F acting against the highest stone in the direction F *c*, and the weight of its arch-stones; the force F answers to the thrust of the opposite half-arch, and it is required, in order that A C D B may stand, that this force should not be so great as to push it over, or backwards, nor allow its weight to overcome, and drive it inwards. Suppose any joint, as *a b*, be taken. Then the force must prevent *a* D C *b* from sliding along the joint in the direction *a b*, that is, outwards, or along *b a*, or inwards; neither allow it to turn over on the inner edge *a*, nor push it round over the upper edge *b*. When the force F is obtained he finds the greatest value required to prevent the sliding down of *b a*, and the least value necessary to push it along *a b;* also the least value capable of causing rotation about the edge *b*, and the greatest value to withstand the tendency to rotation about *a*. It is, consequently, indispensable to stability, that the values or intensity of the force F in these cases should be greater than the greatest, and less than the least value so found.

Fig. 61.



First, then, to investigate the forces concerned in sliding. The agent tending to push the mass *b* C D *a* inwards along *b a* is the weight W of the mass, which through its centre of gravity G presses vertically downwards; it acts, therefore, partly against the plane *a b*, obstructing the motion, and partly along the face, which is effectual: so that if *fh* be taken on the direction of the weight W to represent the weight, and *fg* be drawn perpendicular to, and *g h* parallel with, *a b*, or *a* E, these two lines represent the relative effect of the weight on the joint; *fg* being expended in compressing the materials, and *g h* in the production of motion, and their ratio with respect to the whole weight (*fh*) is $\frac{fg}{fh}$ and $\frac{gh}{fh}$. Hence we find the force pushing the mass down *a b* is

$$W \times \frac{g\,h}{f\,h}.$$

To oppose this there are,—F, acting obliquely against the joint *a b*, and the friction arising from the pressure of F and *fg* on the face of the joint. The force F must be resolved similarly to the weight W; if *a b* be produced to *c* where F *e* produced meets it, and *c e* be taken in magnitude to represent F, then *c d*, drawn perpendicular to *a c*, and *e d*, parallel with it, will indicate the magnitude of the force in those directions, and the ratio each bears to the original force F (*e c*), is shown as before,— $\frac{c\,d}{c\,e}$ and $\frac{e\,d}{c\,e}$, — the former merely compressing the materials, while the latter pushes in the direction *a b*, or the reverse to the direction of *g h*, which is *b a*. Then the first part of the opposing force is

$$F \times \frac{c\,d}{e\,c}.$$

The second part, or the friction, will be, of course, the effect of those components of the resolved forces W and F, which act perpendicular to the surface of the arch-stones at *a b*; these are $\frac{fg}{fh}$ and $\frac{dc}{ec}$ —these bring into operation the peculiar or specific friction of the substance composing the arch-stones, so that the coefficient of friction multiplied by the actual pressures gives us the actual friction; thus we have for this second part of the opposing forces

$$\text{friction} \times \left(\mathrm{W} \times \frac{fg}{fh} + \mathrm{F} \times \frac{dc}{ec}\right).$$

In addition to these resistances there is another, little understood, namely, that arising from the cohesion of the two surfaces in contact at the joint *a b*. This will be according to the quantity of surface in contact; and as every substance has its specific cohesion, like friction, the two elements must be multiplied together, whence this resistance is

$$\text{cohesion} \times \text{surface}.$$

We learn from this analysis of the forces, that to sustain equilibrium the disrupting force acting along *b a* must not exceed the resistances enumerated; gathering them together we should have for equilibrium

$$\mathrm{W} \times \frac{gh}{fg} = \mathrm{F} \times \frac{ed}{ec} + \text{frict.} \times \left(\mathrm{W} \times \frac{fg}{fh} + \mathrm{F} \times \frac{dc}{ec}\right) + \text{cohes.} \times \text{surface}.$$

We now have that value of F in this equation which will prevent the portion of the arch *a* D C *b* from sliding down *a b*, but it must be remembered that the value of the force F, if too great, that is, if $\mathrm{F} \times \frac{ed}{ec}$ be too great, it will move the mass up on the joint; to do so, however, it must overcome

$$\mathrm{W} \times \frac{gh}{fh} + \text{frict.} \left(\mathrm{F} \times \frac{ed}{ec} + \mathrm{W} \times \frac{fg}{fh}\right) + \text{cohes.} \times \text{surface}.$$

The least value of F necessary to overcome this resistance, or to turn the part of the arch over the edge at *b*, is here shown.

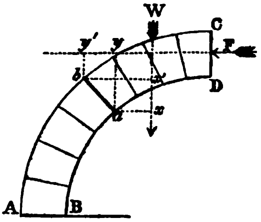
The magnitude of the component forces *c d*, *d e*, and *f g*, *g h*, does not continue the same for all the joints; *c d*, for instance, will be less as the angle *c e d* becomes less, which it will do as each joint below *a b* is the subject of calculation, and greater with each joint above *a b*, until, at C D, *c d* becomes equal to *c e*, or the whole force is effective. The angle *g f h* follows the same law of decrease and increase; but it may be easily shown by geometry that the triangles *c d e*, *f g h*, are both similar to the triangle *c* E D, the angle at E being equal to the angle *d c e*, and the angle *g h f* equal to the angle D E *c*, so that the sides of the triangles are proportional: thus the effects of the forces may be represented by F E and F *c*. From this it will be plain, that as the elevation of the joint *a b*, or the angle it makes with the horizontal line is greater, so will the vertical force of F on the joint increase, that is, F E (*c d*) will become of greater value as compared with F *c* (or *d e*), which is that part of the force tending to push the stone along the joint; finally, at C D, where the force F is perpendicular, we know that the lines *c* E and C E would coincide, that is, *c* F would disappear, and the force would have no power to make the arch-stones slide, it being perpendicular to the surface. Similarly with the weight W, represented by *f h* (or *e* E), the higher the joint the greater will be E F in comparison with F *c*; and E F corresponds to *g h*, or the power of the weight to make the stones slide, so that at C D, *c* E (representing the whole weight) it would be equal to F E, or the stone would be urged to slide with all the force of its weight—this we know to be true; and the lower the joint the greater would F *c* become, or *f g*—the force exerted perpendicular to the joint—would increase, until, at A B, as we are well aware, the whole weight would be perpendicular to the joint, or *f g* would become equal to *f h*, or F *c* equal to *c* E.

We have now considered the probability of any sliding of the parts of an arch in consequence of its weight or the horizontal force applied at its crown; the remaining, and most possible movement that may occur, is a turning of some one or more arch-stones on the edges of the stone immediately below it; this can be readily estimated by the principle of the moment of forces (art. 5, Eq. Struct.). Producing the forces F and W (*fig.* 62), perpendiculars *ax bx'*, *ay by'*, may be drawn from the inferior superior edges of the stone; for about one of these the rotation, if any, must take place. With respect, then, to a falling inwards of the upper part $abCD$, or rotation about the joint at a, the force tending to produce it is the weight of the mass W, and its actual moment of effectiveness depends on the arm by which it acts, or on $a\,x$, thus the moment of rotation about a is $W \times a\,x$; to oppose this there is the force F, acting with the arm $a\,y$, or $F \times a\,y$. There is, however, a resistance arising from the cohesion of the cement, or adhesion of the surfaces; what this is, is not very certain, but it has been supposed to vary with the distance of any adhering point from the point at rest (or a). By the calculation in the note it may be found on this supposition to be equal to $\frac{1}{3} \times$ adhesion $\times$ square of the length of the joint $a\,b$*. This, added to the force $F \times a\,y$, is the whole resistance to the action of the weight, or $W \times a\,x$. The greatest value of F, therefore, which will prevent the overturning, must be taken; it may, however, become so great as to drive the mass over the edge b. In this case the moment will be $F \times b\,y'$, and the resistances are

$$W \times b\,x' + \tfrac{1}{3} \times \text{adhesion} \times (\text{length of joint})^2;$$

and the least value of F must be found which will affect this movement; this value being the greatest admissible.

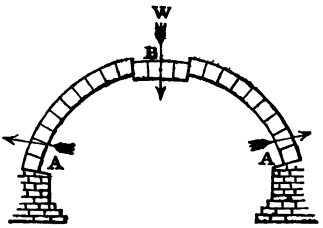
Fig. 62.



Thus Coulomb established the conditions of disrupture for sliding or rotating about some of the edges; the calculation, however, is for one joint only, and would have to be repeated for each joint, making such a process very laborious.

(55.) The most useful and certain source of enlightenment concerning the manner in which forces act in the arch, is the rupture of arches by sliding or overturning. Much has been learned by attending to cases where arches have either settled (or slid, until the forces equilibrated,) or given way under their own or extra pressure. According to the above investigation we may conclude that if the force F, or the thrust arising from the opposite half-arch, be unequal to the effort of the weight W, even after allowing it the assistance of the friction, then a sliding will occur; if the crown be too heavily loaded, W increases, and may overcome, unless the inward pressure at the lower parts A (*fig.* 63), or haunches, is sufficient to withstand it: if W be too great the crown at B will slide downwards, pushing the haunches A A outwards.

Fig. 63.



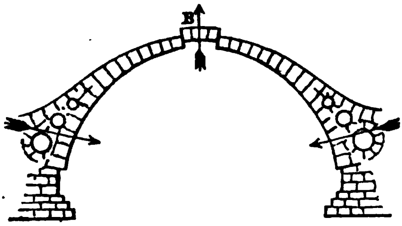
On the other hand, if the haunches are of too great a weight in comparison with the weight of the crown, the arch will give way by the expulsion of the crown, and the inward motion of the haunches, until the whole falls; this occurred with a fine bridge called Pont y prŷd (*the beautiful bridge*), built by a self-taught mason, William Edwards, over the Taaf, in Glamorganshire; when the arch was completed the crown was forced up and the haunches slid inwards; he rectified the structure by making

---

[*] If an elementary portion dv of the joint $a\,b$, be taken, its distance from the edge a being equal to v, the resistance of the element will be $\frac{v}{l}\,C\,dv$, C expressing the specific adhesion of the substances, and l the length of the joint; the moment of this resistance will be $\frac{v}{l}\,C\,dv \times v$, and the sum of the moments of all these small resistances will be $\int \frac{v}{l}\,C\,dv \times v = \frac{C}{l}\int v^2\,dv$, this is readily integrated, and its value may be found for the whole length of the joint, or $\frac{C}{l}\int_0^l v^2\,dv = \frac{1}{3}\,C\,l^2$, the whole adhesion.

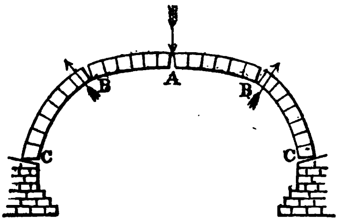
three holes in the lower parts, which greatly relieved the upper parts from the pressure. The improvement is shown in the figure by the dotted circles. This derangement evidently results from the insufficient value of W, or the arch-weight, at the higher portions, in comparison with the lower.

Fig. 64.



These two cases include the chances of sliding among the parts of the arch, but the rotation may arise from similar causes. The general appearance of these movements are shown in the following figure. Sometimes an arch will open at the under side of the crown, or A; the stones turning about their upper edge, this is accompanied by a corresponding separating of the arch-stones on the outside at some distance from the crown, as B; while at the abutments C, the motion is similar to that at A; the upper parts consequently are thrust inwards, while the haunches, turning round their external lower edges, fly outwards; this is owing to the excess of the weight or loading at the crown, while the parts B C are insufficiently heavy.

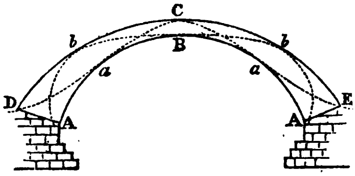
Fig. 65.



The motion may occur in the opposite direction, that is, the parts at A, turning upon their lower edges, may be impelled upwards, while the haunches, turning about the edges C and the outer edges at B, fall in.

Experiments have been tried with a view to determine when and where this movement should commence. Dr. Robison tried some chalk models, loading them until they fell. The phenomena occurred as the last figure represents; there was always a chipping or crushing of the inner edges of his model arch-stones between B and A before the arch fell; and he supposed that, to exhibit the propagation of the pressure, a straight line should be drawn from the upper edge at A, where the stones touch, through as many arch-stones as it will pass without cutting the internal or external surfaces; along this line the pressure is supposed to act, but later calculations have given it a curved form, as shown in the next figure. Here the dotted lines indicate the line of pressure in both cases: where the pressure on the crown is increased the line of pressure rises until it touches the extrados of the arch at C, sinking to the intrados at *a a*, and again meeting the external face at D; if the weight cause it to pass beyond the edge of the arch-stones, then the arch will open at B, *b b*, and A A, the edges at C, *a*, and D sustaining the pressure. Increase of weight at *b b*, will raise the line at those points, depressing it at the crown and the abutments; A *b* B *b* A will therefore be the limit of the change, so that the arch may stand; if greater, the arch will open at C and about *a a*, and D E, the edges at B, *b b*, A A, propagating the pressure.

Fig. 66.



An admirable observation of M. Perronet, when building a bridge at St. Edme de Nogent, testifies in favour of this view. He drew on the face of the arch-stones, while the bridge was building, three lines,—one horizontal, over the crown, and the others, joining it, passed over the face of the stone-work to the abutment; when the centre was removed, the horizontal line sank through its whole length, but most at the key-stone, while the others moved inwards at the upper part, and curved outwards at the lower, similar to the line C *a* D; the settlement of other arches has exhibited a similar movement.

Dr. Robison took an observation of an arch which fell by the rotation of the arch-stones on each other; he remarks that some time before it fell chips were observed to drop off from the joints about ten feet on each side of the crown, or about half-way between B and *a* (last figure); the joints at B, *b b*, and A A, then opened, and chips dropped from the points *a a;* immediately afterwards the crown sunk, the haunches rose, and as soon as the edges at C were broken off, the arch fell in pieces. In this instance the curve of pressure evidently changed its position as the destruction of the edges went on, at first touching the intrados, midway between B and *a*, but, as these edges gave way, the point of contact was carried farther down, finally assuming a form probably similar to the line D *a* C *a* E.

M. Gauthey observed a similar process, when he tried the experiment with a bridge which was to be destroyed; it separated in falling into four parts, the edges chipping at *a*, *a*, before the final movement. M. Boistard and others experimented with models to discover the points of rupture, and the results obtained are these:—

In semicircular arches the points of rupture were at 30° from the springing or abutment.

In oval arches, the figure being made of three circular arcs at 50° from the abutment, measured on the smaller circle.

With flat arches, the point of rupture was at the springing, which also occurred in the circular arc, the height of which was less than one-quarter of the span.

In all cases the whole mass of the arch tended to separate into four portions, turning on the extrados at the springing and crown, and opening at two intermediate points.

Gauthey made a calculation of the position of these important points in the arch, taking the span at 65½ feet (20 mètres), and the thickness of the arch at the crown 3¼ feet, with a level extrados or roadway.

	Point of rupture.
Semicircular arch . .	27°
Flat arch, rise = ⅓ of the span	45
Do. rise = ¼ do. .	54
Circular arc of 60°, on piers 16·4 feet high . .	0

CHAPTER VII.—BRIDGES.—*Stone, Wood, and Iron.*—*Stone:*—*Forms of the Arches.*—*Foundations.*—*Piers and Abutments.*—*Laying the Arch-courses. Spandrels.*—*Thickness of Key-stone.*—*Bridges of London, Chester, Dora Riparia, SSma. Trinita, &c.*

(56.) WE conclude, from a theoretical examination of the principles of the arch, that it is a very delicate structure, which, with a very small variation in superacting forces, would totter and fall; in fact, an arch would be a thing, when built, only to look at and admire its gracefulness of form, but not to touch, much less to sustain heavy weights continually passing over it: practice, however, tells a different tale, and reveals the wonderful capabilities of the arch, insomuch that often, where a solid piece of brickwork or masonry might be employed, relieving arches are preferred, not merely because of the saving of material, but for the superior strength obtained. Thus the arch, if constructed with any moderate amount of care and knowledge, is a very stable structure.

(57.) Its use, in consequence of these well-known qualities, is very general: it is, however, in bridges, that the arch has been so much studied. In a bridge, by which is meant a structure spanning a river or other obstruction to intercourse between places, there are so many circumstances calling for the exercise of skill in arch-building, that bold attempts have been made to try to their limits the strength of the materials and the ability of workmanship; various materials have been successively tried in order to surpass, if possible, former works; and various forms adopted to increase convenience, or add new elegance.

The materials now in general use for building bridges are stone, brick, wood, and iron. There are advantages and disadvantages in each substance, but stone has the preference, for convenience in working, durability, and neatness of appearance, taken together; brick is a most useful material, nevertheless, and is much used for bridges over canals, for small roads, and the numerous arches which are required for the passage of railways, but there are disadvantages known to the practical man, arising from the form of brick, which is unsuited for voussoirs; while the inner edges meet, the outer must necessarily be somewhat apart: this deficiency is made up with mortar, which is, unless very good, an unsafe substitute, requiring considerable time before it dries; the porosity again of brick militates strongly against its use in situations where it would be constantly exposed to water. The superiority of stone, however, is limited to the form of arch where the parts are sustained by a mutual crushing force, as in ordinary bridges; where a tensile force is likely to occur, stone is dangerous, from its brittleness and frangibility; wood and iron then becomes invaluable, especially the latter. Wood is very extensively used for bridges, being generally easy to obtain, and is readily worked; its defects, in comparison with stone, are the changes likely to be produced by alteration in the heat and moisture of the atmosphere, and its endurance. Wood also is destroyed by animal and vegetable agents; but by preserving processes, the wood is fortified greatly against these destructive influences. Iron is a material, the value of which was little known some time since, neither is it fully understood at present: its utility in bridge building has been sufficiently shown of late years; it being highly tenacious, and capable of resisting great crushing pressure, fits it for all kinds of bridges, and where stone would be useless. Its great liability to oxidize or rust much injures its value, although by giving it a protecting coat, such as of paint, this evil can in a great measure be remedied; it is also liable to a great change of bulk, owing to variations of temperature, the expansion in length of a cast-iron bar, 100 feet long, being 1½ inch between the freezing and boiling points of water. The introduction of iron for bridge building has brought into practice an elegant, and, at the same time, a most invaluable form—the suspension bridge. In this chapter we shall describe the practice of building in stone, with examples of good stone bridges.

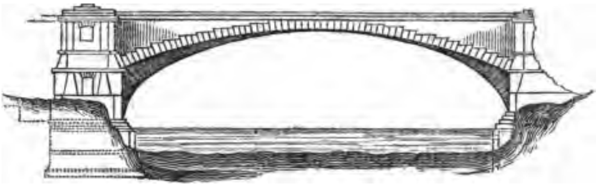
(58.) Various forms are given to the arches of stone bridges, elegance and a level roadway being the chief considerations. There are three general kinds, namely, the semi-circular, the flat arch of an elliptical form, and arches of arcs of circles, variable in magnitude. Of the first description are most of the ancient arches; generally they are not very graceful in appearance, and have the credit of obstructing the current to a considerable extent: they cannot be of very great span, because their height must be equal to half the span, which would give either an insurmountable inclination to the roadway, or require approaches to be made of an extensive kind. We may observe in some old bridges the great rise of the roadway, owing to the loftiness of the semicircular arch bearing it; Pont y prŷd exhibits a very elevated extrados (see *fig.* 64, last chapter); in Blackfriars Bridge, which is nearly of a semicircular form, the rise being 43 feet for a span of 100 feet of the central arch, the roadway presents a gradient or rise of 1 in 15. Westminster Bridge formerly supported an equally steep roadway, but it has been reduced to 1 in 24.

The form of arch denominated flat is either a half ellipse or a figure somewhat resembling it, made by several arcs of circles put together; the latter method is preferred for the greater room afforded to the water current. The curves thus made up are drawn by three arcs of a circle, if the height of the arch is above one-third of the span; if less, a greater number. Perronet used twelve arcs to make the curve for the Neuilly Bridge, the height being one-fourth of the span. The bridge formed of such arches allows of a low and level roadway, at the same time requiring but few arches, as the span may be great. In the bridge just alluded to, the height is 31¾ feet, for a span of 127 feet: had this been a semicircular arch of the same span, its height would have been 60 feet, or the roadway at the crown 30 feet above its actual position. In addition to these important particulars, the appearance of the low arch is elegant.

A great number of bridges have been executed of late years, in which a simple portion of a circular arc has been taken for the figure. Some of them appear well; but others, on account of their abruptly starting from the abutments, when these stand up high out of the water, are not pleasing to the eye. Where the springings of the arch are below the water, a considerable portion of the waterway is blocked up, and the haunches become very bulky; this latter evil has been compensated by filling them up with light earth, or small arches. On the other hand, if the springing or abutments be raised out of the water, and the roadway is not to be raised, the arch must be low, or very flat; in that case the horizontal thrust of the masonry is very much increased, and the crown more liable to sink. The bridge erected over the Dee at Chester is a large arc of 200 feet span, and the rise 42 feet; the great dimensions are, however, the principal points of interest; had the springings been higher, which would have kept them out of the floods, the appearance would have been much more agreeable; or still more so, had the curve been elliptical*. The bridge of a single arch of 147·63 feet span over the Dora, near Turin (see *fig.* 77), has a rise of only 18¾ feet, although the springings are in the way of floods, but the flatness and consequent lightness of the arch, with its architectural simplicity, constitutes it an elegant object.

A French mode of saving the appearance of the arch springings dipping in the water at high tides has been, in some cases, adopted. While the general body of the arch springs from beneath the surface of the water, the archivolts, or arch-stones of the faces, form a segment of a circle whose springings are at a considerable height on the piers. Perronet, in the Neuilly Bridge, adopted this method; his elliptical arch was marked by a front arch, of a circular arc, having a radius of 160 feet. The Gloucester Over Bridge, built by Telford, is an elliptical arch of 150 feet, but the outer faces are formed by the arc of a circle of 220 feet radius. *Fig.* 67 exhibits this arrangement. It has been thought by Telford that the opening is thus altered "into the shape of the entrance of a pipe," suiting the contracted passage of the water, and thus lessens the "flat surface opposed to the current of the river whenever the tide rises above the springing or middle of the ellipse."† Perronet also considers that the introduction or passage of the water is facilitated. However, in any form of arch, the care exercised to give free passage to the water is well spent, as many bridges have been destroyed through neglect of this particular.

Fig. 67.



GLOUCESTER OVER BRIDGE.

(59.) In building a bridge, after a suitable site is determined, and the bed of the river examined, both as to its variation of surface, and the state of the bed, the next and most important work in the whole structure is, laying the foundations for the piers and abutments. The piers must have a foundation sufficiently strong to support its own weight and that of the two half arches resting upon it: this strength may be obtained by a small base or foundation if the earth on which it stands is strong enough, or, if the soil be weak, the base must be proportionally larger. The earth under the stream on which the foundations for piers are laid must be well examined, not merely superficially, but for some depth, as a sound seat for the masonry must be obtained; and although the superficial earth may appear good and firm, it may continue only for a very small depth. On a French railway a viaduct was built apparently on a good base of clay, yet some of this being dug out to make way for a layer of concrete (a mixture of lime and gravel), and the pier erected thereon, it was proved by the falling in of the viaduct at this part that the earth was insecure; beneath the clay bed, which was not thick, it appeared there was one of sand, and the cutting away a portion of the bed for the concrete completed its weakness. In some cases the ground is rocky, and the engineer is much assisted in his work; but where movable sands or muddy beds are presented for the foundations, great skill and labour is required to form them. Where bridges are thrown across roads or other localities where they are not affected by currents of water, a layer of concrete, which forms a hard and firm bed, is sufficient to trust the piers upon; but in large rivers all resources are required. Where the bed calls for strengthening, piles of 8 or 9 inches square are driven in until they are almost immovable after several blows of a ram or monkey; these piles are placed 3 or 4 feet apart, and extend over the area intended for the foundation. Another method, called by the French *encaissement*, is to encompass the area by a series of piles, filling up between them with sheeting piles, so as to form an enclosed space, which is dug out for the depth of a foot, and the hole so made filled with concrete. Such a method is serviceable in a quicksand or movable sand bed, as the sand of itself is firm, and when protected by the encaissement and concrete from the drifting power of the tide, has been found to form a sufficiently strong base for the masonry; unprotected, however, sand has proved very treacherous to the engineer. In the Hexham Bridge, built under Smeaton's direction, the gravel bed, owing to the floods, which he thought were properly provided for, was removed from under the foundation of a pier, and the bridge fell down. A fine bridge at Plymouth, over the Lary, was found to be in imminent danger in consequence of the drifting away of the sandy bottom upon which the piers stood; and it became necessary to protect the bed with a layer of clay, and an overlayer of stone, for a distance of 60 or 70 feet on each side of the bridge (see *fig*. 68, D, E).

---

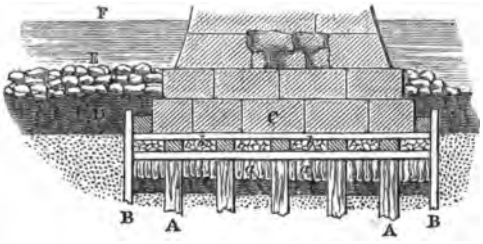
* Hosking (Practical Treatise), in *The Theory, Practice, and Architecture of Bridges.* London, 1843.

† Telford's Life and Works, p. 261.

When the piles are thus driven in, the tops are cut off level, and the earth between them excavated for the depth of a foot, to admit a layer of broken stones, with lime and sand; upon the levelled pile heads a platform of oak, beech, or elm planking is laid, followed by a second laid across, and the whole strongly jointed; on this stage the masonry is commenced.

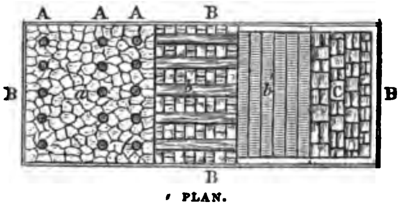
The following figures show the foundations of a pier of Lary Bridge, near Plymouth, which was set on a sandy bed; a layer of clay was placed on the sand between the piles A, A (*figs*. 68, 69), and above that a bed of rough stones *a*, *a*, and a series of sheeting piles B, B, inclosing the area; the tops of the piles being cut level, a caisson was used to deposit the masonry, and the flooring which it left for the courses is seen at *b b*, the lower planking immediately on the pile heads being 4 inches thick; above this were five beams passing longitudinally, 12 inches by 8 inches, in section; uppermost was another layer of planks 3 inches thick (seen at *b'*, *fig*. 69). Directly upon this the masonry C of the pier was laid: F (*fig*. 68), shows the waterline, and D, E, mark the clay and stone work which was laid, in order to

Fig. 68.



SECTION.

Fig. 69.



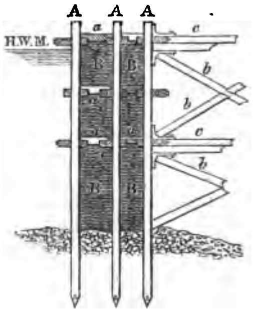
PLAN.

counteract the action of the water. The principal piles A A were 35 feet long, the depth of sand being very great; the sand was excavated by a large spoon for a depth of 3 feet between the piles, for the clay and rough stonework. A considerable portion of the work was done by the protection of a diving-bell in an admirable manner*.

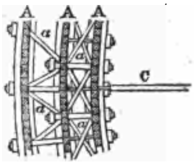
To give additional strength where there was any doubt, inverted arches have been used, spanning from pier to pier, which were laid on a course of planking; this plan was adopted by Telford, who also, for a base, laid courses of rough large stones on their edges extending along the whole length of the bridge, and forming a thick pavement.

(60.) Much of the foundations is sometimes done under the water, and, notwithstanding the apparent difficulty of thus working in the dark, excellent foundations have been laid for various works under water, the stones being let down and the spot noted or found by probing with a rod. An old method of laying the foundations was by the use of a rafter, bearing a quantity of masonry, which was floated to the proper place and then sunk; this was modified into the caisson, a large chest capable of supporting many tons of stone-work. Westminster Bridge was thus built up to the water level. Sometimes, where the circumstances permitted, a new channel has been cut out for the river, in which the foundation could be easily laid. The most expensive and difficult, though necessary, method of proceeding, is the construction of coffer-dams. These are formed by a double row, at least, of piles, reaching above the water line, between which a puddle or filling-in of earth is closely packed; the water in the space inclosed is then pumped out by a steam engine, and the operations can be carried on under this shelter. *Fig.* 70 exhibits a section and plan of part of the coffer-dam used in building the London Bridge; three rows of piles A A A, about 50 feet long, and shod with iron, were driven into the bed of the river, and firmly bolted together, and the space between B B, filled or puddled with clay; struts *a a*, were introduced between the rows of piles, and bolts tied them in; braces *b b*, abutting on longitudinal beams *c e*, supported the piling in the interior space. The line H. W. M. denotes high-water mark.

Fig. 70.



SECTION.



PLAN.

(61.) The piers thus commenced should have their foundations of a larger area than the body of the pier; the best arrangement of a pier is a broad base with a superstructure diminishing in a curved line to the springing of the arch. It is recommended that an offset or projection of 6 inches for each course beyond that above it should be given for the first 4 feet in height, and in the first and second courses the stones are generally larger than in the overlayers; the piers are solid throughout, the blocks of stone being secured by cramps, cement, &c., and the ends provided with angular projections for cutwaters; these necessary guards to the piers, to prevent any destructive effect from the concussion of floating masses, as ice, and large vessels, are generally formed by a portion of a circular arc. Smeaton recommends two arcs of 60° each, as best suited to divide the current; some have recommended triangular and semicircular forms. In Telford's bridges the triangular form is principally used. The piers of bridges are, on an average, from one-eighth to one-seventh of the span of the arch in thickness; those of the Neuilly Bridge are one-ninth only; the piers of Waterloo and London Bridges under one-sixth of the span, and Westminster and Blackfriars Bridges reach so great a thickness as one-fourth of the span.

---

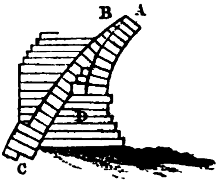
* Transactions of the Institution of Civil Engineers, vol. i. p. 100.

(62.) Great care is bestowed on the terminal supports of bridges, or the abutments. The massiveness of the piers has been less considered, because they have been supposed to transfer the thrust, and not sustain it, whereas the abutments have no similar horizontal support; they are therefore made of great weight, and, generally, in the best specimens of these structures, arranged with a view to convert the horizontal into a vertical thrust, or to transfer much of the pressure to the base of the abutment, where means are provided for sustaining it.

Where the abutment of the terminal arches of a bridge is on solid rock, little trouble is necessary; but, as is generally the case, should the site present a loose and compressible bed for the masonry, piling and platforms must be used. The material of the abutments is generally masonry or brickwork, and they are of such a weight as to require no aid from the earth behind them.

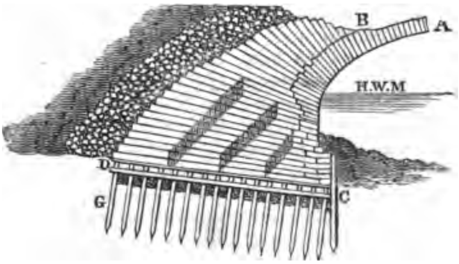
The thrust sustained by the abutments varies much with the form of the arch; thus semicircular arches give less horizontal pressure than elliptical or flat arches, and these again less than those formed of a small segment of a circle; a great portion of the vertical pressure arises from the filling in of the spandrels or space between the haunches of an arch and the roadway. This weight can act serviceably only in assisting the abutment against a sliding motion through the thrust of the arch; otherwise it acts injuriously in overloading the foundations, and is not compensated by the former effect, for the courses of stones are frequently laid in a radiating direction, which is highly beneficial. In an elliptical arch the direction of the greatest thrust is when the tangent to the curve of the arch is about 60° to the horizon; it is at this point that the curve turns most rapidly, so that the abutments are recommended to start from this point, the surfaces or faces of the abutment stones being kept at right angles to this tangent line, down to the platform or base. In many cases the courses of the arch-stones are carried on through the abutment; the following figure represents an abutment of a bridge over the river Ribble (Lancashire).

Fig. 71.



The arches are elliptical and of 120 feet span, with about 32 feet rise. The arch proper suddenly turns, as usual in these arches, at A, or where the tangent to the curve is about 60° inclined to the horizon; here the internal arch B begins, carrying the thrust from this point to C, or the foundation, which lies on a rocky bed: a great deal depends on the settling of the masonry; but if the settlement happened to be less in this additional arch than the other portion of the abutments, a great pressure would be exerted on the small base which C presents: in this instance, if the settlement be equal, the pressure will be divided between the segment B and the front part D of the abutment. In some bridges the arch abuts on masonry with a face inclined similar to the arch-stone meeting it, and the faces continually approach to the horizontal towards the foundations; this is the case with London Bridge. This complete and instructive bridge presents a very general example of bridge-work; one of the abutments is shown in the following figure. The elliptical arch A is met at B by the masonry of the abutment; the courses are gradually inclined and extended, so that at the base C D the face is but a few degrees inclined to the horizon; the last course lying on a platform of carpentry, which finally rests on the levelled heads of the piles; the space between them is strengthened to a little depth by made ground, and they are not driven vertically, but somewhat inclined, so as to be perpendicular to the face of the lowest course; this gives them much greater effect; behind this mass of solid masonry is an overlaying bed of concrete E, against which the common earth was thrown up. This enormous mass of stone-work is, as seen in the following figure, at the lowest course, 73 feet from the large pile C at the foot of the arch to the opposite extremity D; some of the interior fastenings of stone to stone are shown, the shaded parts running up from the foundations. The greater part of the abutment is below the level of high water, which is shown by the line H. W. M.; this is 34 feet above the foundations.

Fig. 72.

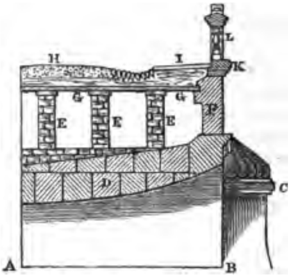


Many bridges have been built where the abutment courses are thus radiating; the great Chester Bridge presents a similar order; except that there is but a small approach to the horizontal in the lower courses, in comparison with the section of the London Bridge abutment; backing those inclined courses is a small arch, under and beyond which the courses are laid horizontally; the abutments of this bridge had the advantage, for the most part, of a rocky bed. In a great number of cases, however, a common horizontally laid mass of materials is supplied to the arch; this method allows of all the chances of fracture between the courses of stone to a great extent, as, when the arch abuts in a direction near to the horizontal, a very small portion of the thrust tends to press the courses together,—it is exerted in tearing them asunder, because no regard is paid to the extent of the angle of resistance. The bridge of St. Maxence presents an example of this form; the arches of this bridge being small segments of a circle, and springing from the abutments at an angle of about 20° only, appear almost to be huge lintels, and with a mass of stonework spanning 76 feet 8 inches, exert an enormous thrust nearly horizontal; to meet this there is a quantity of masonry, extending about 64 feet inland from the springing of the arch, 34½ feet high, and 40 feet wide, independently of the indirect assistance of the side works. All this masonry is laid in horizontal courses; less material, disposed so as to make the transition form a direction nearly vertical to the horizontal, and giving a spreading base to distribute the pressure on the foundations, would have been more in accordance with mechanical principles.

It would appear, from observing the most recent practice in bridge building, that far greater massiveness is given to the substance of arches and their supports than is required. Some engineers, considering that each arch should stand of itself, have given but a moderate mass of abutment, and large piers, in comparison with others, who have allowed small piers, and supposed that there must consequently be an accumulation of pressure for the abutment to bear. Some very old bridges are quite monsters in this respect; appearing to be all support with very little to be supported; a river would sometimes be nearly dammed up with stonework intended to bear the weight of a number of arches, varying in size from one *grand* arch (probably 70 feet span, as in Old London Bridge), to openings which appear like the vents of a sewer. The advance made of late has been very great in this respect, but the call for quick and cheap, yet strong work, which there has been through the extension of railways, shows that much more may be done with materials than architects have hitherto supposed. The foundations of some railway bridges are, in comparison with former practice, exceedingly slight, yet they support the railway under the pressure of enormous weights,—weights also moving with high velocities, and producing a destructive vibration. A fine arch carrying the Great Western Railway over the Thames at Maidenhead, reaching over a space of 128 feet, rests on benches lying simply on the chalk stratum. In some bridges, the abutments are smaller than usual, and counterforts or buttresses are placed against the back part of the abutment to strengthen the support. In the Chester arch the principal part of the abutment extends about 40 feet inwards from the springing; more masonry is added, but it is lightened by the introduction of a small arch, 20 feet wide and 35 feet high. Where the foundation is very solid, little or no masonry except a continuation of the arch is required; in an ancient bridge which stretched across the river Adda, at Trezzo, Milan, the arch-courses abut on the rocky banks of the river; this was not a small arch, for it appears to have been an arc of about 140°, having a rise of nearly 88 feet, and the enormous span of 251 feet.

(63.) When the centres are erected, the laying of the arching courses begins on each side simultaneously; the first row or course of stones laid on the piers is the *springing* course, the upper face being cut in the direction of the radius of the arch, to form a properly inclined bed for the regular wedges of the arch. Sometimes an attachment is made between the springing and the next superior course, as in the Dora Bridge. The upper courses are laid either directly with mortar spread evenly over the faces, or, where particular care is taken, lead or iron is sometimes inserted to keep the proper form or give an even bed for each stone; these, as well as other peculiar circumstances, will be illustrated in a few subsequent descriptions of bridges, which are always highly instructive: in some cases the arch-stones have been laid dry; and being then accurately placed, the joints have been filled with mortar after all the courses were laid. Great care is taken in giving the arch-stones the proper inclination, so as to form wedges fitting each other closely; the external or facing stones being generally cut so as to meet the horizontal masonry between the haunches (or the spandrel) neatly; sometimes the radiating order is carried up to the cornice or upper part of the bridge, but the appearance is not nearly so proper for such a structure, especially where large arcs are used, giving a great depth of the spandrel: this is shown in Westminster Bridge; the spandrel or space between the pier and roadway is sometimes filled up with rubble or rubbish, which presses upon the back or extrados of the arch and the pier directly, producing a great and unnecessary pressure for the foundations to support. It would appear that the sinking of a pier of Westminster Bridge occurred partly through the pressure of the rubble filling-in of the spandrel. Cylindrical openings have been adopted to relieve the weight, which was Smeaton's method; but Mr. Telford introduced the system of longitudinal walls, which he practised in Tongueland and other bridges; he filled up the space for some distance with close rubble, and upon it erected walls of from 18 inches to 3 feet thick, and 2 or 3 feet apart; they were steadied or tied together by laying long stones from one to another, the outer or visible walls being thicker than the rest; long stones rest upon the spandrel walls and support the roadway. *Fig.* 73 is a section of Wellesley Bridge, Limerick, taken very near the springing, the line of which is seen at A B, B C being one of the piers; the section of stones D is therefore part of the arch, and they form a curved instead of a straight line, because the arches in this bridge are of the complex form mentioned at page 153. Immediately over these stones is seen the upper part of a filling-in of rubble, which extends from the pier at the line of springing to a height of 12½ feet; there is therefore a partially solid spandrel, but above this height the courses cease: the walls E are carried up at a distance of 6 feet from each other, and are 18 inches thick, and 6 feet 6 inches high; these also are built of undressed or rubble stone, but the outer wall F is of dressed stone and 2 feet thick; above these walls flag-stones G G are laid, upon which the materials for the road H and the foot-path I rest; K and L are, re-

Fig. 73.



spectively, the cornice and parapet of the bridge. This bridge was built by Mr. Alexander Nimmo, a pupil of Telford.

(64.) With respect to the size of the arch-stones requisite in any particular case, little can be said to guide any engineer in his undertaking: in all cases former practice is alluded to as a rule, but generally there appears to be a much greater depth of the arch-stones than is requisite, if judgment be given by the known strength of the materials, and the weight they are likely to bear. Calculations tedious to detail, but the results of which may be stated, point this out most strongly. It has been estimated* that in the bridge of Neuilly the horizontal pressure against the keystone, arising from the weight of the arch, filling-in, pavement, and possible load of carriages and passengers, amounts to 408,013·8 lbs.; now the masonry of the bridge is of Saillancourt stone, of which Rondelet states a cubic piece 1·968 inches in measure, requires a pressure of 7719 lbs. to crush it. Then, as the arch-stones at the crown are 5·328 feet deep, the force required to crush them is 5,017,467 lbs., or about twelve times as much as it is probable it will ever be called on to sustain. This is not a singular instance; nearly all bridges might similarly be shown to be unnecessarily massive in this as in other respects; Blackfriars Bridge was proposed by Mr. Mylne to have arch-stones 6 feet deep at the key; this was pronounced by Mr. Simpson, examined before a committee concerning the subject, to be more than requisite by a foot or more; and in the central arch the thickness actually given is less than the smaller limit given by the above witness. The evil arising from this excessive strength is not merely the expense of useless material and building, and the greater time necessary to erect the bridge, but the injurious effect produced on the foundations of the structure. The principal, and certainly a powerful reason offered for it is, the impossibility of fairly bringing the whole face of the stones into contact with each other, because of bad workmanship: in some cases this is notorious; it was found, on repairing Westminster and Blackfriars Bridges, that, in various places, even in the external masonry, chips of stone and wood, and pieces of slate, had been packed in at the joints, which gaped through careless working. If the mason's labour can be trusted, or is closely looked after, much improvement might be effected in this particular; thus it is that a collection of the practice of different engineers exhibits a considerable difference in the substance given to arches. The following table shows the proportions of the thickness of the arch-stones at the key to the span and rise of the arches; the materials of construction at the key are named, although they vary very much in their strength, as some limestone and sandstone approaches the strength of granite, while others are not much stronger than good brick.

	Span.	Rise.	Material.
Neuilly (127 feet span), elliptical	24·5	4·7	Sandstone.
Tongueland (118 feet)	32·7	10·5	Do.
Victoria (160 feet), Durham Railway	34·7	15·6	Do.
Hutcheson (70 feet), Glasgow	20·0	3·9	Do.
Gloucester Over (150 feet), elliptical	33·3	7·7	Do.
Chester (200 feet, sandstone below)	50·0	10·5	Limestone.
Westminster (76 feet)	15·2	7·6	Do. (Portland).
Blackfriars (100 feet)	20·0	8·0	Do. do.
Pont y prŷd (140 feet), Glamorganshire	46·6	11·6	
London (152 feet), elliptical	32·0	7·9	Granite.
Over the Dora, near Turin (147 feet)	30·0	7·9	Do.
Waterloo (120 feet), elliptical	24·0	6·0	Do.
Toulouse (112 feet)	42·5	15·7	Brick.
Over the Lea Cut, Eastern Counties' Railway (87 feet)	23·2	4·2	Do.
At Maidenhead, Great Western Railway (128 feet)	24·5	4·7	Do.

It appears, from these few examples, that the practice of different bridge builders widely differs; the granite structures have about the same proportion as those of sandstone or even brick: indeed, the latter contrast strongly with the rest, when the quality of the substances is considered. By a comparison of the average qualities of these materials, we find that if granite be called 100, the comparative strength of limestone is 57·2, or rather more than one-half; of sandstone, 44·4; of brick, 6·5, or one-fifteenth the strength of granite; and mortar about 5·0, or one-twentieth. If the Neuilly and Maidenhead Bridges be compared, we find the proportion of thickness to the span and height are the same; they are both, moreover, of the same actual span, but the former is of a very strong sandstone, and the latter of brick, or rather of the mortar which separates the bricks, and gives the curve to the arch; so that if comparative figures, given above, be taken, the strength of the Neuilly Bridge is nearly nine times that of the Maidenhead Bridge, although the latter has been proved to be sufficiently strong; this result contrasts favourably with the calculation at page 159, respecting the Neuilly structure, by which it appeared that it was twelve times stronger than could be required. An old bridge at Toulouse far surpasses that at Maidenhead for thinness at the crown, being but one forty-second of the span. Waterloo Bridge, somewhat less in span than the brick arch we are comparing, is a little thicker at the crown, although of a very strong material. Pont y prŷd, built by a genius, though an unscientific man, is surprisingly bold; though of great span, its thickness at the crown is but about one forty-seventh of the span. Chester Bridge, however, surpasses them all: the actual depth in this—the largest stone arch existing,—is but 4 feet, —while London Bridge is about 5 feet; the Dora, rather more; and Waterloo greater than either.

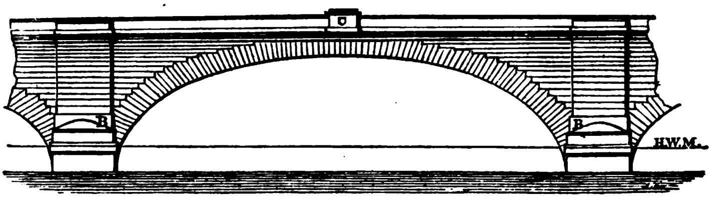
---

* Gauthey, Traité de la Construction des Ponts, vol. i. p. 195. (1843.)

(65.) The great number of large and beautiful stone bridges which have been erected in modern times, and stand as monuments of industry and science, prevents a description of all, but the following details present striking examples of bridge building:—

I. London Bridge.—The present bridge particularised by this name is as remarkable for its elegance and the size of the arches, as the old bridge was for its ugliness and the smallness of its arches, some of which appeared as mere perforations through a wall. The old structure had lasted above 700 years; and in the last century it became a matter of serious debate; the repairing of the bridge cost about four thousand pounds a-year, and the stopping of traffic in consequence of the waterfall up or down stream, as the tide happened to be, by reason of the damming up of the river, in addition to the loss of life and boats from the same source, made it necessary to improve this bridge. After some ineffectual attempts to keep up the old building, in 1800, on a Parliamentary inquiry concerning the improvement of the Port of London, a number of plans were produced for a new bridge. A noble iron bridge was proposed by Messrs. Telford and Douglas, to be 600 feet in span; but the opinions expressed by men referred to on the subject differed amazingly, some from inability to handle the subject, others for personal reasons, and the better class for different theoretical views. Finally, in 1823, an act was passed for the building of a new bridge, a plan of Mr. John Rennie's being specified, which allowed five granite arches, the middle being 152 feet, the two side arches 140 feet, and the land or extreme arches 130 feet span. The original purpose, to erect the new on the site of the old bridge, was abandoned, and a new spot chosen, a little farther up the river, saving the steep approach of Fish-street Hill: much of the money, however, was spent in making the approaches, the whole expense being about 1,426,045*l*., of which the bridge itself, by contract, cost 425,082*l*. The contractors began the work on the 4th of March, 1824, by an examination of the bed of the river with a diving bell; the first pile for a coffer-dam was driven on the 15th of the same month; the first dam completed on the 1st of April, 1825; and the water was pumped out by a steam-engine, the river at the place being 29 feet deep; the excavations for the foundations were then commenced, and when the stiff blue clay, which forms the bed of the river, was reached, piles were driven about 20 feet into the clay; two rows of sleepers, about 12 inches square, were laid horizontally on the heads of the piles, and above them a beech planking six inches thick, on which the masonry was laid. After the ceremony of laying the foundation-stone, the works rapidly proceeded; by the end of the year the south abutment and two piers were much advanced, and in 1826 the centering was begun, a description of which is given at page 138. During this time, the water-way being obstructed, it became necessary to throw two of the arches of the old bridge into one, and to remove a pier; the traffic was not arrested by these alterations, and they were effected in six weeks. By the 4th of August, 1827, the first arch was completed, the masonry, on the striking of the centre, sinking only 1½ inch; and, at the close of the year, two arches, and part of the centre arch, were carried up; also four piers and the north abutment foundation were very forward: the work so rapidly advanced, that on the 10th of November, 1828, the last arch was keyed; from that time until the opening, the cornice, paving, parapets, and lamps were fixed, and the expensive approaches, after much delay in discussion, completed; finally, on the 1st of August, 1831, the ceremony of opening the new bridge took place, or seven years, four months, and twenty-seven days after the commencement of the works.

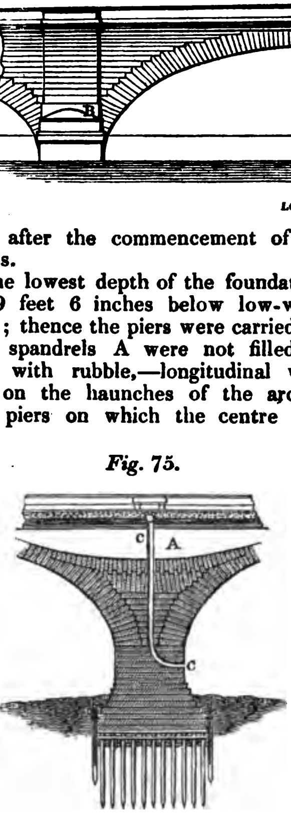
Fig. 74.



LONDON BRIDGE.

The lowest depth of the foundations is 29 feet 6 inches below low-water mark; thence the piers were carried up. The spandrels A were not filled up solid with rubble,—longitudinal walls rest on the haunches of the arches. The piers on which the centre arch rests are 24 feet thick at the springing, but increase to 38 feet in thickness at the lowest course: those next to the abutment are 22 feet thick, the former being rather less, and the latter about one-sixth of the span, which is at this time considered large, although they do not appear heavy. There are, altogether, 2092 piles under the piers and abutments; the starlings B (*fig.* 74) are nearly of a circular form, their upper surface being of a curve, agreeable with the waving line of the water. To prevent the injurious action of the water from the roadway, a pipe C (*fig.* 75) is carried from the road through the piers, and opens below the springings. The abutments are of solid masonry, 73 feet thick at the base (see *fig.* 72, *ante*), and rest on slanting piles; they are backed by rubble.

Fig. 75.



SECTION OF A PIER.

The arches, five in number, are of an elliptical form, which gives much lightness in the appearance of the piers, the centre being 152 feet 6 inches span, and 37 feet 10 inches rise from the line of springing, or 29 feet 6 inches in height from high-water mark to the soffit, the contingent arches, 140 feet span, and 32 feet 6 inches rise; the remaining two, leaning on the abutments, 130 feet span and 30 feet 6 inches rise from their springings; the key-stone of the large arch is 5 feet ½ inch deep, and 1 foot 6½ inches thick; at the springings 10 feet ½ inch deep, and 1 foot 10½ inches thick. The spandrels, according to Telford's improved method, are not solid, but the roadway is supported on walls (see page 158) by flags, on which is laid a stratum of puddle, followed by another of gravel, and upon that the paving. The parapets, which are very plain, and close, are 4 feet 1 inch high; the roadway, by reason of the flat arches, has the small rise only of 1 in 132.

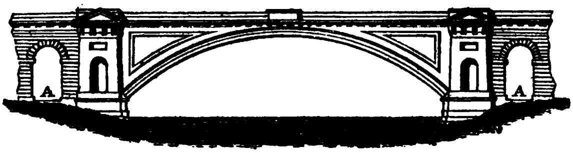
The height of the bridge from low water is 60 feet, and the length between the wing-walls 1005 feet, allowing a clear water-way of 690 feet: its total width is 56 feet, being 6 feet greater than was intended; this alteration was made after the coffer-dam for the first pier was completed, which, with another enlargement of 2 feet in the height of the abutment arches, cost 42,000*l.*; this width is distributed between the footpaths, which are 9 feet from parapet to curb, and the carriage-way—35 feet wide. The approaches are by land arches, extending over Thames Street on the north, and Tooley Street on the south side; they are elliptical also, and of brick, the former being 38 feet span. The granite for the bridge was selected from Aberdeen, Heytor, and Penrhyn, and the total weight used was 120,000 tons.

This bridge is considered as unequalled in size and appearance; the arches are very beautiful in figure, and the plainness of the facing, which should always be observed in such structures, gives an idea of grandeur and strength. The span of the centre arch is great, but has been exceeded; it is very flat, the height being but 0·25, or one-fourth of the span: it is, however, surpassed by many others of an elliptical or segmental form; thus the elliptical arch of Wellington Bridge, at Leeds, 100 feet span, has a rise of 0·15, or a little more than one-seventh; Gloucester Over Bridge, within 2½ feet of the same span, three-thirteenths; the bridge of the Holy Trinity, at Florence, 95½ feet span, under one-seventh; and the rise of the Dora Bridge, a circular segment, is but one-eighth of the span. The bridge, however, is not only both a great ornament to the river, and one of the most perfect specimens of bridge architecture, but it has plainly shown how much more may be done with materials than has hitherto been attempted.

II. **Chester New Bridge.**—This bridge, although not the largest in span which has ever been erected, is the greatest of stone-built arches now standing. It connects the banks of the river Dee, across which lies a main road of communication between Wales and the north and central counties of England. An old bridge, of seven arches, in sandstone, supported on large abutments, existed, but it was found to be very inconvenient, notwithstanding various improvements: it was built in the year 1280; and, for that time, it was doubtless more than sufficient. In 1825 an act was obtained to build a bridge, which had been proposed about twenty-five years since by Mr. Harrison of Chester; and the works commenced in 1827. The bridge is founded upon the rock, in which is the valley of the river, but which rises rapidly on the city side; the foundations of the abutments are consequently simple, except that in working at the north or city abutment it was found that a dislocation of the strata had occurred, where the farther end of the abutment would insist, forming a dyke filled with soft material, so that piling was indispensable, and even the piles sank 5 or 6 feet at each blow, for a considerable portion of their length. They were driven at a distance of about 3 feet one from another, the earth between them being dug out for a foot in depth to make way for rubble stones, the interstices of which were filled with mortar, and the whole rammed down. It was also adjudged advisable, from this occurrence, to commence the arch a foot lower than it was purposed, in order that the body of the thrust might meet the rock. On these piles a floor of stone was laid, and the abutment carried up of large dressed courses. According to the specification, the foundation of the abutment on the north side was to be sunk 21 feet below the springing of the arch, and that on the south side 16 feet 4 inches below the same level. The courses are not horizontal, but were laid with an inclination nearly as great as the first course of the arch, for a distance backward of above 40 feet, whence they are carried on horizontally; at this point there is a relieving arch A, of about 36 feet high and 20 feet wide. The inclined courses in the abutments must be considered as taking the thrust of the arch.

The centering upon which the arch was carried up was quite peculiar, and acknowledged to be the invention of the contractor, Mr. Trubshaw. It consisted of six trusses, each of which was formed of timbers, resting, on one end, in cast-iron shoe-plates on piers of stone, and

Fig. 76.



CHESTER NEW BRIDGE.

spreading upwards to the intended line of the arch, like a fan; there were three piers thus bearing fans, which extended the whole span of the arch; three horizontal beams, 13 inches deep, tied the fans together between the piers and their upper ends, which were bound by 4-inch planking, bent round so as to describe the curve of the arch: at this part advantage was taken to place the wedges for striking the centre when the arch should be completed, and above them was placed the covering or planking for laying the arch-stones; iron bolts connected each rib near the upper part. This centre is very simple and effective; the failings which occur to centres, as mentioned in page 137, do not apply here; indeed, half the arch is said to have been built up before the centre was quite finished; on the other hand, the obstruction to the waterway must be considered, if comparison be made with those centres which have allowed free navigation. When the springing course was laid, a wedge of lead, 1½ inch thick, and tapering to an edge at the back, was placed between the springers and the first course of arch-stones. This was intended to prevent any change of form in the arch when the centre should be removed; and strips of sheet lead, 8 or 9 inches wide, were put in the joints up to that part where the line of pressure was considered to pass from the front part to the back of the arch-stones; this was taken to reach over two-thirds of the soffit. When the centre was loosed, it is reported that the opening of the joints did not occur, and the pressure seemed to be spread evenly over the bed of the stones; neither is cracking perceptible in any part of the arch. The key-stones were similarly treated, thin strips of lead being hung down on the stones between which they were to be driven; the stone was covered with a putty of white lead and oil, and driven in by a small pile engine. When the centre was lowered, and the arch allowed to settle, it sunk 2¾ inches only. The arch-stones were laid by the traversing machine, now so much used in building, consisting of a carriage, to which the stones were suspended, and which was movable on a railway laid on a frame of timber, from 45 to 50 feet long, stretching from one side of the bridge to the other; this frame also had wheels, resting on a railway fixed to another frame, extending from one end of the bridge to another; thus, a stone suspended from the upper carriage could be borne to any required spot within the area or compass of the lower frame. There were two cross frames on the longitudinal railway, one for each side of the arch, so that the work rapidly proceeded. The first course of arch-stones laid on the springers was 6 feet in depth, which was gradually reduced until at the key the stones were 4 feet deep. The spandrels were not filled up, according to the old practice, with rubble, but walled, the tops of the walls being joined by a pointed arch; and then another tier of smaller arches above them, on which finally rested flagstones to carry the materials for the roadway, consisting of rubbish, rubble stones, and gravel. The masonry of this bridge is of a mixed character; the facing or visible parts of the abutments towards the river, up as high as the spring of the arch, and the two courses of arch-stones immediately above the springers, are of granite; the key-stone, and course on each side of it, are limestone, of the variety called Anglesea marble; the remainder, including the abutments, the other arching courses, wing-walls, and other parts, are of new red sandstone, which is found abundantly in the county. The width of the bridge within the parapets is 33 feet, and its length about 340 feet, the span being 200, and the rise 42 feet; thus it is a portion of a circular arc having a radius of 140 feet. The parapets are

4 feet high, and 15 inches thick. The whole cost of the structure was 49,900*l.*, of which 7500*l.* were laid out on the approaches.

This bridge stands unrivalled amongst stone arches for its span; owing to the individual magnitude of this one arch it has many points of interest, and, as a building considered in reference to others, it has several defects. It is instructive to observe the span, rise, depth of key-stone, and the material which was chosen; the rise is but a fraction above one-fifth of the span, and with so large a span, must produce an enormous crushing pressure upon the keying course; this, however, is of limestone, and but 4 feet in depth, or one-fiftieth of the span; it is of less actual size than the granite key-stones of London Bridge, and the bridge over the Dora, both of large span, but not above three-fourths the size of the Chester arch. The bridge is eminently light as respects the actual quantity and kind of masonry in it; some say its appearance, when viewed from the old bridge, which is a little further up the river, is grand; this must arise from the mere extent of the curve, and the great height of the road-way — sufficient to allow of vessels sailing under it. It is a fine-looking structure, with a somewhat graceful appearance, which is much increased by the white archivolt or facing arch-stones, contrasting with the dull red sandstone of the other parts; the haunches or outer spandrels present a great surface to the eye, and its face does not accord with the ornamental facings of the abutments, which are fanciful and paltry. It was at first intended to erect large Doric columns, with a corresponding entablature; this, however, was relinquished for the present niche, and panel above it, which is certainly but little improvement. If, as has been remarked by Mr. Hosking (page 45, treatise before mentioned), the springings of the arch had been raised up higher out of the reach of floods, and the arch either flatter, or of an elliptical form, it would relieve the dead haunches, and offer a more graceful curve to the eye: the very flat circular curve in the bridge over the Dora (see *fig.*, page 166), or in the ellipse in the beautiful bridge over the Arno at Florence, with its plain and consistent abutment facing, are instances. A bridge should not, in an architectural point of view, present an unrelieved and bare mass of materials; yet it is proper that the beholder should feel impressed with an idea of its boldness and strength; also house ornaments and other architectural fineries, suitable enough where prettiness only is required, should be prohibited. A large arch is essentially a sublime object, particularly in situations where bridges are generally placed; and a leading characteristic of a sublime object is plainness, with bold and striking points.

III. *Bridge over the Dora Riparia, near Turin**.—This noble bridge, carrying the main road from France, called the road of Italy, which crosses the Alps, is of one large arch of 147 feet 7½ inches span, spanning a river generally shallow, but subject to heavy floods, which, as the bed is considerably inclined, rush along rapidly. It was constructed with a remarkable attention to scientific principles, and, certainly, the result has fully repaid the great labour expended. As the road at the entrance to the town meets the direction of the stream obliquely, it was necessary to construct an oblique arch, or make a new road through the suburbs; the former kind of arch the engineer (Chevalier Mosca) considered too much of an experiment with so large a span, and if piers and two or three arches were used, it might prove dangerous in such a stream as the Dora; the latter proceeding, therefore, was adopted.

In proceeding with the erection of the bridge, dams were constructed each side of the river, and drained by a temporarily-made channel, and the soil excavated nearly 6¾ feet below low-water mark. Oak piles, 12 inches thick, from 30 to 40 feet long, protected by an iron shoe, were then driven from 3 to 4 feet apart, at the finish the heads being cut off level, to form an even surface for oak beams, 12 inches by 10 in scantling, which were laid transversely and longitudinally, and spiked to the piles, the rectangular spaces between them being filled with broken stone and a liquid cement of lime and ceroso (pounded tile). Upon this bed the foundations of the abutments were laid; they were made up of granite blocks 21 inches thick; three courses, each 2 feet deep, succeeded, allowing offsets of one foot; five more courses, with a level facing, followed these, bringing up the abutments to the line of springing. This part of the work was then allowed to rest and settle for a whole season, during which time the figure of the arch-stones, the centering, and other particulars, were laid out. An admirable plan was adopted in these proceedings. A platform of 5000 square feet was laid down, and having a plane surface, the curve for the arch was drawn; this, being a portion of a circular arc, might have been properly done with a centre; but in the present case it was more scientifically effected, by dividing the chord-line into a number of equal parts, and calculating for the purpose the length of the ordinates or lines erected at right angles to the chord, and reaching up to the curve. Another curve was also drawn for the centering, the height given to it being somewhat greater than that of the intended arch, to allow for sinking: the arrangement of the timbers was drawn in full size on the platform, so that they were readily prepared; thus, when the carpenters on the platform had constructed a rib, other workmen at the site of the bridge fixed it in its place. There were ten ribs in this centre, which in arrangement was very similar to Perronet's centre, represented on page 137, except that it was greatly assisted by the rows of piles in the middle of the river.

---

* Transactions of the Institution of Civil Engineers, vol. i.

From the figure of the arch drawn on the platform, wood models, to direct the stone-cutting, were made; for other parts of the arch-courses, tables were constructed, so as to obtain the greatest nicety of workmanship. Great attention was paid to the probable effect of the settling or movement of the whole body of arch-stones, when the work should be completed. It is a fact, calculable and observed, that the courses at the springing and the crown squeeze each other at the edges when the centering is removed or eased; this, in the preceding example of the Chester arch, was well met at the springing by a wedge of lead; if in any case the pressure thus caused be great, a chipping of the edges ensues. Mosca directed the stones to be cut for the proper size of the arch; but the joints, instead of being merely in the direction of radii, he ordered to be made diverging towards the intrados at the springing, and so on in a decreasing progression, till they became parallel; and, towards the crown, the divergence to be in the direction of the extrados.

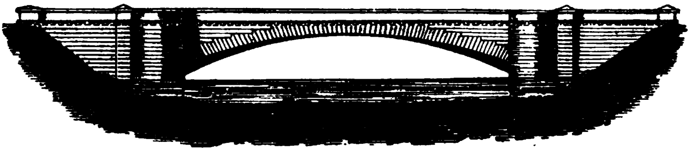
When the laying of the arch-stones was about to commence, two small bridges were erected at each side of the centering; on these the stones were carried by means of upright timbers with tackle,—a method inferior in neatness and handiness to the system of latitudinal and longitudinal railway carriage as used at Chester: the work, however, proceeded quickly, as the stone wedges, which were about 5 tons weight each, and 651 in number, making a total weight of 3250 tons, were set up in seventy-five days. The springing courses, which were from 15 to 18 tons in weight, were bound together; and succeeding courses were cut so as to form the proper angles for the external and internal curves, as this bridge was built with two curves, after the complex arrangement mentioned at page 153, only both curves were circular (they may be seen at the haunches in the figure). After each course had been laid, its position was compared and corrected by means of two timbers—one horizontal, and the other vertical—which had been placed and marked for the purpose; small plates of lead were put between the voussoirs or wedges, according to directions, for keeping the joints diverging, and retaining the stones in their places until the lowering of the centre; between the courses at the crown small iron wedges were introduced for the same purpose; these remained until the arch was completed, when, the position of each stone being found correct, a mixture of lime and clean sand was poured into the joints, and held there by a stuffing of tow at the apertures of the joints, and the wedges removed.

Before removing the centre the plan of Perronet was adopted, in order to determine the amount of sliding which might occur when the centre was lowered. Three lines were drawn on the face of the arch, one horizontal, across the crown, and two oblique, from each side of the key-stones to the springing. The depth given to the key-stone was 4 feet 11 inches. After the lapse of twenty days from the time of keying the arch, the centre was lowered; it had been retained by check-pieces or wedges, 240 in number, and when the checks were removed, the frames commenced sliding downwards uniformly. The arch, being at last left to itself, was found to have descended very regularly from 18·9 feet—the height of the centre—to 18·4 feet, in the course of the five days during which the centre was being removed; the engineer, however, purposed pushing the arch to the greatest extent of settlement, and with that view loaded it with 3000 tons of ballast, evenly disposed over the surface; this was a weight much beyond that which it could ever be called on to bear in practice; it was suffered to remain on the arch four months, and depressed the crown 1·57 inch more. It was now judged proper to proceed with the exterior spandrel walls, the interior space being filled up solid with masonry; the horizontal masonry of the spandrels was, by previous cutting of the arch-stones, brought to meet them angularly with a regular appearance; the upper part of the abutments (above the springing line of the arch), the cornice, and roadway, were successively laid; the cornice being plain, with the appendages denominated modillions. The surface of all the stonework upon which the materials for the roadway were to rest was covered with a layer of bituminous cement, 5·9 inches thick, and beaten down hard: on it was placed another layer, mixed with gravel, 2¾ inches thick; thus the joints of the masonry were saved from the infiltration of water, which is so injurious to the structure. The different parts were now dressed, and the bridge made ready for public use; four years having been occupied in its erection, and the expense to the Sardinian Government being 56,000*l*.

The total length of this bridge is 300 feet, the width 40 feet between the parapets, which are 3 feet 4 inches high; the span of the arch, as before mentioned, is 147 feet 7·6 inches, and the rise, after all the work had been completed, 18 feet 3·2 inches; the height is, therefore, only one-eighth of the span—a proportion much less than other bridges of great span; the arch-stones, ninety-three in number, are all of the same depth—4 feet 11 inches—except the two at the springings. The carriage way is 30 feet wide, and the remaining 10 feet is given to two footpaths. The masonry was finely dressed—even to extravagance—the faces of the granite arch being polished; the joints also were faced with a cement made up partly of granite dust, to correspond with the masonry.

Whether considered in reference to its external appearance, or the scientific manner in which every part was executed, this bridge is a splendid object. It is remarkably plain; the lines which mark out its parts are few, but bold, and accordant one with another; indeed quite a contrast with most other bridges; the segment, being small, gives an elegant lightness, which is much increased by the double curve—the outer curve springing from a higher point than the inner, after the manner of Gloucester Bridge (see *fig.* 67). If, as has been remarked, a projection had been given to the lower part of the abutment walls, instead of the plain face, reaching, as it does, into the water, it would have been an improvement. The impression of firmness, arising from massive abutments, is considerably prevented by the circular or cylindrical form given to their faces, by which it appears as though part of the arch were hidden behind them; they are considered as acting similar to cutwaters before piers, assisting, with the conic form of the arch-opening, to give free passage for the floods; they allow, also, a wider approach for the roadway. The key-stone is certainly much deeper than requisite; the flatness of the arch, doubtless, throws a great pressure upon the stones at the crown, but when it is

Fig. 77.



THE DORA RIPARIA BRIDGE.

compared with some other bridges, and particularly if the nicety with which the work was done be considered, a much less depth would have been safe.

IV.—The Bridge of the Santissima Trinita at Florence is justly considered as a very beautiful structure. It is not very large, the span of the principal arch being but $95\frac{3}{4}$ feet, but the curve of the arches, and the architectural taste generally shown in its parts, is such as to call forth praise from all judges in architectonics. Modern science cannot, however, take the credit of producing this bridge, it having been built in the sixteenth century by Bartolommeo Ammannati Battiferri, celebrated at the time for architectural talent, over the Arno, a river, which, having its source among the mountains, is liable to heavy floods from the melting of the snows; these had repeatedly swept away former bridges. He proposed and executed the present structure with three arches, the central being 95 feet $8\frac{3}{4}$ inches, and the contiguous pair 85 feet $8\frac{1}{4}$ inches. The piers and abutments are supported on piling, different in its disposition to the modern methods; the base of the foundations has a small offset, and, therefore, covers a greater space than the body of the piers; the cutwaters are triangular, extending above 19 feet from the body of the piers—these latter being 26 feet 9 inches thick, and $27\frac{1}{4}$ feet high; the arches spring 9 feet 5 inches above the ordinary water line, and appear to be formed of two portions of an ellipse, of which the major axis is $113\frac{3}{8}$ feet, and the minor axis $31\frac{1}{6}$ feet: the schools at Florence have concluded that the curve is parabolic, but the former is the more general opinion; whichever it be, the arches are somewhat pointed, to an extent, however, which keeps them graceful. The centre arch rises 15 feet $1\frac{1}{2}$ inch, and the two smaller arches 14 feet; the ratio of the rise to the span is, consequently, 1 to 6·5, or $\frac{1}{6 \cdot 5}$ of the span; the material of their face is marble, but by the appearance of the soffit of the arches the interior is of rubble work. The parapets are about $3\frac{1}{4}$ feet to $3\frac{3}{4}$ feet high, and 2 feet thick; the whole passage way between them being $32\frac{2}{3}$ feet, of which two foot-paths occupy $11\frac{1}{4}$ feet, and $21\frac{5}{12}$ feet is allowed for carriages. The building occupied from the 1st of March, 1566, to the year 1569; since that time little alteration has occurred in the structure, excepting a slight sinking, which has been principally on the south side, where the curve of the arch has become continuous in consequence.

V.—Waterloo Bridge is a fine specimen, without any rise in the roadway: it reaches over a very broad part of the Thames, its length, between abutments, being 1240 feet: the arches are all of equal size, and elliptical, having a span of 120 feet, and a rise of 35 feet; they rest on piers 20 feet thick, counter arches being placed between the curves, on each pier, to carry the thrust, generally considered to occur at a point of the curve where its tangent becomes inclined about 60° to the horizontal line. The masonry is of granite, weighing about 498 lbs. to the cubic foot; as there are 34,000 cubic feet in each arch, there is, consequently, a weight of about 16,930,000 lbs., or 7559 tons, from the arch alone, on each pier, except those next the abutments; the key-stones are 4 feet 6 inches deep. Brick walls are laid over the backs of the arches, so as to make the spandrels hollow: on them rest the flag-stones supporting the road and footways; the former is 28 feet and the latter are each 7 feet wide. As a piece of architecture the bridge is striking to any beholder, notwithstanding the pillars which are set up at each pier; had the facing of the pier been plain, as in London Bridge, it would have appeared more to advantage. It was designed by Mr. R. Dodd, and built by Sir J. Rennie.

VI.—The romantic Pont y Pryd, spanning the river Taaf, near Llantrissent, Glamorganshire, deserves notice, on account of the circumstances of its erection, as well as the size of the arch, considering the time when it was built. It was the work of Wm. Edwards, in 1751. It is the third bridge which this self-taught architect erected at the same place. The first was in three arches; but, in consequence of a heavy flood, to which this mountain stream is subject, it was carried away two years after its completion. Edwards then attempted a wonder (for his time), to erect a bridge of one large arch, stretching 140 feet, or over the whole torrent; but, unfortunately, not having calculated the thrust arising from the heavy haunches of his arch, before the parapets were set up, the haunches sank, the crown was pushed out, and the whole fell into the river. Undaunted by these disasters, Edwards re-commenced, lessening the weight of the haunches by leaving three horizontal cylindrical holes in each of them (see *fig*. 64, page 150); this alteration had the desired effect, and all stood secure. As the former, so this arch was 140 feet span, rising 35 feet; the arch-stones, at the crown, are only 3 feet in depth, or less than a forty-sixth of the span. The lower holes or cylinders are each 9 feet in diameter; the middle 6 feet, and the upper 3 feet diameter. Although so light and large as to be a masterpiece at the time it was built, it has withstood the action of many floods, and the disintegrating effect of the changing seasons.

CHAPTER VIII.—*Bridges of Wood.—Character of Wood.—Simple Beam.—Trussed Girder.—Town's and Long's System.—Straight Timber Trusses:—Bridges of Kandel, Palladio's, Schaffhausen, &c.—Curved Timber Arches:—Price's, Mahavillaganga, Wiebeking's, Schuylkill, and other Bridges.*

(66.) THE utility of wood for making a line of communication between two places, appears to have been thought of before any other material, and led the way in bridge building. It is the most simple, as well as cheap, substance used in these structures, and may be built up most readily; not only does it possess these advantages, but, next to those commonly called suspension bridges, it has, at present, been used for the greatest spans[*]. There are, however, some practical objections urged against the use of wood in exposed situations — it is very liable to decay, and destruction by fire. Wood cannot, of itself, withstand the effect of alternate wet and dry; this certainly need not occur with the materials of the superstructure, but wooden bridges frequently abut or rest upon piling, or posts on piling, part of which, as the tide ebbs and flows, must be alternately dry and wet; the timber work of the arch should never be within reach of the tidal waters, though, in several instances, it has so occurred, where there are sometimes very high tides. Wiebeking, a German engineer, who built many wooden bridges, soaked the posts which were to dip into the water in hot oil. Several methods have been put forth and patented by different persons for the protection of timber against the weather and the destructive effects of the small cryptogamous plants which are apt to grow over dead wood[†]. In many bridges great care has been taken to inclose the important parts by close planking, which is injurious. However much interested parties may exhibit the faults of timber work, there are sufficient long-standing examples to prove the efficiency, economy, and durability of wooden bridges.

---

[*] The iron tubular bridges over the Conway River and Menai Straits excepted.

[†] The growth of plants on wood appears to be the consequence of the decay of wood. All dead wood, when subjected to the action of varying temperature and moisture, commences a slow decay or eremacausis, for the proximate elements of organic matter containing nitrogen are easily acted on by the above agents, as the nitrogen, amongst other negative properties, has no particular affinity for any of the other component elements, but facilitates decomposition; when this commences, the germs of cryptogamous plants, which had been naturally deposited in the pores of the wood, are rapidly developed, and materially assist the decomposing agents; the *Merulius lacrymans*, a fungus well known as connected with what is called dry-rot, powerfully aids in the destruction of wood, by insinuating itself between the fibres, and rapidly spreading. Open situations and currents of air are, however, inimical to the growth of these plants, which fact points out the impropriety of boxing up the woodwork of bridges too much, as has frequently been done by bridge carpenters. Mr. Kyan patented, some years since, a method of preserving wood by soaking it in bichloride of mercury (corrosive sublimate); some chemists supposed that it had a preservative effect, because the bichloride ($Hg\ Cl_2$) decomposed when it met with the albumen (a proximate element) of the wood—one part of the chlorine being evolved, while the remainder, then protochloride of mercury ($Hg\,Cl$), combined, with the albumen, forming an insoluble compound, and neutralizing its tendency to decompose. Liebig considers the bichloride to combine with the lignine or woody fibre. The solution of bichloride has, however, been detected by chemical tests, to a depth of from ⅛ to ¼ of an inch in various woods, and, by electrical test, somewhat deeper; it appears, however, to penetrate fir less than some other woods (Faraday). The wood thus treated becomes of less specific weight, less flexibility, but more brittle. It has been much used. M. Bréant proposed, in 1837, the use of sulphate of iron, which did not alter the qualities of wood like the previous substance. Lately Sir Wm. Burnett has asserted the claims of chloride of zinc as a preserving agent: trials of it have been made, with a satisfactory result.

The modes of applying timber to bridge building may be classed into two:—the truss of simple straight beams; and series of curved beams, forming ribs.

(67.) The most elementary form is a simple beam or girder, stretching from point to point, several of which would support transverse planking for a roadway. Out of a close consideration of the effects produced on such a beam through resting its ends on a solid bed, while weights are on the upper side, the general system of trussing, or adding extra supports of struts or braces, is completely developed. In the first place, from the principles of the strength of materials, it appears that the sustaining power of a beam will increase as its width increases, or twice the width will give twice the strength, but the increase is as the square of the depth; that is, twice the depth gives four times the strength; it is evident, therefore, that width is of little importance in comparison with depth as a means of supplying greater supporting power, not merely on account of the economising of material, but of uselessly loading the beam; for instance, beam A, 1 foot square at the ends, bolted to the side of another, B, of similar size, or scantling, twice the strength is obtained, as before remarked; but the weight of the compound beam is double of the single strength; if, however, we proceed by taking a beam B $5\frac{1}{2}$ inches by 12, or not one-half the depth of that to be strengthened, and bolt it underneath, or on the upper surface of A, the strength being as the square of the depth, which in feet is now 1·42, is equal to 2, or as much as the two equal beams in the former case, while the weight of materials, instead of being double, is not one-and-a-half, so that, in fact, there is more available strength in the latter combination, because it has less weight of its own to bear. There is also another physical principle concerned in a beam supported at each end. Even by its own weight a tendency to bend is shown, if the length be considerable in proportion to the breadth and depth, and superincumbent weights, according to their nearness to the middle of its length, act more powerfully in bending it; this leads at once to the conclusion that greater strength should be allowed to the middle than to the ends of the beam. The form for equilibrium can be calculated, and we obtain the following general figure of such a beam*; we have still some part of the beam idle, while the other parts bear the burden; when such a beam as the above bends it must be that the lower side is in a state of tension, and the fibres are preparing to break asunder; but the upper side, by descending, becomes compressed, and the fibres have to resist a force tending to push the ends together; between these extreme and opposite actions the intensity decreases until some line of neutrality *a b*, is reached; the parts on either side of this neutral line, consequently, do little work, except that of assisting to load the beam with their weight. In cases where very large and long beams would be required, it is a matter of importance to attend to these observations of the action of forces: they show us that if the upper and lower parts of the beam could be retained,

Fig. 78.

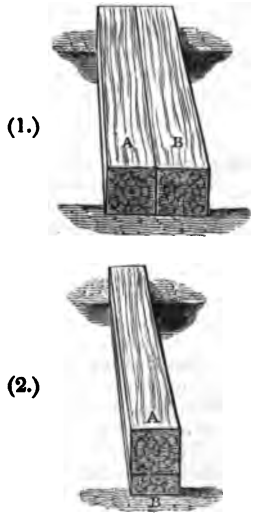
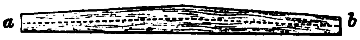


Fig. 79.



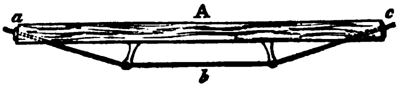
---

* See *Strength of Materials*, *seq.*

while the other could be cut away, a very valuable improvement would be effected: this has been done in various ways, one of the most simple being that of the trussed girder or beam.

(68.) The general form of a trussed girder is shown in *fig.* 80. A wrought-iron rod *a b c* is screwed into plates at each end of the wooden beam A, and passing obliquely downwards is held out by the struts abutting against the under side of the beam; this rod is, therefore, the counterpart of the lower half of a common beam; while the wood is principally engaged by the crushing effort of any incumbent force, the rod takes up the stretching effect, the horizontal portion *b* having the greatest tension; thus the beam cannot bend more than the iron rod allows by its extension under pressure, which is an exceedingly small amount. By this contrivance a beam half the depth of that taken for example in the preceding paragraph (or 6 inches) may become a very much stronger support; or if, in the second case, an iron tension rod were substituted for the extra depth given to the beam, a girder many times more powerful and lighter in appearance might be constructed; wrought iron is very much heavier than fir, but it is about five-and-a-half times stronger in resisting a stretching force. The trussed girder has been frequently used for small bridges; one over the Whitadder, at Abbey St. Bathan's, Berwickshire, is of the considerable span of 60 feet, supported by trussed girders, which are composed of two timbers, forming a beam 11 inches deep by 6 inches wide; the tension rods *a b c* of the girders are only 1 inch in diameter, and are tightly screwed by nuts at the extremities *.

Fig. 80.



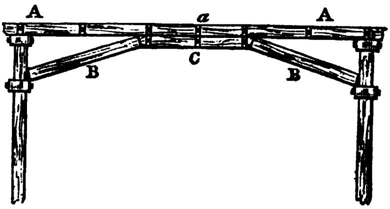
A form of bridge which may be properly classed with the improved girder is that proposed by Ithiel Town, an American; their lightness, simplicity, and cheapness of construction, are surprising, in comparison with most other structures of this kind. These bridges consist of two rows of horizontal ties or stringers, one above the other, but separated to a distance of 15 feet or more by lattice-work, or inclined pieces crossing each other; there are generally three tie-beams or stringers, with a line of lattice-work between them; the inclined pieces form an angle of about 50° with the horizontal stringers; between each of those inclined one way there is a distance of 4 feet 6 inches, the piers themselves being from 10 to 12 inches broad by 3 to 3½ inches thick, for a truss of 17 feet high; the whole is firmly secured together by wooden tre-nails. The height of the truss has been proposed to be generally about $\frac{1}{10}$th of the span. It is, however, greater in some bridges built on this principle. The roadway rests either on the lower or upper stringers, the latter being, doubtless, the best position, as the stringers are brought into a more even state of tension. This combination may evidently be called a trussed and improved girder, the ineffective central parts being removed, while the two important parts are effectively connected and the system stiffened by the intermediate lattice-work. A bridge was erected over the James River Falls, at Richmond, (U.S.), in 1838, on the Richmond and Petersburgh Railway, where these simple truss-frames support spans of from 130 to 153 feet; they were 20 feet deep, the railway resting on the upper stringers. The whole length of the bridge is 2900 feet, the railway being suspended at a height of 60 feet above the water. The expense of the bridge evidenced the great economy resulting from Mr. Town's method: it cost about 24,200*l*. Another frame bridge was suggested by Mr. Long, in which a number of the inclined pieces were left out, so as considerably to lessen the quantity of short timbers; vertical iron ties are introduced, at convenient distances apart, serving the place of the omitted timber. A bridge of 1260 feet in length was erected over the Connecticut River, for the Western Railroad (U.S.), in which the spans were 180 feet each, the timbers being, however, very small. The lower ties are 1 foot deep, and about 5 inches broad; the inclined braces are 8 inches square, which is also the size of the upper stringers. According to either method a great saving of time and material is effected in their construction, when compared with other bridge-work in timber.

---

* An American, Mr. J. R. Remington, proposed to build bridges with very much less timber than usual. His principle has not been very clearly stated; but a bridge was built by him of 150 feet span, the support being tie-beams or girders 5 inches deep and 5 inches wide at the ends, tapering towards the middle to 2¼ inches square; these beams or stringers have a fall of 24 inches from the chord-line in the middle of the bridge, so that they act by tension: it would thus appear to be a description of suspension bridge.

(69.) The most varied alteration and improvement upon the simple beam or girder is that of wood-trussing with struts and ties. It can be effected by adding timber work above or below the principal beam; the most simple deduction from the equilibrated beam (*fig.* 79) is that of the simple roof truss (see *fig.* 15, page 121), of two rafters. Bridges frequently allow of an horizontal thrust, which is borne either through the piers or by the abutments; a simple and economical trussing of three additional timbers under the main beam is then applicable when the span is not too large. A bridge built over the Clyde, at Glasgow, in 1832, is so composed. Main beams A were laid on piles, while the beams B, B, C, gave support to the middle or weakest part; these beams resist by compression. Any weight at *a* stretches the upper beam, which, meeting the straining piece C, imparts to it the force; but this piece acts as a wedge between the two struts B; most of the pressure is thus given to the struts which abut on the piles, and the resistance must here be supplied; with a number of spans it is found in the thrust of the analogous struts in the next series, or, finally, the abutments. All the pressure does not, however, act against the succeeding strut, because B is inclined; a portion, depending on the obliquity of the strut (art. 11, page 119), is carried by the pile; pressure has, however, little influence, if the ground be firm, as we learn from the principles of the wedge. In the Glasgow Bridge the span or distance from pile to pile is 34 feet, the struts B are 13 inches by 12 inches in section, and the other timbers 12 inches square. This elementary form has been frequently used when an horizontal thrust could be sustained. The Utica and Syracuse Railroad, United States, is supported over the Onondaga Creek Valley for a distance of 600 feet by 20 of these frames (or trestles) of 30 feet span each, the struts being mortised into the beam directly, without any intermediate straining pieces.

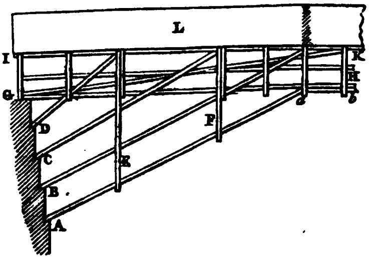
Fig. 81.



GLASGOW BRIDGE.

In France rough round timbers have been used for bridges, disposed similar to the above; one over the Durance, at Bon-pas, is above 39 feet span, and another over the Var above 49 feet span. When the same arrangement is used for larger spans the beams are apt to bend, even by their own weight; this circumstance prevents the use of long timbers in such structures; braces are then added, holding together the struts and tie-beam, the whole system being thus stiffened; the bridge of St. Clair, over the Rhone, at Lyons, having a span of 44 feet, and many others, are so constructed. Another development of this method is exhibited in the bridge over the Kandel, in the canton of Berne, built by Jos. Ritter. In the following representation of half the bridge, the parts corresponding to the lower beams in the Clyde bridge are A, *a b*, being one strut and part of the intervening

Fig. 82.



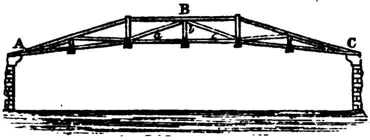
BRIDGE OVER THE KANDEL.

beam; the strut would, however, be too long to support itself steadily, being, according to the figure, above 70 feet long; it is, therefore, tied up at two places by the braces E, F; the superior struts B, C, D, do not play against the lower tie-beam G H, but they meet the upper beam I K with the vertical braces, which is a mode of suspending the roadway resting on the lower beam G H; the three horizontal beams are tied together by the short braces, and, with the diagonal strut G K, form a truss quite independent of the timbers A, B, C, D, and their attendants; it might, if of sufficient stiffness, stand of itself. A roof L, common to wooden bridges, protects the covered wood-work from rain, and is a great convenience. This bridge is stated to have a span of 166 feet; although well disposed, there are, doubtless, more timbers than is requisite; this, however, is a common error in all kinds of bridges, and one, in most cases, on the safer side.

To obviate the difficulty of using very long beams, instead of making the lower truss of three pieces, as in the above cases, four, five, or more pieces have been braced up to the tie-beam, forming a polygon; this, however, of itself, is in a state of unstable equilibrium, similar to the catenary (see art. 20, page 126), and on account of the obtuseness of the angles the downward pressures act with a most destructive intensity on the joints. In several instances failures have occurred with bridges constructed after this manner.

(70.) Where horizontal thrust could not be allowed, or otherwise was sought to be avoided, the trussing has been partially or wholly built above the roadway. All the conditions of a thrust-bearing tie-beam are then included, similar to roofing in general cases. Palladio proposed and built specimens of wooden bridges of this kind, some of which exhibit excellent mechanical arrangement. The bridge erected under his direction over the Cismone, near Bassano, in Italy, of 108 feet span, is a combination of three ordinary roof-trusses, of tie-beam, rafters, and king-post, *a b c*, surmounted by two great rafters, an intermediate straining-piece, and two vertical posts A B C; the long tie-beam A C is raised in the middle. He proposed another very ingenious form for a bridge, obviating the necessity of long timbers, though the span were great: it was to compose each truss of a number of small frames, similar to the figure in the chapter on framework (page 120); these, like a number of arches, might be secured one to another, and would form a very strong and good-looking arch.

Fig. 83.

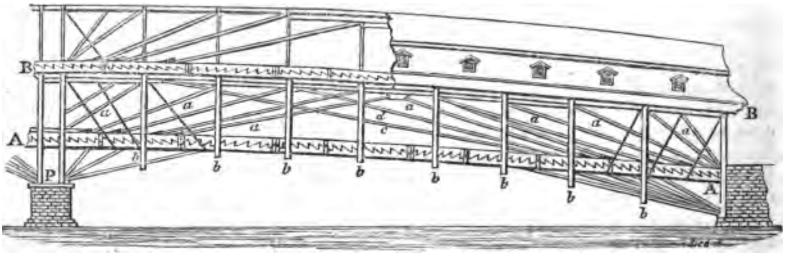


BRIDGE OVER THE CISMONE.

The most remarkable example of wooden bridge built with the trussing above the tie-beam was the bridge over the Rhine at Schaffhausen, built by Ulrich Grubenmann, called a common carpenter, in 1757; it was, however, burnt by the French army in 1799.

The diagram represents one-half of the bridge, the resting-place P being a pier belonging to an old bridge, left near the middle of the stream, in order to assist in supporting the great truss. The principal parts were the enormous

Fig. 84.



SCHAFFHAUSEN BRIDGE.

tie-beam A A, the rafters or struts *a*, *a*, and the vertical ties or braces *b*. The tie-beams were made of fir beams, two in depth, scarfed and bolted together in the manner shown by the serrated line; their width being about a foot-and-a-half, and united depth nearly 3 feet. This beam was shored up by a few struts, abutting on the pier and abutments, and carried above the system of rafters *a*, *a*, which, on examination, will be found to form, with the small horizontal beams against which their upper extremities thrust, a series similar to the roof (art. 19, page 126), the two struts *c*, *d*, being left out at present; this large truss was stiffened and held together by the eight braces *b*, *b*, which were placed at a distance of about 18 feet apart. Although the rafters meet the upper tie-beam B B at various places, yet their lower ends, carrying a thrust down to the beam A A, are very properly collected towards the extremities, or the parts most able to sustain pressure. With the beam B B, the upper series of struts and braces constitute another and lighter truss; one-half of the length is left with the roofing. This certainly formed an excellent truss; indeed, it was said that no piece could be removed without endangering the whole, showing that the structure was encumbered by no useless material. When built it appears that the pier sunk, so that Grubenmann had to strengthen his truss by additional rafters *c*, *d*, the interior rafter being of very great length; with these beams it would appear as though the final thrust of the trusses was transferred to the abutments only. This appearance probably gave rise to an opinion that the pier was unnecessary to support the bridge: it evidently must have borne some pressure; indeed, the two spans were not in the same straight line, the pier being above 8 feet out of a straight line between the abutments; the construction, however, was so truly on scientific principles that, with some strengthening, there is no reason, as Telford remarks, why the pier could not be safely removed. This bridge is not the greatest span built after this method. Its total length was 365 feet, the greater span being 192 feet.

The same ingenious carpenter and his brother built several other bridges after this model, and of very large span, one at Wittengen being 390 feet span. A very neat bridge of 150 feet span, upon Grubenmann's principle, was built in 1838 on the line of the Baltimore and Ohio Railroad, over the Patapsco River, with the judicious introduction of the cross-strutting between the vertical braces, according to *fig*. 13, page 120.

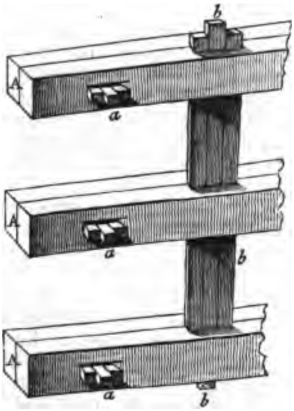
Sieur Claus exhibited in Paris a model of a bridge on Grubenmann's principle, which was intended to be built over the Dery, extending between the abutments the enormous distance of 959 feet. It differed slightly from the construction of Schaffhausen Bridge, in having an even series of rafters, equally inclined, and of equal length, in place of the variable lengths and inclination which Grubenmann adopted; diagonal as well as vertical braces were also added.

(71.) The introduction of curved timber in wooden bridge-building is a great improvement, from the mechanical ability of the material, and the aspect of the finished structure. Timber used in a curved form is generally bent into the required shape, and two or three pieces or planks, laid one upon another, are bolted together while so bent; otherwise the same form may be given by cutting across the fibres; in the latter operation, however, a great portion of strength is lost through the injury to the fibres; by the former method, small pieces, requiring little trouble to bend them, can be built up, by proper fastening, into a sound rib.

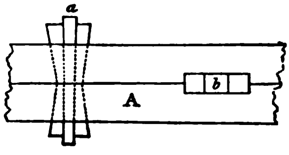
(72.) The effect of any weight acting on a curved beam will be somewhat analogous to that of pressure upon a stone arch, as represented by *fig*. 66, page 150. If the weight act on the middle of the crown C, a line of pressure C *a* D will be formed, the fibres being strained at *b* and A, while they are compressed at C, *a*, D. And pressure near the extremities would effect at *b* and A a compression, while at D, A, and E, an extending strain would occur. These suppositions must be uncertain, from the nature of the wooden arch; the conditions are not nearly the same; the rib has all the benefit of resisting, by its arrangement, any bending inwards, or in the direction of its curvature; it is, in nearly all cases, subjected to a strain which acts, more or less, over the whole length, whether the roadway be superimposed or suspended; the ties or struts, which are placed at different parts of the rib to support the roadway, prevent the simple action of the force which theory supposes, and the line of pressure (page 150) is at any time a doubtful matter. It has been called a parabola, but as the weight of the rib is frequently variable, and the other parts of the bridge are also so variable, the curve of equilibrium cannot be any regular curve.

(73.) Mr. Price revived the application of curved ribs, which were used at an early date;—Trajan's Bridge, over the Danube, in Lower Hungary, is an instance, which seems to have guided the above carpenter in his plan: he proposed that ribs should be made up of planks, varying, of course, in size, according to the size of the bridge intended; they were to be made up in two thicknesses, which should be bolted together with wooden keys. A bridge on this plan was built over the Piscataqua, near Portsmouth, United States, with a span of 250 feet. It consists of three ribs, each formed of planks 12 and 15 feet long, with keys and wedges of hard wood fastening them together. The ribs were connected with braces of hard wood, passing through holes cut in the ribs. The structure has been approved, although, if the ribs had been bolted together, forming one thick rib, more strength would have been gained, as the cutting required to insert the vertical brace, must have greatly weakened the rib. A bridge on a similar plan was constructed over the Mahavillaganga River, near Kandy, in the Island of Ceylon: it is 205 feet in span, and rises about 25 feet. It consists of four horizontal rows of compound ribs, each compound rib being, as in *fig.* 85, made up of three pairs of beams, A, A, A, which are about 12 inches wide by 14 inches deep, held together by the dovetailed wedges *a*, *a*, *a*; the connecting keys *b*, *b*, *b*, were cut so as to dovetail in the beams as they pass between each pair; these keys or braces were about 8 feet apart, and the whole compound rib about the same in depth. Radial struts, notched on the ribs, supported the roadway on each side of the centre. This bridge withstood, in 1835, a great flood, which rose 20 feet above the line of springing of the arch, the rapid current bringing with it trees and masses of jungle*.

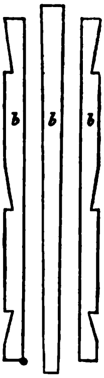
Fig. 85.



MAHAVILLAGANGA BRIDGE.



PLAN.



KEYS.

(74.) Wiebeking, director-general of roads and bridges in Bavaria, built several bridges with ribs of very large span and light appearance. His method was to bend long planks, instead of short pieces formerly used; the framework was very simple, each arch being of three ribs, with intervening struts, where requisite, between the ribs and roadway. By experiment this engineer found that the rough wood was more flexible than that worked up or squared, for a beam of fir 54¾ feet long and 1¼ thick, could be curved so that the depth of the curve was 1-36th of the length, whereas, in an unwrought condition, it was increased to 1-13th of the length; pieces, also, which had been cut into regular rectangular forms, were, when joined together, susceptible of greater curvature than a piece alone. He found that pine had more elasticity than fir wood; larch than pine; and resinous woods more than oak, which, when undried, allowed a depth of curvature so small as 1-26th of the length. His wooden-rib arches are said to have been tested, on completion, with a weight of about 149,000 lbs., or 66½ tons.

---

* Papers of the Royal Engineers, vol. iii. p. 154.

The bridge over the Regnitz, at Bamberg, is one which illustrates his practice on a large scale, the span being 208 feet—the greatest span, except in one case, erected on his principles; the rise of the arch is 17 feet. It was built of one arch, in place of a stone bridge, which, by the resistance of the piers to the current, was carried away by a flood. There are three ribs—two on each side, and one in the middle of the public way, which passes over the ribs; the outer ribs are composed of two widths of timber and five depths at the springing of the arch, but at the crown the depth is reduced to three beams; the middle rib is one width of the regular beams, but on each side are a triad of extra beams, making the whole width three beams; two inner ribs were placed diagonally to assist this combination in opposing any tendency to lateral motion. Cross struts and horizontal bridles or ties connect the ribs and carry the struts which bear up the timber work supporting the roadway. The abutments are very slight in comparison with those of other bridges, the weight of the materials and the proportionate thrust being much less than in arches of heavier materials. The rise of Wiebeking's wooden bridges is also small as compared with stone arches; in the Bamberg structure it is a little above 1-12th of the span, and in another bridge at Freysingen, over the Isar, of 153 feet span, the rise is about 11½ feet, or less than 1-13th of the span; while it may be seen from the table (at the conclusion of "Bridges") that, in the most considerable flat arches built of brick and stone, the rise of the curve is, in no case, less than 1-8th of the span, very frequently 1-4th. Wiebeking states, as the result of experience, that ribs from 100 to 150 feet span may have a rise of 1-20th of the span; 1-18th for 200 feet; 1-15th for 300 feet; and 1-14th for 400 feet span; these numbers give flatter arches than he adopted, but are limits allowable where the arch is required to be flat. There appears to be a sinking after the arch is constructed, through a slight yielding and settlement of the fibres—a circumstance to be expected; it is very small in amount—less than the ordinary settling of stone arches; Wiebeking expresses it as equal in inches to

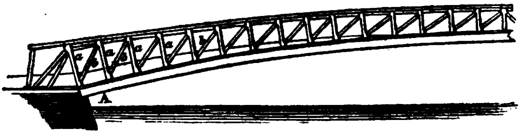
$$0{\cdot}806 \times \frac{\text{rise}}{\text{span}},$$

the rise and span being numerated in feet.

The same engineer offered to erect a wooden bridge at Munich, having a span of 273½ feet, and also one with the very great span of 554 feet.

(75.) A numerous class of bridges have united in them the ordinary straight timber truss and curved rib; in some cases the curved portion is intermixed with the other parts of the truss, the ends generally resting on an horizontal tie-beam or stringer; otherwise the rib is placed beneath the straight timber truss. Of the latter kind are several large and tasteful structures. The celebrated bridge over the Schuylkill, at Philadelphia, and a bridge near to it, of smaller span, are so arranged. The former, called, from its great length, the Colossus, is one span of 340 feet, with the very small rise in the middle of 20 feet, or 1-17th of the chord-line, so that

Fig. 86.



SCHUYLKILL BRIDGE.

the curve is a portion of a circle 1465 feet in diameter, comprising an arc of 26¾ degrees. There are three ribs of equal size, the planks composing them being 13 inches wide by 6 inches thick, cut to the proper curvature—a method calculated materially to weaken the power of the timber;—seven of these planks were put together, forming a rib 3 feet 6 inches deep by 13 inches wide: in joining the lengths it was so arranged that one plank terminated between two others at their middle. On these ribs a truss frame was erected; it is about 7 feet deep at the middle of the bridge; the radial posts, as the figure shows, are partially strengthened by the slant struts, which, had they been crossed by others in an opposite direction, would have been much stronger: as it is they do not constitute with the posts a stiff framework; over all is a curved tie. The roadway is laid upon bearers lying across the ribs, and joined to the posts of the upper framing. This bridge, though containing comparatively little timber for its size, is, if the ribs alone were brought into action, about three times stronger than could be required, supposing the bridge crowded with people. It stands, however, as one of the best specimens of wooden bridge-building: it was arranged by Wernwag, in 1813. Another bridge, similar to the above, with a few additions, was erected over the same river some years previously, at Philadelphia: it crosses the stream in three arches, two of 150, and the middle 194 feet span, the latter having a rise of 12 feet, or about 1-16th of the chord-line.

A bridge was erected in 1828 over the river Ottawa*, which marks for many miles the separation of Upper and Lower Canada, to make a communication between the banks, near the Chaudière Falls. The span was 212 feet, with a rise of 12 feet, or nearly 1-18th of the chord; three ribs, 30 inches deep by 12 inches wide, formed the under support, while above them there were a series of braces and king-posts, in some respects similar to the Schuylkill Bridge, except that instead of the pieces being inclined all in one direction they were crossed; the struts supplied at the ends of the bridge were very inefficient, as the result proved; the king-posts passed through mortices cut in the ribs, and thus considerably weakened them. It had not been in use for three months before the diagonal braces showed signs of a settlement, and, notwithstanding various mendings, in 1835 the bridge gave way: in the process of dislocation it was observed that the diagonal pieces, although bolted together, and to the posts, and additionally tied by iron straps, tore away from the upper rib, until the lower rib at last presented a concave instead of a convex upper surface. It has been considered that the failure was owing partly to the want of abutment of the upper rib, but the order of the destruction shows that the lower rib was not efficiently tied to the upper, as the oblique pieces left the latter, which was therefore little or no burden, but it certainly was not brought into service; the lower ribs were deprived of their strength by the mortices for the king-posts, which appear to have been very ineffective; a great improvement would have been to substitute for them iron rods, which would have tied the upper and lower ribs safely. The curve given was very small, and by a bad disposition of the roadway an unnecessary strain was thrown towards the weakest or central part of the arch.

There is a very good specimen of this class of bridges called the Ladykirk and Norham Bridge, over the Tweed: in this instance the ribs vary in depth, being eight planks at the springings, and three planks at the crown or highest part of the curve; the planks are 6 inches thick and 18 feet long, so that at the ends the ribs are 4 feet deep, but at the middle 18 inches; by the arrangement at the termination of each length of plank from the ends to the middle, one is omitted; at every such place a radial brace holds the rib and an horizontal compound beam above the level of the roadway; struts, inclining towards the middle of the arch, are placed diagonally between the radial braces, similar to *fig.* 85, of the Schuylkill Bridge; but each opposite pair has a straining or horizontal piece between their upper ends; by this method the horizontal beam, made up of all these pieces, is deepest at the middle, and tapers to the ends. The appearance of the bridge is somewhat elegant; though there is a great quantity of timber employed. It consists of two arches, each of 190 feet span, and 17 feet rise, and the roadway being 18 feet wide, two trusses only are given to each arch; the whole is a combination of small timbers, none being above 28 feet long.

---

* Papers of the Royal Engineers, vol. iii. p. 158.

(76.) There is another method of using curved timber or ribs as prime means of support, in which the rib is placed above the roadway, and its extremities abut near the ends of a tie-beam or stringer, extending across the whole opening; posts or other ties, with intermediate bracing and strut beams, connecting the rib and beam at different parts; by this plan the rib stands in the position of rafters in the common roof, although by reason of its distribution of any forces acting on it the rib is a much better supporter of the long horizontal beam than straight timbers could be, where the span is great. In this form the thrust is taken away from the abutments. A fine bridge on this principle was erected over the Delaware, at Trenton, United States, the largest span being 200 feet, with a profusion of timber. There are five ribs, forming two lines of carriageway and two footways; the two outer ribs are nine planks, the three inner seven planks deep, the outer having an actual depth of 18 inches and width of 13 inches. The ribs do not abut on the stringers. although the latter are strapped to it where they meet; ties descend vertically from the ribs to cross-timbers under the stringers, on which the flooring is laid. Some ties were placed in a direction slanting towards the centre of the curve, extending from the rib to the stringer,—it has been said to counteract the horizontal thrust of the rib; whether it would or not, depends much on whether the stringer is sufficiently stiff and has an abutment; if this be not provided, there could be no assistance from such bracing: it depends also, to some extent, on the state of the ties; if the vertical be tied up more tightly than the oblique ties, the latter do not act efficiently; and as there are no strutting pieces between the stringer and beam, there is little to prevent an upward flexure at one part while a pressure is acting at another. Several railway bridges, with a profusion of material, have been constructed on the principle in this country, having a cross bracing between each tie or king-post.

Chapter IX.—*Iron Bridges.*

(77.) Owing to the advance of iron manufacture, this substance has become a favourite and valuable material in bridge-building; and, judging by the varied and bold attempts made with it in this, as in all other branches of constructive art, the most extensive and admirable bridges will be those of iron. Until of late, however, very little indeed was known of its mechanical character —of its ability to sustain or propagate forces acting upon it under various conditions, which must guide the workman in his disposing of the material in any case. At present our information is greatly increased by the experiments which arose from the manifest uncertainty of the opinions about cast-iron beams,—by the inquiry how a vessel could be made of rolled iron or boiler plates riveted together, and possessing sufficient stiffness, as well as general strength and lightness, for a sailing or steaming boat; and also in an eminent degree by another inquiry, which the success of the last-mentioned appears to have prompted—the capability of rolled-iron plates, in the form of a tube, to span wide and deep waters, or other difficult places, where all ordinary means would fail, and sustain safely great weights constantly passing through it.

In comparison with wood and stone, some kinds of iron possess great powers of resistance to crushing pressures; others bear a great tensile strain, but the extensibility of iron is much less than that of wood, fir being about two and a half times more extensible than cast iron, and pine still more; cast iron, however, is the kind of iron least extensible,—a fact militating against its use in positions where a small sufferance of extension would be advantageous. The remarkable strength and durability of iron, as a means of support, is both evident from well-known experiments made with it, and many of the now numerous structures in which it has been used. A couple of iron bars, 5 inches deep, 2 inches broad, and 12 feet long, placed 5 feet apart, the ends simply resting on two piers or supports, with planks laid across them, would support twice as many persons as could at once stand upon them, thus forming an efficient foot bridge of nearly 12 feet span; this example is founded on the known general strength of ordinary cast iron, the safety of which for such a form of bridge is far below wrought or rolled iron; cast iron has, nevertheless, until of late, been considered as the most eligible on account of the cheapness and the ease with which a frame or rib of any size and shape could be obtained.

(78.) There are objections to the use of iron in the ordinary forms of bridges, and difficulties arising in its use,—as the expansion of the metal, its rusting or oxydizement, the brittle nature of cast, and imperfect character of much that is called wrought iron.

Although the expansion of iron by reason of increase of temperature is great in comparison with wood or stone, it is, in reality, very small; in a bar of cast iron it is 1-168,000th of the length for every increasing degree of Fahrenheit's therm.; for wrought iron 1-152,000th of the length, which shows in favour of cast iron in this particular; thus, a bar of cast and wrought iron, each 70 feet long in the winter, would become respectively 70 feet $\frac{1}{8}$ inch nearly, and 70 feet $\frac{1}{6}$ inch long in summer, taking the mean variation of temperature at 30 degrees. Owing to this effect of temperature, short pieces are to be preferred in construction, as the expansion can be better compensated. In order to ascertain some practical information on this subject, Sir J. Rennie made a series of observations on the effect of temperature on the iron work of Southwark Bridge, one of the largest in span of ordinary iron bridges. He found that the arch on the Southwark side of the river (230 feet span, and 18 feet 10 inches rise) rose 1-40th of an inch for every increasing degree of temperature, or, taking a variation of 50°, it rose one inch and a quarter, which is about half an inch less than the amount obtained by calculation with the expansibility of cast iron, as given by Lavoisier*. This is certainly a small quantity, and, as respects the elevation of the line of roadway, of no consequence; but the expansion and contraction to this amount, year after year, must have a considerable effect on the connected parts of the bearing framework, for not only does each rib in an arch expand, but they do not all expand equally; those parts exposed to the sun's rays will expand most, and the difference of temperature between the inner and outer ribs is frequently above 50°: the injurious effects of such movements on the joints must be considerable. Some attempts have been made to dispose the pieces forming the supporting rib so that expansion might be allowed for in the joints.

Another and very serious misfortune is the rapid destruction of iron by rust or oxydizement when exposed to the weather; this corresponds, as far as effect is concerned, to the rotting of wood in bridges made of that material; the process of destruction, however, is the opposite in the two cases, in wood being decomposition, in iron it is composition—the gradual combination of oxygen with the elementary iron forming the rust or oxide; this action is also much increased by the unfavourable position in which the metal is placed when composing a bridge over a stream.

These are evils common to all kinds of iron, yet they affect in practice much less than might be supposed; expansion cannot be prevented, but the slight extensibility, elasticity, and compressibility of the whole material forming a bridge and its abutments, appear to meet the case in a great measure. Oxydation cannot be compensated, but, fortunately, it may be prevented to a great extent by coating and recoating the metal to protect it from atmospheric influence.

In addition to these common properties which distinguish cast and wrought iron, and point out that service for which they are respectively best qualified, cast iron being of a crystalline and brittle character renders it a doubtful supporter where cross strains are concerned, or any exercise of tensile force; where, however, a uniform crushing pressure is acting, and great stiffness required, this description of iron is most useful. Shocks and twists exert a disastrous influence on the parts of cast-iron beams; also anything which causes vibration, for the particles soon separate beyond the circle of their mutual cohesive attraction. Many and serious accidents have proved the danger of using cast iron under such unfavourable circumstances, although, when it is considered how carelessly this material has been applied, as though it could not be too severely tried, the surprise is that more accidents have not occurred. Wrought iron, by the processes through which it passes, becomes fibrous, and is not marked by that brittleness which characterises cast iron: it is, at the same time, less able to resist compression in the ratio of about 4 to 5, but through the possession of flexible and slightly-extensible qualities, it is much more trustworthy: the objection to its use is principally one of the immediate expense. Experience has abundantly confirmed the conviction that wrought or rolled iron is the most proper, and, indeed, only safe description of that metal for those purposes in which intermixed and irregular crushing and tensile forces are likely to occur; for, generally, they are not completely met by judicious arrangements, because not thoroughly preconceived. There is one fact militating against iron, which is at present little understood, and possibly on that account generally neglected;—it is, the molecular change which occurs in a mass of the metal when subject to continual vibration, making wrought iron brittle and cast iron more so. Some have attempted to account for it on ordinary mechanical principles, but from actual experiment, as well as observation, it would appear that the change is brought about by magnetic (polarising) action: whatever more decisive information may point to as the cause, it is certain that the effect is a destruction of the most valuable property of iron.

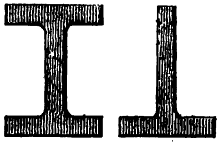
---

* This philosopher stated it to be ·00000618 of the length for 1° F. Daniell (Phil. Trans., 1831) gives it ·00000595.

Iron is used in bridge-making as a girder or simple beam, which is either solid or hollow, the latter being the latest and most improved kind; or as a series of plates or frames, somewhat similar in the conditions of their equilibrium to masonry arches; frequently, however, the frames are in long pieces, merely disposed in the arched form; or, lastly, as a series of connected bars or wires, forming an articulated or flexible chain or line, hanging between two or more piers, from which a roadway is suspended.

(79.) An astonishing number of bridges where the span is not very great are made of girders, being comparable with the common beam mentioned in the last chapter, as influences of any superincumbent pressure are similar; the inquiry into the subject has, however, been far more extensive with cast-iron beams, owing to their great social importance; and the varieties of shape and arrangement of the material in girders arising from these inquiries, or evidently, in many cases, from a strange caprice, are surprisingly numerous. Simple girders are very commonly made of the following sectional forms:—

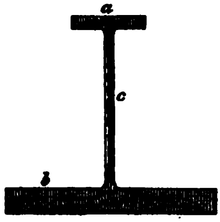
Fig. 87.



No. 1. No. 2.

No. 1 is called the double T or I or I girder; and the second form, used either as in the figure or with the broad side uppermost, is the T girder: in each case it is evident that the disposal of the metal is such as to obtain a considerable breadth, or rather area of section to resist extension or compression, or both; while the middle parts, being about the neutral axis, are cast thin in comparison, or sometimes partially left out, which may be done with very little sacrifice of strength, but with considerable relief as regards the weight. In calculating the strength of such beams to sustain loads placed upon them, there is no need of deducting the strength of the left out portion as though that were a full working portion of the beam, for the broad extreme parts or flanges bear the greatest part of the whole force exerted by any load—the lower flange being in a state of tension, while the upper flange sustains the crushing pressure. The improvement effected by the addition of a flange to the under side of the common rectangular beam evidently raised the question how much more might be gained by a further alteration in the form of the girder, and has led to the calculation of the strongest form of beam with a given quantity of material. Mr. Tredgold, from considerations respecting the nature of iron, which proved in some measure to be incorrect, proposed the double T, or the form shown in No. 1 of the above diagrams, the flanges being equal in area of section. Mr. Hodgkinson investigated the subject in the only safe way, by direct experiments on cast-iron flanged beams; the results will be treated more particularly in the subsequent chapter on the strength of materials; the great point gained, however, was that the strongest form of section is according to the following figure:—

Fig. 88.



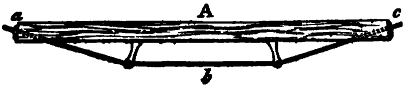
The width of the flanges in his experiment being—top, 2·33 inches; bottom, 6·67 inches: depth—top, 0·31 inch; bottom, 0·66 inch; while the intermediate part is only 0·266, or little more than a quarter of an inch thick; thus, we have the following areas of the flanges:—

Area of top flange = 2·33 × 0·31 = 0·72 inch.

„ of bottom flange = 6·67 × 0·66 = 4·40 inches.

These areas are in the proportion of 1 to 6⅛, the latter being the area of the lower flange: with this disposition a gain of 2-5ths of the strength is obtained over the best common ⊥ formed girder; and at the same time shows that this ratio is nearly that which expresses the ability of cast iron to resist crushing and tensile forces. Girders of a simple kind are much used, especially for railway bridges where the span is about 40 feet or less. Owing to the acknowledged insecurity of these girders, trussing bars have been employed, to assist against the tensile force; the tension rod is then carried more or less below the cast-iron beam, as indicated in *fig*. 80, page 170, the extremities being fastened to the extremities of the beam. Whether the girder is strengthened by this addition depends greatly on the depth allowed between the beam and the rod; little or no assistance is given if the rod be confined within the limits of the beam; it is evident that any stretching force which may be exerted on the under side of the beam A will stretch much more, the rod *b* being farther from the neutral axis: if, however, it be level with the under side of the beam it will stretch only as much as the material of the beam; if, therefore, the beam A be of cast iron, which allows but a very small extension of its parts without breaking, and the rod *b* of malleable iron, which allows of a much greater extension (about $\frac{1}{10000}$th of its length for every ton of pressure), be placed on a level with the under side, it cannot be of any service when a trial occurs, except to assist the load in breaking the girder. An instance of this faulty trussing was exhibited in a bridge near Chester, carrying the Chester and Holyhead Railroad over the river Dee, and attended with the lamentable result of a failure, while a train was passing over it on the 24th of May, 1847, by which the train was precipitated into the river from a height of above 30 feet, and several persons were killed. The construction of this girder bridge, the largest erected of its kind, was as follows:—there were three spans of 98 feet each, the girders being 3 feet 9 inches deep and 109 feet long; each girder was composed of three lengths of casting, which were bolted together, the depth at the joints, by the addition of cast-iron joint plates, being increased 6¾ feet; the lower flange was nearly 5½ times the area of the upper; the malleable-iron truss bars were placed in a series of four on each side of the girder, and keyed up tightly at the end of the girder by wrought-iron keys; in consequence of the length of the girder and their reaching only to the under side of the cast-iron girder, they were very nearly horizontal, in addition to which some part of their length (near the ends) was above the neutral axis, so that altogether it is probable they were never brought into action; indeed, after the accident, they were found to be uninjured. Many and conflicting opinions were given at the inquest concerning the cause of the fracture of one of the outside girders, which brought about the accident, the engineer of the line contending that it was the result of the tender, belonging to the locomotive which drew the train, getting off the

Fig. 89.



rails, striking the outside girder, and breaking the thin rib between the flanges. If it was so, still the girder must have been defective, to give way through such a blow against it; had the tie-rods been under tension the shock would have effected only a momentary extension, and the parts directly recovered themselves; but the cast iron, already strained by the train upon it, could not recover from the blow; and possibly the circumstance of its having been loaded with twenty waggons of ballast but an hour or two before the train came up considerably strained the beam. It was stated that the girders were proved capable of supporting more than could be placed upon them, but this plainly is an argument little to purpose; testing weights are dead weights —they are applied gently, whether actual weight or the hydraulic press is used—but that is not to be compared with the rapid passage of train loads, which produce a series of concussions at all times; and besides these things, the continually-increasing brittleness of iron caused by vibration (art. 78) no doubt made these girders less able to bear this impulsive force.

The bridge is now strengthened by an under trussing of two struts and intermediate straining beam, similar to the method shown in *fig.* 81.

Many girders of as many forms have been trussed by malleable-iron rods, in a proper manner, varying in length from 40 to 90 feet; in some instances the cast-iron beam is very judiciously made of a curved form. In such a compound girder, if the tie-rod is keyed up tightly, there is a greater chance of the whole of the cast iron being brought into a crushing pressure, the neutral axis passing between the beam and tie-rod; however, probably from the dislike frequently evinced in practice to inquiring into the principles of anything undertaken, much objection, which will not stand before inquiry, has been offered to any improvement on the simple girder; truss-rods have been denounced in general terms as useless—for what reason has not appeared; again, the difference of expansion of cast and wrought iron has been urged, but this difference is only 1-2,460,000th of the length, so that in the longest girder—the Dee Bridge—the rods expanded by a change of 30° of temperature 1-60th of an inch more than the cast-iron beam! That one extends more than the other is very true, but when the truss-rod is placed at a sufficient distance from the neutral axis, they will extend greatly before they allow the girder to deflect at all. That the tension-rods should have a support independently of the girder to assist it at all is shown to be a fallacious idea, from the fact that if the girder could receive all the pressure as a crushing effort it would be most safe, owing to its great strength in resisting a force thus acting; but if the tie-rod be properly applied, as before stated, its function is exactly such as to convert the tensile into a crushing force on the beam, for when the load tends to deflect, it strains the rod first; it exerts a corresponding pull on the ends of the girder, which tends to crush the latter, and, supposing the rod to be so strong, it could not give way, except the cast iron were first crushed.

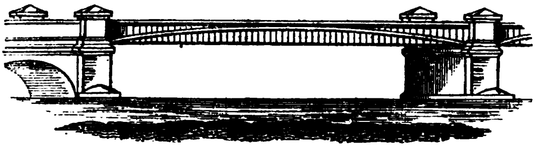
(80.) The trussing of a cast-iron girder for the sake of security, however well effected, still appears like patching a piece of bad work; and as prevention is far better than cure, and the greatest safety is not to be exposed to danger, many plans have been offered to the public within the last few years for applying wrought iron alone in the formation of girders; the early notions of this kind being to rivet an opposite pair of *angle* irons (ᒣ) to the top and bottom of iron plates: the angle irons, which are pieces bent at right angles, thus formed an upper and lower flange similar to the cast-iron girder (*fig.* 88), and, consequently, was the important part of the girder—the plates serving the purpose of keeping the flanges at a distance from the neutral axis; a fault, however, was committed in their construction from want of more information at the time—the bottom flange was made considerably the larger. Mr. Fairbairn's experiments on the tubular bridges have since proved the ability of wrought or rolled iron to be above twice as great, as crushing forces, to resist tensile, for a solid girder; so that the upper should be above twice as large in area as the lower flange.

(81.) There are two forms of construction, one of which is, we believe, at present a proposition, in immediate principle very closely allied to the ordinary girder;—one is that called the lattice bridge, and the other an arrangement set forth by M. Busse. The lattice girder is analogous to the wooden lattice bridges of Mr. J. Town, mentioned at page 170; they are composed of longitudinal upper and lower ties, with thin bars of iron intervening, and crossing each other. There is a bridge of this kind on the Dublin and Drogheda Railway, of 84 feet span, the lattice being 10 feet deep, and formed of flat wrought-iron bars 2½ inches wide and 3-8ths of an inch thick; they cross each other at an inclination at an angle of 45°, or so as to form the diagonal of a square. The roadway is sustained at about midway between the top and bottom of the lattice, upon cross beams two feet apart.

The bridge proposed by M. Busse is one wholly composed of wrought iron, being, in effect, a trussed T girder. The arrangement is intended for ravines or other places inaccessible for erecting centres, or building arches. Several girders, made up of two ⅂ pieces with two or three plates between them, stretch from pier to pier, while as many bars, linked together, pass underneath, and are kept at a distance below the girder by two pieces of iron acting like two legs,—a screw joint in the middle giving opportunity to tighten this truss-rod at pleasure. Above the girder there is a lattice work extending some 20 feet in height, affording additional security; the inventor adopted it for another purpose—that he might suspend a second roadway near the top of the lattice, thus making one way for railroad and another for common road traffic. The piers are another and more novel portion of his proposed structure; they are arranged with a series of plates, meeting on edge, and forming a kind of pyramid, about one-fourth as broad at the base as it is high. Piers or pillars of this description can be quickly built up, to heights of 200 or 300 feet, while their weight is small, and, if in rivers, they offer very little obstruction to the current. M. Busse designed his truss-work for openings or spans of 100 feet, and 24 feet wide, with a series of five trussed girders; and very properly submitted this idea to experiment, with a model 1-12th the size in every respect of the intended bridge; so that it was 2 feet wide, and 100 inches, or 8 feet 4 inches long. On this little bridge he placed 40 cwt. or 2 tons, which indicates considerable strength, if it was a load placed in the middle; whether it was so, or distributed over the surface, and whether it was the breaking weight, does not appear. M. Busse supposed that as his model was in cubic dimensions 1-12th smaller than the intended spans, the latter would be in strength proportional to the cube; that is, the strength of the full-sized bridge might be thus calculated, $2 \text{ tons} \times 12^3 = 3456$ tons. Upon such a supposition the inventor reckons that two trains of heavy locomotives on each line (which would weigh 144 tons) would be but the twenty-fourth part of the load it could carry: this, however, is certainly not the case; the weight of the bridge truly increases as the cube of the size, but the strength appears to be about as the square: on this supposition his bridge would have a strength of $2 \times 12^2 = 288$ tons, which is equal to double the load of locomotive engines proposed as a maximum: the weight of the materials has first to be deducted from the above strength, which, therefore, would give useful or available strength somewhat less than the above amount. It may seem that such a calculation exhibits the bridge as inefficient; but much depends upon the manner in which the model supported the two tons; if it was placed on or hung from the middle, the result is greatly modified; in that case the locomotives are not equal in weight to one-half the strength, because they are distributed over the whole length of the span, and such a load is known to be equivalent in its effects to one-half its weight, or 72 tons, placed at the middle; then, as the 288 tons would represent a load placed at the middle, it would be equal to four times the greatest probable load. M. Busse is no imitator in applying wrought iron for supporting a bridge, if we may credit his remark that he knew of no bridge having been previously built of wrought or rolled iron; probably he was not aware that experience had brought many to use it some years before his invention appeared. It is, nevertheless, from its simplicity and capabilities, well deserving attention.

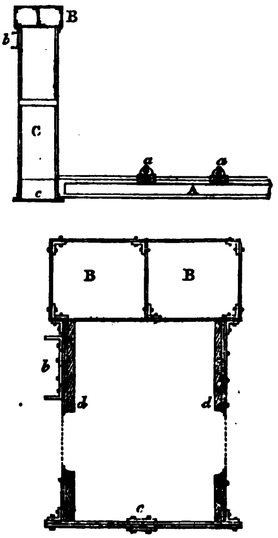
(82.) At an early period of the introduction of plate iron for girder-making, the idea of tubular or hollow beams appears to have been developed, although the construction of the early tubes does not evince the least attention as to the proper distribution of the material. The great improvement in these useful forms has been made by Mr. Fairbairn, who patented his invention near the end of the year 1846. It was one of the results of his experimental inquiries to discover the best form of tube or girder for crossing the Conway River and Menai Straits, which he began during the previous year. It will be sufficient to describe the parts of this new girder, without entering into the inquiry concerning its strength, as the subsequent detail of the experiments made for the tubular bridges will supply the information. The following figures exhibit a bridge erected on this principle over the River Trent at Gainsborough, for the Manchester, Sheffield, and Lincolnshire Railway. There are two spans, each 154 feet wide, according to the elevation (*fig.* 90), which is a side view, but presents an inclination to the direct line of vision, owing to the oblique angle at which the line crosses the river. There are for each opening two girders, one on each side of the rails, with a width of 26 feet between them; the peculiarity of their formation is shown in *fig.* 91, and may be divided into three parts, the cellular top, sides, and double-plated bottom. Bearing in recollection the character of a load-supporting beam sustained at each end, we observe that the provision here made for resistance to crushing and tensile forces is the principal feature in the beam. The upper side is composed of two rectangular cells or tubes, B B, of 36½ inches total width, and their depth is 15 inches; the plates are well united at the corners by rivets and angle-irons. The lower side *c* is simply a double thickness of plates, 36 inches in width, presenting a less sectional area to meet the force of tension than the upper cells to meet that of compression, according to the fact discovered by the inventor—that the rolled-iron structures can be crushed much more readily than torn asunder. The sides C, which serve principally to keep the parts in their intended form, are of thin plates, strengthened or stiffened on the inside with ribs *d* of T iron. The whole is indeed, for the most part, a miniature representation of the celebrated tubular bridges. Between the girders extend cross-beams A of iron plate, and common rectangular section, carrying longitudinal beams of wood, whereon the rails *a* are laid; these cross-beams are placed 4 feet apart. An addition is made to the external face of the girders in the shape of an arch, seen in the elevation and section at *b*; it is professedly for mere ornament—to relieve the broad surface of plating; as such it is but an incumbrance, but plainly it might be turned to use against compression. The following are the general measures of the girders:—

Fig. 90.



GAINSBOROUGH BRIDGE.

*Fig.*91.



Clear length . . .	154	feet.
Whole depth . .	12	,,

Depth of cells . . .	1·25	feet.
Total width of cells .	$3\frac{1}{16}$	,,
General width of girder	$2\frac{1}{2}$	,,
Width of bottom plates	3	,,

These girders are both light and strong, allowing but a small deflection with the heaviest loads; one of 60 feet span and 6 feet deep was deflected only 0·4, or less than half an inch, by the load of three heavy locomotives driven at a slow and quick pace. The greatest span hitherto erected with the tubular-celled girder is 200 feet.

(83.) All bridges where the upholding parts rest at each end on some supports, and sustain a roadway by compression of the upper, and more or less tension of the lower side, are essentially girder bridges; their value is calculable on the same mechanical principles, and they must be treated with like care and adjustment. The chief value of the girder, besides ease in construction, is the opportunity it affords for throwing a way across any open place without the annoyance of a rise in the roadway, or blocking up a portion of the opening with heavy haunches or spandrels, which occurs generally in arches. In addition to these inconveniences, there is the erection of piers at intervals, probably of not more than 90 or 100 feet, because of the otherwise great rise of the arch at its crown, even in the elliptical or flat curve; but where girders such as the last mentioned can be applied to the extent of 200 feet span, much of the labour of constructing foundations for piers is avoided. The value of a girder form of support, in all these respects, has received a remarkable exemplification in the celebrated and, at present, most gigantic tubular bridges, over the Menai Straits and the Conway River. It was a most fortunate circumstance for science that these structures had to be designed and erected,—the series of experiments undertaken in order to discover a means of passing over a wide and deep river and strait, without obstructing waterway or maritime traffic (even for any length of time), having enlightened us greatly concerning the character and capabilities of malleable iron, when in plates riveted together. The Chester and Holyhead Railway, amongst other difficulties, in its course through North Wales, had to pass over the River Conway and Menai Straits; the river at Conway, which is close to its mouth, and where the railroad crosses it, is a deep and rapid stream, the level of the rails being little more than 20 feet above the water-line, while the opening to extend across the whole current must be at least 400 feet; an arch, therefore, which should have the unheard-of span of 400 feet, with only 20 feet rise, was out of the question. The straits also are, in many parts, very deep, with a rapid current; the rail level, however, to within a few feet of the shore, on the Carnarvon side, is about 100 feet above the water; this would permit the rise of an arch, and it was proposed to venture two enormous cast-iron arches, of 450 feet span each, the middle abutment or pier being the Britannia rock, which is seen at low water about midway between the shores. This project, though bold, is not equal to that of Mr. Telford, who offered to replace old London Bridge by an iron arch 600 feet span; and there is no reason why it should not have answered. The engineer was compelled to abandon this idea, owing to the opposition of the Commissioners of the Admiralty, who required a clear way above high water of 105 feet, that vessels might freely sail under it. As a suspension bridge had been found to be quite inapplicable to railway traffic, owing to the great undulation during the passing of a train, some untried means of obtaining supports for openings of 450 feet had to be discovered. Mr. Stephenson, the engineer of the line, then considered the possibility of adopting a modified suspension bridge—or a tube of a circular or oval form, through which the trains were to run, with side-suspending chains; thus, while the chains gave support, the tube would impart that rigidity to the roadway which is requisite. Under these circumstances the matter was referred to Mr. Fairbairn, who proposed a self-supporting tube, founded on the efficiency of steam vessels, made of thin iron plates, to support loads in them without injury *. From this time (April, 1845) a series of most interesting experiments were conducted by Mr. Fairbairn to

---

* In the evidence before the Railway Committee of the House of Commons, Mr. Stephenson said that Mr. Fairbairn had told him he was building an iron vessel, 220 feet long, which, when finished, should, being supported at each end only, carry a weight of 1000 tons of machinery in the middle.

determine the best shape of tube, and best distribution of the material.

It is impossible to enter into the whole series of inquiries, with notices of the fears, failings, and successes, in all cases interesting and instructive, which attended the earlier stages of the proceeding*; we shall, therefore, give a short notice of the order of experiment, showing the principal steps in developing the magnificent result.

(84.) The operations first undertaken were on cylindrical tubes of thin iron plate, the tubes A during the trials being supported, at each end, on firm blocks B, (*fig.* 92,) which were cut out so as to give the tube more bearing than the flat surface could offer; a hole was also made midway between the ends of the tube, that the trial weights D might be suspended from the inside. Nine experiments were made on cylindrical tubes, the general numerical results of which are shown in the following table:—

Fig. 92.

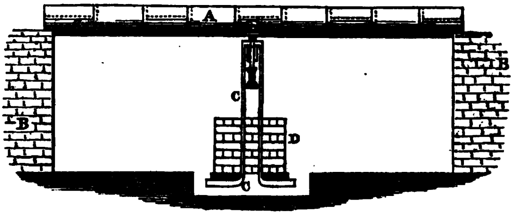


TABLE I.

No. of Experiment.	Distance between supports. ft. in.	Diameter. in.	Thickness of plate. in.	Ultimate deflection. in.	Breaking weight. lbs.	Remarks.
1	17 0	12·18	·0408	·39	3,040	Crushed top.
2	17 0	12·00	·0370	·65	2,704	Crushed top.
3	15 7½	12·40	·1130	1·29	11,440	Torn asunder at bottom.
4	23 5	18·26	·0582	·56	6,400	„ „
5	23 5	17·68	·0631	·74	6,400	„ „
6	23 5	18·18	·1190	1·19	14,240	„ „
7	31 3¼	24·00	·0954	·63	9,760	„ „
8	31 3¼	24·30	·1350	·95	14,240	„ „
9	31 3¼	24·20	·0980	·72	10,880	„ „

The first and second experiments indicated the greater weakness of the top, while the remaining seven were torn asunder at the bottom. That the former should have readily puckered is not surprising when the thickness of the plate is remembered; for a tube of 17 feet long and one foot in diameter, a thickness of material about 1-25th of an inch is surprisingly small; the thicker plates, in the subsequent experiments, successfully resisted the crushing pressure. The first of the second class (Exp. 4) was torn across the middle, at the place where the hole was made for the suspension of the weights; all the others gave way at the rivet joints. The nature of the results in these trials is more clearly observable from the course of each experiment—the successive alterations in appearance and in deflection which attended the addition of weight. One very objectionable character of the circular tubes proved to be their tendency to change of shape under pressure—the sides collapsing, and, of course, pushing the top and bottom farther apart; and, unless circular blocks were fitted in at each end of the tube, the ends became flattened, the sides spread out, while, in the middle, they were forced towards each other, showing very little stiffness; this weakens the whole tube, while it partially accounts for their inferior amount of deflection in comparison with the subsequently-tried forms. With the heavier weights, therefore, these tubes are rather to be considered as elliptical, or oval, than circular.

These experiments, while they afford abundant evidence of the powers of wrought-iron tubes for the purpose designed, show the impropriety of a circular form. Mr. Fairbairn then proceeded with elliptical tubes, and, by several experiments, obtained the following results:—

---

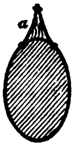
* See "An Account of the Construction of the Britannia and Conway Tubular Bridges, by W. Fairbairn, C.E., &c.," which supplies us with the whole history of this investigation.

Table II.

No. of Experiment.	Distance between supports.	Diameters.	Thickness of plate.	Ultimate deflection.	Breaking weight.	Remarks.
	ft. in.	in.	in.	in.	lbs.	
19	17 0	14·62 9·25	·0416	·607	2,100	Crushed top.
20	24 0	21·66 13·50	·1310	1·35	17,076	Torn asunder at bottom.
21	24 0	21·25 14·12	·0688	·45	7,714	Top doubled up.
22	18 6	12·00 7·50	·0733	·95	6,867	This tube had a fin on the top, which gave way.
24	17 6	15·00 9·75	·1430	1·39	15,000	Both sides ruptured.

In four out of the five trials these tubes gave way by compression; in the 22nd experiment the top of the tube was strengthened by a fin *a*, which extended the whole length; but the result proved it to be of little service, as will be noticed by comparing this tube with the 24th experiment, where a tube a foot shorter, with less than twice the thickness of plate, and no additional strengthening, bore nearly 2·2 times the weight. On the whole, these figures argue favourably for elliptical tubes: the 19th experiment, with a tube equal in length and quantity of material to the cylindrical tube in Exp. 1, certainly sustained little more than two-thirds of the latter tube before it was crushed at the top, but it contrasts also strongly with the 24th experiment, where, with very nearly the same diameters, rather greater length, and not quite 3½ times the thickness of metal, almost 7¼ times the weight was borne. The remainder, however, testify in favour of the elliptical form; although stiffness appeared to be yet wanting, and the upper parts were too weak to prevent crushing.

Fig. 93.



Some time before these latter experiments, and, indeed, at the outset of the inquiry, Mr. Fairbairn perceived the disadvantage of any curved forms; the results of trials with a few cylindrical tubes fully established this opinion; the rectangular section offered itself as the best shape for general strength and stiffness, and it is not a little remarkable that, soon after several experiments on rectangular tubes had been made, Mr. Fairbairn sketched as the probable form which the tube would finally assume, nearly the exact shape and arrangements actually presented by the bridges themselves. At first a common tube 9·6 inches square and 17½ feet long was tried, the thickness of the plates at the top being ·075 inch; the bottom and sides ·0743, or a little less, the whole weight of the tube being 202 lbs.; this tube deflected about ·037 of an inch for every hundredweight, until 3738 lbs. were laid on, when it yielded, the top doubling up, and the side bulging close by. The same tube had a thicker plate = ·252, or above a quarter of an inch, put on the top; then it required 8273 lbs. to injure it, when a joint at the bottom side was torn, near to the place where the weights were suspended. This experiment plainly indicated that the material could better resist tension than compression; it was, however, more strikingly shown in the next trial; the tube being in dimensions as follows: —17½ feet between supports, 9·6 inches square; thickness of the plates—top, ·0757 inch; bottom, ·142 inch; sides equal to the top; thus the bottom side was twice the thickness of the top. When the tube had deflected 0·94 of an inch (by a weight of 3788 lbs.) the top plates could no longer stand the pressure, and doubled up in two places near the middle. The same tube was then laid with the thick side uppermost, when it bore a load of 7148 lbs., also a deflection of an inch and three quarters before yielding, and then by extension—thus the power of support was increased

7148 — 3788 = 3360 lbs., or nearly as much again, by simply turning the tube upside down; afterwards more than double the strength was obtained by the same treatment with another tube, the results of which, with all the experiments on rectangular forms, are shown in the following table:—

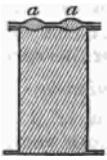
TABLE III.

No. of Experiment.	Distance between supports. ft.	in.	Depth. in.	Width. in.	Thickness of plate. Top. in.	Bottom. in.	Ultimate deflection. in.	Breaking weight. lbs.	Remarks.
14	17	6	9·6	9·6	·075	·0743	1·12	3,738	Broke by compression.
14 a	17	6	9·6	9·6	·252	·075	1·10	8,273	(Reversed). Extension.
15	17	6	9·6	9·6	·075	·142	0·94	3,788	Compression.
15 a	17	6	9·6	9·6	·142	·075	1·76	7,148	(Reversed). Extension.
16	17	6	18·25	9·25	·149	·269	1·03	6,812	Compression.
16 a	17	6	18·25	9·25	·269	·149	1·73	12,188	(Reversed). Compress$^{\text{n}}$.
17	24	0	15·00	2·25	·260	·260	2·66	17,600	Compression.
18	18	0	13·25	7·50	·142	·142	1·71	13,680	„
23	18	6	13·00	8·00	·064	·064	1·14	8,812	„ fin on top.
29	19	0	15·40	7·75	·230	·180	1·59	22,469	„ corrugated top.

By reversing the tube in Experiment 16, a weight of nearly 12,188 lbs. was sustained; but as the tube still yielded to the crushing force, a stronger plate was riveted to the top, when the load was increased to 13,867 lbs., and after bearing it some minutes the plates were torn asunder at the rivets, 4 feet from the middle. Another character developed in this experiment was the superior value of greater depth than width; this tube was less wide than the former, but nearly twice as deep. These experiments were such as to inspire confidence in proceeding with the inquiry, at the same time that they pointed out several practical points of interest and guidance. In Experiment 17, the depth was above six times the breadth of the tube, with top and bottom plates of equal thickness, sustaining a weight of nearly 8 tons, with a surprising deflection of above 2½ inches; as might have been expected, the tube yielded to compression. A trial was made in this experiment to prove the enduring power of the tube—when the load had been increased to 14,240 lbs., it was allowed to remain on all the night; but the only effect was to increase the deflection from 1¾ inch—when the weight was at first suspended—to 2 inches, or merely a quarter of an inch, and when this load was lifted off the tube it was found to have lost little of its elastic power.

An idea arose in consequence of the continual buckling of the top plate by compression, to bring, if possible, the top side into a state of tension, for which purpose Experiment 18 was designed; the tube was 37 feet 8 inches long, two supports being placed 18 feet apart, so as to allow of a portion of tube extending beyond them to balance the middle span of 18 feet; the lower side was horizontal while the upper was curved, giving a depth of tube in the middle of 13¼ inches, but at the two supports 17¼ inches. The result of a trial proved the impracticability of this idea, the top still buckling up; yet the experiment showed that the effect of the two portions of tube beyond the supports or piers was to relieve the middle part of some of its weight — a circumstance which has been turned to account in the Britannia Bridge. From the insufficient stiffness of the fin on the top of the elliptical tube, Mr. Fairbairn was led to try another compound top, shown in the section, *fig*. 94, the two plates of the upper side being bent out to form two cavities or tubes *a a*, extending along the tube; with this corrugated top Exp. 29 was made, the upper plates being each ·115 inch thick, or ·230 together, the bottom ·180 inch, and the sides ·070, or about 1-14th of an inch. The little tubes *a a* were 1·65 inch in diameter, or about 1-9th the depth of the tube. On comparing this tube with No. 17 in the table, its supe-

Fig. 94.



riority is manifest; allowing for the difference of length between the supports, and the top and bottom being very nearly the same distance in both tubes from the neutral axis, we have a breaking load for No. 17 a little less than for No. 29; but in the former the plates of top and bottom were each ·260 of an inch; in the latter ·230 at top and ·180 at bottom: the former tube being above one-fifth greater in weight than the latter. Moreover, No. 17 gave way by compression, while No. 29 failed only in consequence of the sides tearing away. This satisfactory experiment showed an approaching adjustment in the thickness of the plates; for just "previous to the tearing of the sides from the top at the rivets, that part had begun to assume a slightly-undulating appearance on one side, arising from the weakness of the side plate, which gave way near the shackle [where the weight was suspended]. This was not the only part that suffered under the strain, as the opposite side was tearing from the bottom plate at the same time, evidently showing a rapid approach to rupture on both the lower and upper sides of the tube."*

(85.) At this period of the investigation all the results were collected and made up into a report, which was laid before the directors of the company. At the same time Mr. E. Hodgkinson, whose assistance had been given in the inquiry, made a reduction of several of the experiments, showing the resistance per square inch (in section) offered by the different tubes at the time of rupture. These results it will be interesting to notice, although from the different behaviour of some experiments to that supposed in theory, and the complex arrangements of other tubes, the results do not confirm or deny, to any great practical value, the experimentally-found form of tube for this special purpose.

" *Cylindrical Tubes.*

The strength of a cylindrical tube supported at both ends, and loaded in the middle, is expressed by the formula

$$w = \frac{\pi f}{a\,l}(a^4 - a_1^4).$$

where l = distance between the supports.
a, a_2 = the external and internal radii of the tube.
f = the force sustained by one square inch of section at the top and bottom of the tube.
w = the breaking weight (including half the weight of the tube).
π = 3·14159.

From the above formula we find f—the force borne per square inch to be

$$f = \frac{w\,l\,r}{\pi\,(a^4 - a_1^4)}.$$

Computing the results given in the first table on cylindrical tubes, we find the following values:—

$$\left.\begin{array}{ll} \text{Exp. } 1, & f = 33456 \\ \;\;,, \;\; 2, & f = 33426 \\ \;\;,, \;\; 3, & f = 35462 \\ \;\;,, \;\; 4, & f = 32415 \\ \;\;,, \;\; 5, & f = 30078 \\ \;\;,, \;\; 6, & f = 33869 \\ \;\;,, \;\; 7, & f = 22528 \\ \;\;,, \;\; 8, & f = 22625 \\ \;\;,, \;\; 9, & f = 25095 \end{array}\right\} \begin{array}{l} \text{Mean } 29887 \text{ lbs.} \\ = 13\cdot34 \text{ tons.} \end{array}$$

Fracture in all cases took place either by the tube failing at the top, or tearing across at the rivet-holes; this happened on an average, as appears from the above, when the plates were strained 13⅓ tons per square inch, or little more than half its full tensile strength.

" *Elliptical Tubes.*

The value of f in an elliptical tube, the transverse axis being vertical, is found by the following formula:—

$$f = \frac{w\,l\,a}{\pi\,(b^3 - b_1\,d_1^3)},$$

where d, d_1 are the semi-transverse external and internal diameters; b, b_1 the semi-conjugate external and internal diameters; and the rest as before. Computing the results from Mr. Fairbairn's experiments, we have from

$$\left.\begin{array}{ll} & \text{lbs.} \\ \text{Exp. } 20, & f = 36938 \\ \;\;,, \;\; 21, & f = 29144 \\ \;\;,, \;\; 24, & f = 45185 \end{array}\right\} \begin{array}{l} \text{Mean } 37089 \text{ lbs.} \\ = 16\cdot55 \text{ tons.} \end{array}$$

" *Rectangular Tubes.*

If in a rectangular tube, employed as a beam, the thickness on the top and bottom be equal, and the sides are of any thickness at pleasure, we have

$$f = \frac{3\,w\,l\,d}{2\,(b\,d^3 - b_1\,d_1^3)},$$

d, d_1 representing the external and internal depths respectively; b, b_1 the external and internal breadths; the rest being as before.

---

* Fairbairn's "Account," &c., p. 247.

Exp. 14 gives by reduction—

$$f = 18{,}495 \text{ lbs.} = 8{\cdot}2566 \text{ tons.}$$

This is however much below the value which some of my own experiments give, as will be seen further on.

The value of *f*, which represents the strain upon the top or bottom of the tube when it gives way, is the quantity per square inch which the material will bear either before it becomes crushed at the upper side or torn asunder at the bottom. But it has been mentioned before that thin sheets of iron take a corrugated form with a much less pressure than would be required to tear them asunder; and therefore the value of *f*, as obtained from the preceding experiments, is generally the resistance of the material to crushing, and would have been so in every instance, if the plates on the bottom side (subjected to tension) had not been rendered weaker by riveting.

The experiments made by myself were directed principally to two objects,—

1. To ascertain how far this value of *f* would be affected by changing the thickness of metal, the other dimensions of the tube being the same.

2. To obtain the strength of tubes, precisely similar to other tubes fixed on,—but proportionately less than the former in all their dimensions, as length, breadth, and thickness,—in order to enable us to reason as to strength from one size to another, with more certainty than hitherto, as mentioned before. Another object, not far pursued, was to seek for the proper proportion of metal in the top and bottom of the tube. Much more is required in this direction.

In the three series of experiments made, the tubes were *rectangular*, and the dimensions and other values given below.

No. of Experiments.	Distance between supports.		Depth of tube.	Breadth of tubes.	Thickness of plates.	Breaking weight.	Value of *f* for crushing force.
	ft.	in.	in.	in.	in.	tons.	tons.
1	30	0	24 nearly.	16 nearly.	0·525	57·5	19·17
2	30	0	24 „	16 „	·272	22·75	14·47
3	30	0	24 „	16 „	·124	5·53	7·74
						lbs.	
4	7	6	6 „	4 „	·132	9·976	23·17
5	7	6	6 „	4 „	·065	3·156	15·31
6	3	9	3 „	2 „	·061	2·464	24·56
7	9	9	3 „	2 „	·030	·672	13·42

The value of *f* is usually constant in questions on the strength of bodies of the same nature, and represents the tensile strength of the material; but it appears from these experiments that it is variable in tubes, and represents their power to resist crippling. It depends upon the thickness of the matter in the tubes, when the depth or diameter is the same; or upon the thickness divided by the depth when the latter varies. The determination of the value of *f*, which can only be obtained by experiment, forms the chief obstacle to obtaining a formula, for the strength may be made as in the 'Application de Mécanique' of Navier, part I. art. 4, or as in papers of my own in the 'Memoirs of the Literary and Philosophical Society of Manchester,' vols. 4 and 5, second series. In the last table of experiments, the tubes were devised to lessen or to avoid the anomalies which riveting introduces, in order to render the properties sought for more obvious. Hence the results are somewhat higher than those which would be obtained by riveting as generally applied.

The tube 31 feet 6 inches long, and ·272 inch in thickness of plates, was broken by crushing at the top. This tube was afterwards made straight, and had its weak top replaced by one of a calculated thickness, and by this small addition of metal applied in proper proportions to the weakest part, the tube was increased in strength from 22·75 tons to 32·53 tons, and the top and bottom gave way together."

This report, while it points out several interesting facts, shows of what little utility known theoretical principles were to the subject under discussion. It points out a superiority of the cylindrical and elliptical over the rectangular tubes; but this is not practically true when the experiments are examined, for it will be remembered that, long before breaking, the curved tubes greatly changed their shape, evincing their unfitness for useful purposes; so that the real strength of the tube, usefully considered, is that which it could bear without change of shape; this failing is not exhibited in the rectangular form. Again, the calculation according to the equation for rectangular tubes supposes the thickness of top and bottom to be equal, a condition the experiments have abundantly shown to be disadvantageous for obtaining the strength of all the material, and finds the value of *f* in Experiment 14 to be only 8¼ tons, but we observe in Experiment 14 *a*, which was with the same tube, but a stronger top plate, more than double the strength was gained. Such a case is, however, not contemplated in the above equation for the rectangular tube. Mr. Hodgkinson's experiments show that as the thickness of metal is greater, so the value of *f*, or the force borne per square inch, increased, which evidently happens in consequence of the superior stiffness of the thick plates; but all this is more than gained with their top plates disposed as in Experiment 29, Table III., or in a tubular form; thus Mr. Hodgkinson recommended that suspension chains be used as an auxiliary in supporting the tube, because otherwise great thickness of metal would be required to produce sufficient stiffness and strength; nevertheless, the tubes are now up, withstanding heavy gales and the pressure of heavy loads, without suspension chains, and the thickness of the plates little more than that of the small tubes used by Mr. Hodgkinson in his experiments!

(86.) The investigation had now elicited facts and principles allowing of a serious consideration respecting the probable ultimate form and other conditions of the tubes which should span the river Conway and Menai Straits. From a contemplation of the experiments on rectangular tubes, Mr. Fairbairn proceeded to discuss the proportions of a tube of 400 feet span; he compared the top and bottom areas of the tubes in the following cases:—

Experiment.	Area of top in inches.	Area of bottom in inches.	Ratio.
15	0·142	0·075	1·893 : 1
16	0·269	0·149	1·805 : 1
29	0·230	0·180	1·278 : 1
Mean . .	0·213	0·135	1·659 : 1

or very nearly as 10 to 6; in Nos. 15 and 16, the top area is much greater in proportion to the bottom than in the last experiment, but the latter proved to be nearest to the proper ratio. Thus, reckoning that the top of the 400 feet tube would present an area of 655 square inches in section, the bottom would be $\frac{665 \times 64}{100} = 426{\cdot}6$ inches in area; by the arrangement of the plates it was found to be a little more—441·5 inches, and with this preliminary step, the probable strength of the great tube could be calculated from the known rule for obtaining the strength of a beam of any known length and depth (in inches), with a given area of the bottom side. The breaking weight is then found thus:—

$$\text{Breaking weight} = \frac{\text{area} \times \text{depth} \times \text{S}}{\text{length}};$$

the quantity S representing the specific strength of any particular form of beam, or disposal of material, which of course must be found by experiment. This was conveniently afforded by Experiment 29, where it appears the bottom area was nearly 2 inches, the depth 15¼ inches, the length 19 × 12 = 228 inches, and the breaking weight was 22,469 lbs. = 10 tons, and a very small fraction over; then S will be equal to the length and breaking weight multiplied together, divided by the product of the area and depth, or

$$S = \frac{228 \times 10}{2 \times 15\frac{1}{3}} = 74.$$

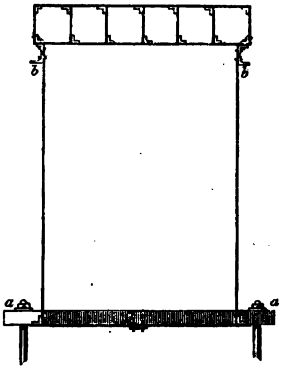
We can now find the breaking weight for the tube, which will be =

$$\frac{441{\cdot}5 \times 312 \times 74}{4800} = 2123 \text{ tons},$$

the load which would be required, according to this calculation, to break a tube of 400 feet span, and 26 feet (= 312 inches) deep, the width being taken at 15 feet. The admirable strength thus predicted from experiment was verified in the first Conway tube of 400 feet span.

(87.) Although the results we have detailed were calculated to give thorough confidence as to the safety of the new bridge, a little more investigation was deemed requisite to determine the proportions of the parts, on a larger scale; a tube, one-sixth of the dimensions purposed for the Britannia Bridge, was therefore constructed, and a few valuable and most conclusive trials of its strength were made. This model tube was 75 feet long between the supports, 4½ feet deep in the middle, and 2 feet 11 inches wide, and of the following figure in section; namely, six cells or tubes in the top, and a plate as before for the bottom; the partition pieces dividing the top into cells were firmly attached to the upper and lower horizontal plates by angle-irons seen in the corners; the sides were strengthened by a line of angle-iron *b*, which was riveted to them in a curved shape, touching the under side of the cells at the middle, but descending to the bottom at each end. The cross-bar *a a*, laying on the bottom plate, and projecting beyond the sides, carried a scale for weights by the vertical rods. It will be interesting to notice the results of each experiment.

Fig. 95.



No. of Experiment.	Thickness of plates.		Area of section.		Ultimate deflexion.	Breaking weight.	Remarks.
	Top. in.	Bottom. in.	Top. in.	Bottom. in.	in.	lbs.	
33	·147	·180	24·024	8·80	4·50	79,578	Torn asunder.
34	·147	·156	24·024	12·80	4·40	97,102	One end twisted.
35	...	...	24·024	12·80	5·79	126,128	Torn asunder.
36	...	...	24·024	17·80	4·94	148,129	Torn asunder.
40	...	...	24·024	22·45	3·86	154,452	Torn asunder.
41	...	...	24·024	22·45	4·89	192,892	Top doubled up.

The bottom side had an area at first of nearly one-third that of the top, which is not according to the ratio of 10 to 6 given in the last article, in order that the bottom might be successively altered until the top was crushed, which would of course show the limit, as it does in the last experiment. The deflexions increased but a small amount when the weights were left suspended for any length of time, which, with the fact that some time generally elapsed, when the last weight was added, before rupture occurred, is a great recommendation as regards safety. The failing of the tube in the 34th trial was not owing to insufficient strength in the bottom or top, but want of stiffness in the sides, a casualty afterwards well provided against; after a × had been placed in at each end, the great superiority of strength from the increase of the bottom area from 8·8 to 12·8 inches, shows that the proper proportion was being rapidly attained. In the next, Experiment 36, the thickness of the bottom plate was increased to about double that in the first trial, without, however, realizing quite double the strength; but it must be recollected that the tube had undergone severe strains in the breaking of the former bottom plates. The area of the bottom side being now augmented to nearly 22½ inches, two experiments were made, one in which the elastic power of the tube was observed, and another to obtain information as to the effect of a strong wind upon the tube; for the latter purpose it was laid on one side, and the weights suspended from the upper side; it was deflected in the middle 0·85 of an inch by its own weight (5 tons 16¼ cwt.), and only 2·36 inches more on loading it with nearly 12 tons, or ·85 + 2·36 = 3·21 inches altogether; but the elastic power of the tube was so unimpaired, that on setting it up in its proper position, it returned to within *one-tenth of an inch* of its former figure*. When the tube was restored to its position, a weight was allowed to remain for several days, constituting Experiment 39; afterwards additional weights were suspended until it again broke by tension, the greater bottom area appearing to effect little improvement; but in reality the area at the part which was torn was only 8·8 inches, the place of rupture being as before near (21·6 inches) to the middle where the weights were hung; after repairs, Experiment 41 was made, when the great weight of 86¼ tons was heaped on this model before it yielded, which was by the tubular top puckering at 2 feet from the middle. The desired result was now attained—the proportion of area was found—and the labour of experiment ended.

These investigations with the model tube are extremely instructive and decisive; they point out the necessary conditions to be observed in building the great tubes, the sources of failing, and how to guard against them. Not a little important is the elastic power exhibited during the experiments, when loads, near to the breaking weight, had been laid on the platform, left on for some time, and effected a considerable deflection. The three following instances show a very great preservation of elasticity:—

Experiment.	Weight in tons.	Time left on.	Final deflection. in.	Loss of elasticity. in.
33	30½	All night.	3·425	·806
37	58½	18 hours.	3·350	·430
38	12	Some hours.	2·500	about ·100

In the second instance the weight of 58½ tons was gradually removed, while the tube recovered itself within less than half an inch; in Experiment 38 the tube was on its side. So small a defection in the resilient power of this tube, when pressed by such great weights, said much for the durability of the intended bridge. Connected with this feature of the malleable iron tube is that of its admirable behaviour under a long-continued strain. It has been considered that a beam should be loaded only with about one-third of the breaking load, to be safe, the elastic limit being supposed to extend thus far; more accurate practical information, however, has shown that the idea of an elastic limit is an assumption not borne out by experiment, as any weight affects the elasticity to some extent; the following examples prove the greater enduring powers of the material:—

Experiment.	Weight in tons.	Parts of breaking weight.	Time left on. h. m.	Increase of deflection. in.
33	25¼	7/10	1 0	·15
34	20¼	½ nearly.	16 0	none.
36	45	⅔	21 0	none.
39	60⅓	17/20	9 days.	·05

Thus the same tube, after being subjected to the previous trials—pulled, pushed, torn, crushed, mended, and remended, bore in Experiment 39 nearly 9-10ths of the breaking weight for nine days and nights, with an insignificant alteration of 1-20th of an inch. To such tests as these the real bridge could never be subjected in ordinary practice.

(88.) After collecting all these data, the question was what would be the probable strength of the great tubes themselves; taking the method of calculation given at page 190, with the specific strength (74) exhibited by the corrugated tube, it was found that the model tube answered very well to the predicted strength; for, trying with the 34th Experiment, we have the breaking load, according to calculation,

$$\frac{74 \times 12{\cdot}8 \times 54}{900} = 54{\cdot}83 \text{ tons};$$

in the actual experiment 56·3 tons were laid on before rupture, or rather more than the anticipated strength. Mr. Hodgkinson, considering 12 tons per square inch to be the strength of the tube, concluded its strength would be 1485 tons, which, deducting one-third for joints and rivets, reduced it to 990 tons; then subtracting the weight of the tube—450 tons, 540 tons would remain for work, or double that amount spread over the length of the tube (as a train load would be), finally leaving 640 tons to resist any strains, blows, and vibration. Safety would be equivocal with such nice proportions of strength and load; but, fortunately, Mr. H. had taken a low estimate of the strength,—14 tons being admissible. Again, the superior riveting adopted by Mr. Fairbairn required an allowance of about one-seventh instead of one-third the strength. These improved views of the tube's good qualities brought its actual strength up to 1009 tons in the middle, instead of 540, or above four times stronger than requisite for supporting the greatest probable load. It became also a consideration whether the top should be a double or single row of square tubes, or cylindrical tubes, for the investigations of Mr. Hodgkinson proved that while cylindrical tubes showed a strength of 18 tons to the square inch, rectangular only attained from 12 to 14 tons; the requirements of practice, however, soon decided in favour of the rectangular form, which, indeed, was much stronger than the preceding statement supposes, owing to the addition of angle-iron at each corner, as shown in the following sections. The parts of the first Conway tube were now proposed, its length being 424 feet, depth at middle 26 feet, and width 15 feet; the row of cells on the top 1 foot 8¼ inches square, and eight in number, the plates being ¾ of an inch thick; the bottom side to be a row of six cells, with the floor-plates of double thickness. The areas were thus to be—

---

* Of the application of this experiment to determining the effect of a strong wind, we shall have opportunity to remark in the description of the bridge.

Of the top . 565 square inches.
Of the bottom 500 „
Of the sides . 259 square inches;

giving a somewhat smaller proportion for the bottom side than that of the model tube in Experiment 41, which is nearly 24 to 22½: this would make the bottom area 529 instead of 500 square inches.

(89.) The construction of the tubes now rapidly proceeded on the banks of the river Conway and the Carnarvon shore of the Menai strait; we shall not, however, enter into the details of the Conway tubes, since they are similar in all respects to the Britannia tubes, but not quite so large, neither are the circumstances of their erection equally remarkable with the latter: in describing the Britannia tubes, both will be explained. Before so doing, however, one or two remarks are necessary on some observations eagerly made, when the first great tube was completed, and lay on its supports ready for floating to its destined position. They relate to the deflection under proving loads, indicating the probable breaking weight; the effect of the wind impinging on the side of the tube; and the consequence of expansion through increase of temperature.

Mr. Fairbairn thus gives the trial of strength on the great tube:—

" EXPERIMENT XLII.

" Rectangular tube 412 feet long, 25 feet 6 inches deep in the middle, 15 feet wide, and 400 feet between the supports.

Area of top . . 670 inches.
Area of bottom . 517 „

Computed weight of tube, including rails and cast-iron frames at the ends, 1300 tons.

No. of Experiment.	Weight. tons.	Deflection. in.	Remarks.
1	0	7·91	The weight of the tube gave a deflection of nearly 8 inches: 95 tons left in the inside for 4 hours increased it from 9·02 to 9·25 inches, or ·23 of an inch. This weight left on for 17 hours further increased it 0·10 of an inch."
2	95	9·02	
3	154	9·50	
4	201	10·50	
5	301	10·95	

It must be borne in mind that the weights were not all at the middle; the first (95 tons) was distributed over a surface of 70 feet at the middle; the second over 105 feet; the third over 150 feet; and the last over 190 feet. Taking these deflections, and comparing them with the ultimate deflection of the model tube in Experiment 41, it appears (as the ultimate deflections are as the lengths of the tubes) that this large tube would bend 29¼ inches before breaking, and to effect this a weight of 2200 tons would be requisite, which consequently represents the strength of this tube somewhat above the highest amount presupposed.

Effect of Wind.—When the tube was completed, and a short time before its removal to the piers, a severe gale from the N.W. blew against the side at an inclination of about 50°, creating an oscillating motion, with a lateral deflection of nearly a quarter of an inch; when the wind came in gusts, which, if their recurrence is regular, like the beats of a pendulum, are most trying on a body free to vibrate, the side effect did not increase above a quarter of an inch. Although the force of the wind is not stated, a great lateral stiffness was exhibited by this observation. We cannot, however, reason from this single occurrence as to the consequences of the storms which are likely to wreak their fury on the Britannia tubes, exposed as they are 100 feet above the water, and spanning a hollow along which the tempest rushes violently.

Effect of the Sun's Rays.—An observation was made, when the tube rested in its place, to discover the alterations produced by heat. On a clear frosty morning in the spring of 1848, gauges were placed in order to detect any expansion; and at noon, a shining sun, warming the top and one side, bent the tube on that side nearly one inch; also it appears that the tube was lifted upwards in the middle to the extent of 0·7 of an inch, and an elongation of three-fourths of an inch also occurred. What change of temperature produced these alterations, we are not informed, although it is singular that an experiment connected with temperature should be made and the thermometer forgotten!

(90.) The reader has now before him an outline of the whole proceeding by which a mere vague notion assumed a scientifically elegant and thoroughly practical climax, in the working drawings and building of the permanent tubes; he may detect the *gradus ad Parnassum* successively, through experiment directed by thought and thought directed by experiment. It was a sure mode of eliciting a trustworthy result—trying anything, that the way might be shown to something; or, in other words, it was, to some extent, the method of exhaustion. Although great mechanical labour had now to commence, the greater labour was over;—the only exercise of mind henceforth required was care in every successive operation. Some time previous to the wished-for conclusion, many passing thoughts were expressed as to the manner in which the monster tubes were to be transferred from their building platforms to the proper bearings on the piers. Mr. Stephenson proposed a chain bridge, with a railway along it, on which the tube might be drawn across. To this, two objections, amongst others, are serious:—the danger of any oscillating motion while the tube might be moving along the suspended platform,—and the great cost of such a work, considering it would be a mere centre on which to arrange the bridge. This was abandoned, and, after some consideration, it was arranged thus:—1. Each tube was to be constructed on the banks of the Conway and the Menai Straits, so close to the water, that, by a little excavation, boats could be floated under it. 2. A number of pontoons were to be constructed, which should be placed under the tube while the tide was low, so that in rising again it would lift up the pontoons and with them the tube; thus it could be floated to the piers. 3. The tube was thence to be lifted up into its place by hydraulic pumps. It will be convenient, therefore, to explain each undertaking in this order.

1. *The Construction of the Tubes.*—When the plan of operation had been settled, preparations commensurate with the work to be done were busily made on the Carnarvon shore of the strait: a spot as comfortable as the abrupt shore would allow, about 400 yards from the site of the bridge, was selected for the workshops and platforms, while near at hand a numerous colony of workmen and their families quickly appeared, composed of about 800 men for the massive masonry, and 700 for the iron work. The platform extended half a mile along the shore, accompanied by three workshops, filled with plate-cutting, punching, and other machines useful in this particular work, while six neat little steam engines supplied the motive power; five wharves were also arranged for unloading vessels when they arrived with iron plates, &c., from different manufactories.

The principal iron work was of the following dimensions:—

PLATES.

For the top, 6 feet long by 1 foot 9 inches wide, $\frac{3}{4}$ of an inch thick at the middle, and decreasing to $\frac{5}{8}$ of an inch at the ends of the tube.

For the bottom, 12 feet long, by 2 feet 4 inches broad; from $\frac{9}{16}$ to $\frac{7}{16}$ of an inch in thickness.

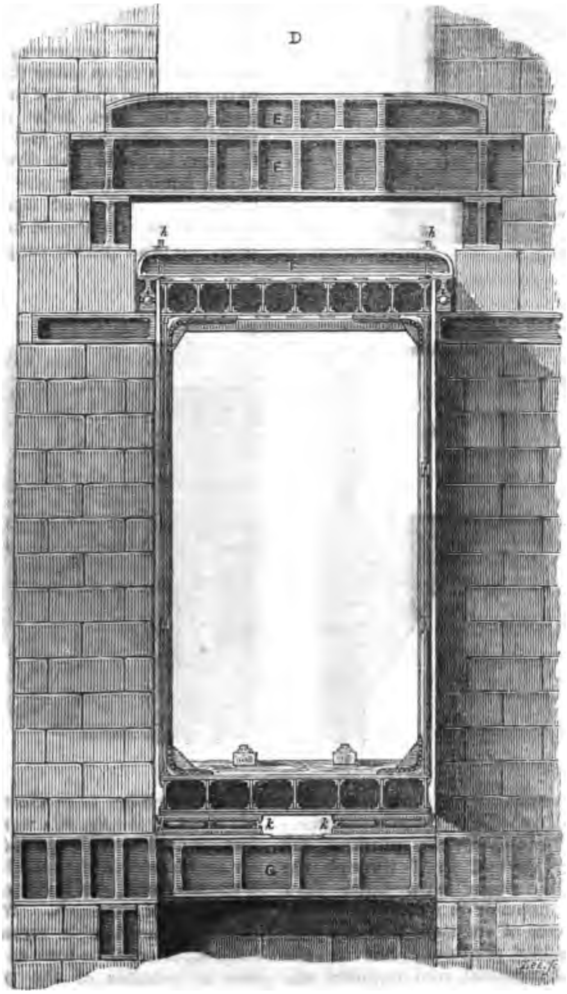
The largest plates weighed 7 cwt.

The rivets were 1 inch and $1\frac{1}{8}$ inch in diameter.

The above are the general dimensions of iron work for the tubes of the Britannia Bridge, which were to be 460 feet in span, 30 feet high at middle, and 14 feet 8 inches wide. A short detail of its admirable construction will receive illustration from the accompanying diagrams, in which the same letters denote corresponding parts.

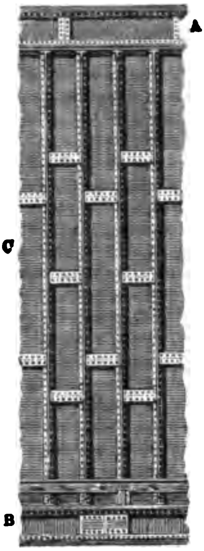
The top A (*fig.* 98) is framed in eight cells, or rather tubes, each of them 1 foot 9 inches square inside, or sufficiently large for a man conveniently to crawl through for riveting or repainting; the upper platform *a* of plates is, externally, very plain, having the longitudinal covering piece *b* over the joints, and transverse pieces, as at the side *c*, connecting the ends of each plate, which, since they have to sustain a crushing pressure, are carefully fitted edge to edge; thus the plates are riveted tightly together into

Fig. 96.



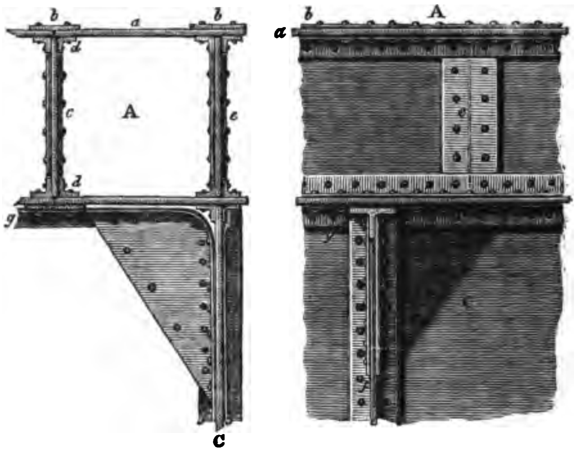
MIDDLE SECTION.

Fig. 97.



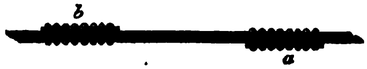
one long sheet; of equal thickness are the division or vertical plates *c*, forming the sides of each little tube; they are firmly joined to the horizontal plates by angle-iron *d*, with rivets an inch in diameter and three inches apart. The riveting was an operation of much importance, since the great source of weakness in an iron-plate structure is the joints. Both machinery and heavy hammers weighing 7 lbs. and more, were used, which effected good riveting. The hand riveting was necessary in operations on the tubes in some cases; thus, when the top plates had been laid, the holder-up, who pushes a heavy hammer against the head of a bolt while the riveters are hammering on the other side, had to creep along the small tube, to the required spot, and putting through one of the rivet holes a hook from which a noose hung, and using it as a fulcrum, he could push down the handle of his hammer under one leg, and thus press the heavy head against the end of a bolt when it should be placed in its hole. It was quickly supplied, red-hot, by the attendant rivet boys, through a small hole, of which there were many at different places for this purpose, and as quickly showed itself through the proper hole, when blow upon blow fixed it immoveably in its place.

Fig. 98.



The *bottom* of the tube had also cells and a double platform; but as the quantity of sectional area required was less than that for the upper part, only six cells or tubes were allowed. Also the plates as at B (*fig.* 97) were longer—12 feet, or double the length of the top side; for as this part of the tube had to sustain a tearing force, the fewer the joints the less loss of strength. Mr. Fairbairn devoted considerable attention to this subject, in order to lose as small a portion as possible of the power of his plates; the result was a system which he calls chain riveting, because the rivets stand in a longitudinal line similar to the bolts in a chain bridge. The plates were double for the lowest platform, so disposed that the joints of one series happened at the middle of the other series, thus:—two of the lower

Fig. 99.



rows meet at *a*, and two of the upper at *b* (*fig.* 98), in each case opposite the middle of the others' length; a covering-plate *a* and *b* (*fig.* 99) was placed over the line of concurrence, and a series of four rivets 1⅛ inch in diameter, in a line, bound each of the ends and rows together. With these precautions the joints are considered to be nearly, if not quite as strong as the solid plate.

The *sides* of the tube, although disregarded in estimating the strength, yet claim a great portion of the credit given to the structure for its supporting power: they must be stiff, tenacious, and light, not apt to bend out or in, but to maintain the upper and lower tubular parts separate at the required distance, without which they would be of no effect. The vertical rows of plates C (*fig.* 97), 2 feet wide, are alternately of three and four plates deep. At each vertical joint a rib or little pillar *f* of T iron is riveted, tending greatly to keep the sides upright; these T irons bend round at the top and bottom *g*, giving a further fastening of the sides to both top and bottom plates; the bends of these irons are again assisted by corner plates or gussets. Unlike the bottom and top, the side plates increase in thickness towards each extremity; at the middle they are one-half, but near each end they attain five-eighths of an inch. It was important that care should be taken in stiffening the tube at the ends, which must transfer all the pressure to the piers, for the experiments frequently indicated a tendency to change of shape at these parts; about ten feet from the piers, therefore, strong pillars of a T form replaced the small T iron, making a very firm frame at those parts.

Altogether a tube, when complete and ready for launching, contained about 1500 tons of iron, and was held together by about 327,000 rivets. It was painted of a light colour, and left for some time to settle, supported only on its ends.

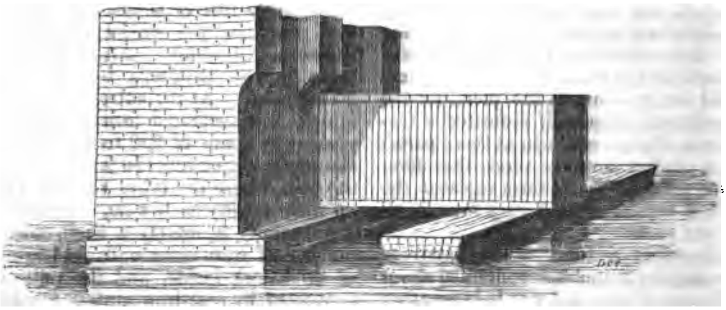
While preparations were made in the iron work, a mighty construction of masonry was rapidly proceeding, to be in readiness for receiving the tubes. This masonry consisted of one grand tower in the middle of the Straits, called the Britannia, from the rock on which it is founded; this subaqueous rock projects upwards to a height which permits the curling stream to hide it for a depth of 10 feet at high water, but as the tide rises and falls about 20 feet, there are 10 feet of its top visible at low water; this circumstance, with its admirable equal division of the strait, made it a desirable foundation for the pier. A base of 62 feet by 52 feet 5 inches was given for this gigantic structure, and successive layers of massive blocks were added, until the tower rose 230 feet above the level of its foundations; the faces or sides are slightly inclined, which reduces the dimensions to 55 feet by 45 feet 5 inches at a height of 102 feet above high water;—at this height the tubes stretch from pier to pier, passing through openings left in the masonry. The other towers, which stand at the edge of the water, on the Carnarvon and Anglesea shores, at a distance from the Britannia of 460 feet, are similarly constructed, except that their elevation is 10 feet less than that tower. Taking the height, breadth, and thickness of the middle tower, it amounts, in cubic contents, to 575,000 feet; but so much of it is left hollow as to reduce the actual contents to little more than one-half this amount. There are 148,625 feet of limestone, quarried at Penmaen—the northern extremity of the Snowdonian range; it is a fine hard material, full of organic remains, and forms all the exterior work of the towers; within are 144,625 feet of sandstone, brought from the new red sandstone beds at Runcorn, Cheshire. Altogether, the stone in the Britannia tower weighs 20,000 tons. At the extremities of the bridge, and separated from the shore-towers, by a gap of 230 feet, stand the abutments, extending 176 feet in length; the ends where the tubes rest are built up with solidity as towers, but beyond the masonry is merely a shell, with a number of transverse walls and arches of brickwork inside, and the rails are supported by cast-iron girders reaching from wall to wall.

Thus four great and deep openings had to be spanned by the preparing tubes; two between the abutments and land towers of 230 feet each, and two over the water, between the land towers and the Britannia rock tower, each 460 feet.

Floating the Tubes.—Much discretion and forethought was required about this novel operation. It was finally agreed to make use of the rising tide, as a lifting and locomotive power, guiding the great mass as well as possible by a series of ropes worked by capstans on shore. The pontoons or barges were about 100 feet long, 25 feet wide at the top, and 10 feet deep, made of iron plate, and internally strengthened by ribs and braces; they also had valves at the bottom, for an important purpose. Each pontoon had a tonnage of 400 tons, and as eight were used,—four at each end,—3200 tons might have been floated on them: the load they had to carry was, however, about 1500 tons, which each large tube weighed. The floating of the first Britannia tube was an occasion of great interest and excitement; a bridge of such unheard-of character had attracted many, and the intimation of a "something" to be done—very extraordinary—brought crowds of sight-seers to the banks of the Straits. It was late in the day when this singular operation was performed; at seven o'clock in the evening of the 27th of June, 1849, all attention was directed to one tube laying quietly at the edge of the water, under which eight pontoons had been floated. Mr. Stephenson, Captain Claxton, and others, ascended the top of the tube, awaiting the rising of the tide to lift the tube off its bearings. In order to guide the mass into its place, two hawsers had been attached to the Anglesea and Britannia towers, between which the tube was to lie, while the other ends were fixed to the shore; these hawsers, in passing over the pontoons, ran through moveable sockets, so that, by merely working a screw, the ropes could be held fast, and the moving tube arrested at any moment. All being ready, at about half-past seven evident signs of motion were perceived; directly afterwards the ponderous tube was free, and moved outwards: after several incidents of passing interest, and various workings of the capstans to loosen or haul in the ropes, according as occasion required, to bring the floating mass into previously-agreed "positions," the tube approached the foot of the towers. One or two movements, and the Britannia pier received one end; soon the Anglesea pier bore the other,—the tide turned,—the pontoons were sunk by opening the valves, and the work was done. The hitherto silent, wondering, waiting multitude, now joined with the less musical sound of cannon, in acknowledging the success of the day's undertaking. Of the nicety required in the last motions, an idea may be formed from the fact that there were not two inches to spare between the ends of the tube and the towers when the former was in its place.

The following sketch will illustrate the appearance of the tube at the foot of the Anglesea tower. The sides of each recess were left unfinished, to permit the entrance of the tube; they afterwards served to guide the tube in its ascent, as well as to bear the cast-iron girders on which the tube should rest when the proper height was attained.

Fig. 100.



FLOATING

The difficulties of carriage were not experienced in the land or end tubes; they were at once built on scaffolding in their permanent places,—no obstruction to the navigation resulting from this arrangement; the great scaffolds rested on piles, and were composed of upright with diagonal bracing timbers, about 100 feet at the deepest parts; they were made of strength adequate to bear two tubes, each about 1300 tons weight, in addition to workmen, instruments, &c. A series of ladders opened communication between the bottom and top of the scaffolds.

Lifting the Tubes.—In comparison with the raising of this tube, which laid at the base of the towers, its floatation was simplicity itself; the care called for in keeping 1400 tons in motion from smashing itself or something else by a collision, and to bring it to an inch where it was desired, was a great achievement; but to lift this massive framework—an unwieldy length of 472 feet—over deep water and a rapid current, without scaffolding, to a height of 100 feet, was, until one expedient had been proved sufficient, an appalling task; that expedient was the well-known hydraulic press, an instrument by which power is obtained from velocity, to an unlimited extent. A description of this beautiful machine is given in the treatise on *Hydrostatics*, p. 9. The pumps used on this occasion were made of extraordinary strength, the cylinder or box of the pump in the Anglesea pier being 9 feet deep, 4 feet 10 inches in exterior diameter, and the cast iron of which it was made was 11 inches thick—making 16 tons of metal; altogether the instrument weighed about 40 tons. In the Britannia tower there were two cylinders, which had been used in lifting the tubes at Conway, the piston or ram being 18$\frac{3}{8}$ inches in diameter, in a cylinder of 20 inches interior diameter, leaving the small but quite sufficient space of nearly 7-8ths of an inch between the ram and cylinder for the water, injected by small pumps 1$\frac{1}{16}$ inch in diameter, through little carrying tubes half an inch bore and three-quarters of an inch thick. According to the hydrostatic principle governing this instrument, the power used at the pump handle is to that given out by the ram or piston as the ratio of the area of the plunger or pump-rod to the area of the piston. This has been stated to be 1 : 355; accordingly, a man pressing with 30 lbs. weight on the plunger would lift nearly 10,650 lbs., or 4$\frac{3}{4}$ tons, laid on the ram. In this case manual exertion was not used; steam-engines of 40-horse power were worked horizontally, and opposite the pumps, by which arrangement the piston-rods of the engines, entering through the stuffing-boxes of these pumps, became the plungers at once. All this apparatus was quietly located in an open space D (*fig.* 96) in the tower, immediately above the opening to which the tube should be lifted. To support it —for it must needs stand out from the masonry so far as to be immediately over the end of the tube, 140 feet below —a pair of powerful girders E E, one on the other, for double security, stretched across the recess before-mentioned, on each side of the cast-iron cylinder; consequently the cross-head or beam attached to the upper end of the ram was able to lift up anything between the girders: these were the means of raising. But the next consideration is the connection between the cross-head and tube below; the latter, that no possibility of injury might exist, was strengthened greatly at each end by a cast-iron framework, two vertical and two horizontal iron beams fitting round the interior; three recesses were left in the vertical beams, one near the top, another at the middle, and another near the bottom of the frame, for the ends of three powerful cross beams, able to sustain above 3000 tons; through them passed the ends of the chains, which extended up to the hydraulic apparatus in the tower, by a series of links six feet long, and as shoulders were given to the heads or eyes of these links, where they passed through the beams, they laid hold of the under side of the upper and lower beams. A succession of links, bolted together as in a suspension bridge, reached the two ends of the cross-head on the hydraulic piston, and rested there by means of the notches or shoulders made on the large ends of the links; pumping commenced, and the tube was slowly carried up between the sides of the recess. At Conway, after the pumping had continued some time, fears were excited in consequence of an increasing vibration which ran along the whole tube, caused, it appears, by the regular and simultaneous action of the pumps, forcing the water into each cylinder, at equal intervals, and raising the tube by jerks. Before the pumps were stopped this motion had produced a deflection, or rather oscillation, of nearly an inch and a half in the middle, or equivalent to about 120 tons load! By an alteration, in which alternate, instead of simultaneous, movements of the uplifting rams were applied, this destructive pulsation vanished. Another accident, attended with no great injury to the structure, happened with the Britannia tube, after a few links had been drawn up: the larger hydraulic cylinder in the Anglesea tower suddenly burst, a casualty common with cast iron, of course letting the water out, and the piston fall. Had it not been for a provision wisely made by Mr. Stephenson against such an occurrence, the tube would have fallen two feet and a half, if not more, a depth apparently not considerable; but with the length and enormous weight of this tube, Mr. Stephenson calculated that it would prove its complete destruction: as it was, although it fell but *one inch*, so tremendous was the effect that iron beams capable of sustaining 500 tons quietly, were broken down. The provision made was a packing of wood planks under the tube as it rose, not allowing at any time a fall of more than an inch: subsequently still greater care was adopted. The part which gave way was at the bottom of the hydraulic cylinder, where there was a thickness of eleven inches of metal; a piece weighing a ton and a half separated, and fell a depth of 70 or 80 feet on to the tube, making a severe indentation, and, in its fall, killing an unfortunate sailor, who happened to be at the time ascending a rope ladder between the tube and press. When the tube at last reached the desired height three strong cast-iron beams G (see *fig.* 96), previously drawn through the outer side of the recess, were driven in underneath it, and the tube was now "home."

As the piston of each hydraulic press was six feet long, the tube could be raised only to this extent at every lift; and while the piston was let down to be ready for another lift, support had to be given to the tube: this was admirably effected with the notches on the heads of each series of links; two cheeks were placed on each side of the chain, and moved towards or from one another by screws. When, therefore, the ram had reached its extreme height, it had brought the notched heads of a row of links just above the edges of these cheeks, which, being immediately screwed together, caught the heads and held them fast until the ram should descend to take the next depth of chain.

(91.) After similar proceedings with another long tube between the Britannia and Carnarvon-shore towers, the works rapidly proceeded towards a completion of this one line of tube. An important operation was that of fastening the four tubes at each tower, so as to constitute one tube, 1523 feet long. The extreme ends of the smaller tubes were lifted up somewhat above their intended level, so that when the large tubes were in their places, and the joining with the smaller properly effected, the latter being let down, tended to stretch, by their weight, the upper part of the long tubes, restoring them, to a considerable extent, from deflection consequent on their own weight.

Expansion through increase of temperature was carefully allowed for in the arrangements. At the Britannia tower the tube laid simply on the masonry and girders; but from this middle point, each way, convenience for expansion and contraction was afforded by rollers *k* (*fig.* 96), on which the under surface of the tube at the piers and abutments rested, while across the top several cast-iron beams F extended, being fastened at the sides of the tube by large screw-bolts *h*; the ends of these beams carried a little trough *i*, extending in a direction parallel with the tube; a similar trough is fixed to girders built in the tower, and between the two are a number of gun-metal balls;—allowing the greatest freedom of motion to the tube, yet bearing a portion—according to the screwing up of the screw-bolts—of its weight. It is calculated that the probable range of expansion will be six inches in each half of the tube.

The under side of the tube is considered to be a level line, except when loads depress it; for in building a rise or "camber" is given to the under side of about 9 or 10 inches, which being nearly equal to the amount of deflection from its own weight, will, if the tube is left free, give an horizontal line; in one tube, however, which had been left on its temporary bearings about a fortnight (ready for floating), we measured a deflection of 12¾ inches, or 2¾ inches beyond the quantity allowed for by the camber. The upper surface is a curve, giving greatest height in the middle or at the Britannia tower, where the tube measures 30 feet; from this point it decreases until at each extremity it becomes 23 feet 9 inches; from these amounts two feet at top and also at bottom are to be subtracted on account of the cells, leaving 18¾ feet at each end, and 26 feet in the middle, more than sufficient for a tall locomotive; in width, 13 feet 5 inches clear are available between the side ribs. On the Britannia tower the tubes have a bearing of 45 feet; on each shore pier 32 feet; and 17½ feet of their ends rest on each abutment. The approaches to the tube by the line are striking; until very near to them nothing is seen, in consequence of the rapid curves taken by the rails; when the turn is made two enormous lions first appear, lying on the ends of two parapet walls, one on each side of the line, resting on the abutments: these gigantic but quiet creatures are 25 feet long, 12 feet high, and 20 tons in weight each, formed out of eleven blocks of limestone. Dignified they must appear, if for no other reason than their size; they are, however, in the Egyptian style—not very remarkable for anatomical accuracy. At a considerable distance farther on, the abutment tower, covering the end of each tube, stretches across the way, presenting two large holes—the entrances to the tubes; on a nearer approach the cellular top is visible, and then a long dark tunnel with a central opening into light at the further end;—sometimes little rays of sunlight peep through a series of ventilating holes, two or three inches in diameter, which served the purpose of passing red-hot rivets in and out during the construction. Small windows have since been added.

(92.) The following is a summary of the principal dimensions of the Britannia and Conway Bridges, principally from Mr. Fairbairn's work on the subject:—

	BRITANNIA.		CONWAY.	
	ft.	in.	ft.	in.
Total length of each tube	1523	0	424	0
Greatest span in the clear	460	0	400	0
Height of tubes:—At the middle	30	0	25	6
At intermediate piers . . .	27	0	——	
At ends	23	9	22	6
Above high water	102	0	18	0
Extreme width of tubes	14	8	14	8
Number of rivets in one length of tube	882,000		240,000	
,, ,, whole bridge	1,764,000		480,000	

An idea may be formed of the quantity of iron used in the bridges from the subsequent calculated figures:—

BRITANNIA BRIDGE.

	Tons.
1 tube, 274 feet long . .	689
3 tubes ,, ,, . . .	2,067
1 tube, 472 ,, . . .	1,400
3 tubes ,, ,, . . .	4,200
2 tubes, 32 feet long, for joining on middle pier . .	204
Cast-iron frames and beams	2,000
Total weight of Britannia tubes	10,570

CONWAY BRIDGE.

1 tube 424 feet long . . .	1,146
1 tube ,, , . . .	1,146
Cast-iron frames and beams	600
	2,892

Altogether about 13,500 tons are calculated to have been used in the two tubular bridges, incurring an expense stated by Mr. Fairbairn to be about 500,000*l.*

(93.) The Britannia and Conway Bridges are certainly great novelties; that they are reducible to well-known principles, is true; but if all inventions and discoveries be treated with similar logic, there proves to be no novelty under the sun. No previous work, no precedent in architecture, gave birth to these tubular bridges, unless, indeed, through those habits of mental movements by which a real object becomes invested with fleeting shadows, until, like a dissolving view, the former is lost in the latter picture: that such was not the case with the rectangular cellular tube, ample evidence is afforded in the experimental investigation by which it was brought to light. As a new thing, questions naturally arise with reference to its endurance, and the effects of heavy trains and tempestuous winds. Any argument on such inquiries must be to some extent speculative; but the principles by which the structure is governed, the character of the material, as far as understood, and relevant comparisons with former experience, supply, in this instance, very ample evidence for judgment.

The tubes are decidedly not tried to their utmost, consistent with safety; as they are, no possible load can at all reach two-thirds of their strength; it has been stated that the pressure may amount to six tons on every square inch of section—a quantity not half the proved strength; but this must arise from a very heavy load. The deflections indicate that, with ordinary trains, a small comparative strain is exercised on the plate-work;—the Conway tubes are said to bend but one-eighth of an inch—an amount absolutely insignificant—when the usual trains are passing through them: so able are they for support, that a tube such as the Britannia, to break with its own weight, must be extended to the enormous span of nearly 2000 feet, a length far greater than the whole length of the existing bridge, which also has three intermediate supports. Indeed, were the three towers removed, and the tube left with support from the abutments alone, which are 1489 feet apart, there would be no danger of its falling. The specific qualities of the material, giving elasticity, yet stiffness, suffering many injuries and sudden shocks without destruction, and when that is impending, affording such timely notice that preventives may be applied; its exemption from decay, if properly coated, as manifested in wood and stone, all testify in favour of its endurance against the adverse influences of wear and tear from expected agents.

High winds were certainly not forgotten in precautionary arrangements; although the experiment made to test the tube sideways, or as the wind would act with greatest effect, was certainly irrelevant to some extent; a dead weight gently hung from the tube could not be compared with the action of winds; unless, out of consideration for the new bridges, they were to blow quite regular all the time, and quietly subside. Seldom, however, are the atmospheric currents so orderly, and less so at the Menai Straits; fearful storms have been known there of a character calculated to try the tubes in the most distressing manner; to which fact the shattered condition of the suspension bridge one morning, after a single night's contest with a raging storm, bore undeniable evidence. This peculiar action appears to be a pulsation of wind at regular intervals, which, should they happen just equal to the time occupied by an exposed chain or free body in vibrating, will have an effect similar to that of a person pushing a child's swing—every successive oscillation will increase, until the position of the moving body or some injury prevent farther motion. These dead pressures, well borne, can argue no safety under a trial of this description, any more than the fact of a body resisting a considerable weight can point out its ability to cope with a small weight rapidly impinging on it. In an early calculation made to estimate the probable power of a tube, taking its weight at 1000 tons, to overcome any aërial effort, the results of Experiment 38, mentioned at page 92, are brought forward for data, which returns an admirable-looking result; the pressure of the wind being supposed at 50 lbs. per square foot, we find 280 tons to be the pressure against the tube, while there are 1000 tons as antagonistic force or power in the tube, half its weight being included in this quantity. But what would a dead weight of a thousand tons avail against two hundred and eighty or even half that amount of tons, of appulsing air, acting with a series of successive intermissions? Experience predicates its inefficiency. If great Snowdon could be hung up, it would oscillate like a clock pendulum, with the next gale. It must be the stiffness of a structure, preventing incipient vibration, on which all hopes of safety can depend. That the Conway tube was little disturbed by a severe gale, on one occasion, resulted wholly from its stiffness—not weight; the same may be predicted of the Britannia tubes; so admirably are the sides stiffened, and particularly near the piers, as to constitute an almost rigid framework. No doubts need therefore be entertained of endurance under the adverse energy of pulsatory winds.

This triumph of science must be classed among the "wonders,"—when complete, for at present but one line of tubes is in use; the south line is rapidly proceeding, its land tubes on their scaffolds, and the larger tubes, waiting for work, stand on their temporary piers below. The bridge is certainly not a beautiful object, consisting as it does entirely of straight lines and rectangles, broad and deep flat surfaces. An astonishing erection every one who beholds and considers will confess it to be; but he must consider as well as behold, for the parts are so proportionate in their greatness, that an ordinary travelling view from, for instance, the suspension bridge, is not calculated to excite the surprise which might be supposed. But a descent to the water's edge, at a stone's throw from the foot of either shore tower, at once unveils its gigantic proportions; the enormous mass of stonework close at hand, or that, still more enormous, shooting up out of the water to a height not well calculated by the eye, but which previous instruction affirms to be 200 feet; the extraordinary distance, too, the spectator is separated from this tower by the deep waters, while at the same time connection is maintained with it by a buff-coloured beam, hanging over the stream, in apparent readiness to fall, 100 feet above his head;—all silently demonstrate that an object unusual and magnificent, to a high degree, is the subject of his contemplation. Comparison of the unknown with the known is at all times inwardly recognised with satisfaction, and this is true in regard to the bridge; length being so remarkable, the height of the longer tubes diminishes in our estimation; but should it happen that painting is required underneath the tube, a little board, apparently unsupported, on which a diminutive creature has settled, pushing about a delicate feeler against the tasteless-looking beam, may be observed by ocular concentration. It proves to be a "child of larger growth," standing upright and at ease, on a strong platform, securely swung under the tube, reaching out a stout pole with a brush full of paint at one extremity, by which process the iron is preserved from premature decay. Illusion at once vanishes; we perceive that it would take five tall men, standing one upon another, to reach from bottom to top of the tube. Comparison, however, must be confined to the ordinary—to the well-known and everywhere-seen objects of daily communication. If we extend a glance to ascertain its character in reference to local remarkable things, the

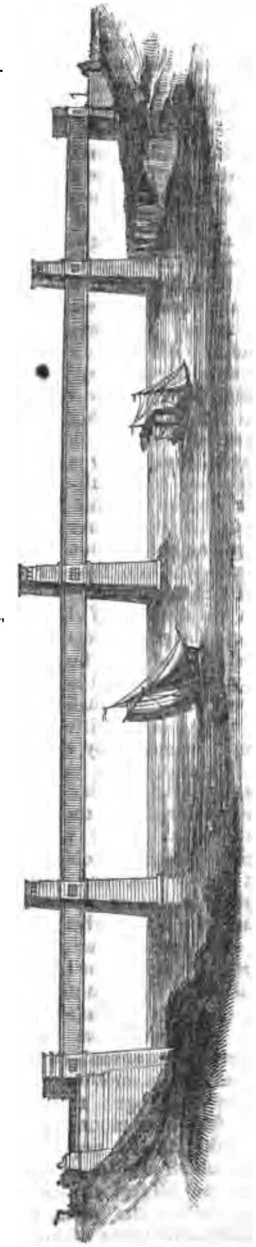


Fig. 101.

BRITANNIA TUBULAR BRIDGE.

impression of grandeur ceases; eyeing the tubes from an elevation, such as the suspension bridge, or the rising ground on the Anglesea shore, we have laid out before us a scene at once beautiful and sublime. Beneath, bearing craft and ships of all sizes, runs the eddying tide of the straits, coloured in unison with the soft green banks which restrict its channel, now wide, now narrow, and, with its little islands dividing the current, afford a delightful irregularity; the shore of Anglesea Island slopes to the edge of this important stream, but the opposite land rises abruptly, though not bare, a hundred feet and more, and so continues northward for a few miles, when it gives place to the confined waters, which suddenly spread out into the wide expanse of the Irish Sea; eastward, in a direction pointed to by the line of tubes, a magnificent range of hills projects into the sky its highest peaks, generally associated with clouds, which stream lazily along their green sides; from north-east to south-east is the earth-view limited by this chain—but eight miles off in nearest distance, roughly ending with Penmaen Mawr dipping suddenly into the sea on the north, and with the notable cloud-capped Rivals on the south. With such associate objects the great bridge is located; it is indeed a grand sight, but the sights below, beyond, north and south, all are more grand, and such qualities are remarkable only to our intuitive judgment, when they become so by comparison; over the Thames, or in equally plain situations, the height of the work would appear astonishing, and the stretch of the tube terrific. Here we see the hollow bridge merely stretching out from the bank before us, and this bank, though high, is yet as the sand-ridge thrown up by a wave, when compared with the elevations beyond; these elevations are so great that the highest peak —well called in the country's tongue Ywyddfa (*the conspicuous*)—venerable Snowdon, might still soar highest, if beside it were placed seventeen Britannia piers heaped one on another. Not only does height disparage; the not less remarkable feature of mountain views—amazing bulk, equally deprives of its wonder the labour of man. How little a large tower not 60 feet square appears to the eye when the same circle of vision includes the bases of mountains, each spreading over several square miles! Indeed, in passing by the northern extremity of the range, whence the white stone for these towers was quarried, we look again for the alteration made in the rugged mountain by the withdrawal of some thirty or forty thousand tons of its body: *Peter* certainly has been robbed to pay—his debt to man's necessity, but he is none the worse for it. Such is the difference between objects raised near to each other, one by the voluntary, long-continued efforts of a progressive rational being in an age of recognised progression, the other by an involuntary, normal, repulsive power, acting possibly for a few hours only, in past ages of great natural changes.

The tubular bridge at Conway is not equally remarkable; its elevation above the tide is but eighteen feet, and the abutment-towers are architecturally designed to harmonize with the old castle, on the precincts of which one of the abutments intrudes itself. Like patches of new on old work, however, there is little real harmony between them; the castle, as an object of artistic excellence and beauty, for which it is celebrated, has severely suffered by the addition, if the fact of occupying one side of it with rails, iron carriage-wheels, and occasional other things quite novel in the inventory of castle furniture, be left out of the question; there is also a complete spoliation of the beautiful suspension bridge, by the proximity of the huge tubes, which have been hoisted within arm's-reach of it.

(94.) A number of iron bridges, where forms similar to the stone arch have been imitated by large castings in iron, do not nevertheless admit of such interesting inquiry into the nature of their equilibrium as the former; for though an arch be imitated, there is little of the scientific character of the arch in respect to resistance against pressure; frequently the materials suffer useless and severe strains, because of their ill-managed arrangement. A bridge at Colebrook Dale—the first constructed in iron—is a semicircular arch of 100½ feet span, with ribs in triads, that is to say, each rib is made up of three ribs, one above another; they were made of castings 70 feet long, each being a quarter of a circle, or two were required to form one rib. Covering-plates rest upon these primary supports, and carry the roadway. These ribs might have been ordered to act as shores or equilibrated arches, in either case containing the whole line of pressure within them; as it is they cannot be treated as one or the other: they are not at all similar to shores, because of their quarter-circle form, neither to a common arch, both on account of semicircular form and small depth of each rib—only nine inches. All pressures must therefore pass without them at some points, being resisted merely by superabundance of material—not a proper disposition. This bridge was constructed in 1777, over the river Severn, and appears to be first in the list of iron bridges built in this country. The roadway is sustained on five ribs, which stand upon stone platforms laid on each bank; 378½ tons of cast iron was used in this bridge. An accident, showing how very little horizontal thrust or resistance was afforded by these half-arch castings, occurred not long after its erection; a bank walling, placed to prevent any slipping amongst some loose strata backing the abutments, proved unequal to its task, was pushed in, and carried these ribs easily before it towards the river.

Another bridge was erected a few years afterward within three miles of the Colebrook Dale arch, having a combination of three ribs under, and two ribs, one at each side of the roadway, extending to a few feet above it, by which arrangement suspension was intended to be given as well as under support. Long castings were again used, some being 50 feet. The inner or under ribs are 130 feet span, 17 feet rise, and 15 inches deep; the outer or suspension ribs of equal span stand 34 feet high; seven feet of which rise is above the public way. As a subject of mechanical or scientific concern these suspension ribs exhibit an admirable method of employing the strength of cast iron, but their combination with the flatter ribs is injudicious; they are a much greater arc of a circle, and larger than the under ribs, and when expansion occurs there must be a greater rising of the outer than the under ribs, which will have the effect of lifting up the roadway off the inner ribs; when contraction occurs the outer ribs sink most, leaving the roadway to be sustained by the three under ribs. Thus alternately each class of ribs have to support every weight.

A bridge was erected over the river Wear, at Bishopwearmouth, near Sunderland, which though so early in date as 1796, merits attention and praise, not merely for a great span attempted so soon in the practice of iron-bridge building, but on account of the admirable manner in which every part was ordered, with but few exceptions. This arch springs from the abutments at a height of about 60 feet above high-water level, which, added to its rise of 34 feet, affords ample room for large vessels to sail freely beneath. Between the abutments there is a space of about 236 feet, which opening is spanned by a row of six cast-iron ribs, each composed of 125 small frames, delineated in the following figures; each frame is about two feet long and five feet deep; three bars *a*, four inches square, forming the curved lines seen in the ribs, and transmitting all pressures, are connected by two others *b*, serving to keep the former at a proper distance apart; flat grooves are made in *a a*, three inches broad and three-fourths of an inch deep, to take wrought-iron bars, or rather straps of equally small breadth and depth, but long enough to slide in a similar groove in another similar frame, which should be placed alongside; by this means all the frames were fastened together. Six of these simple frames were fixed to an abutment, to make a beginning for six ribs; the succeeding frame, like a voussoir or stone of an arch could thus be placed against the first fixed, and both united by a wrought-iron strap; by proceeding in this manner a fine arch—in principle as well as appearance—was easily erected. Between the ribs iron tubes were attached to make one frame of all, and diagonal braces were afterwards added to give more strength in this particular. The depth of these frames is sufficient to include the probable line of pressure, and so bear it

Fig. 102.

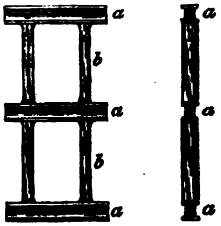
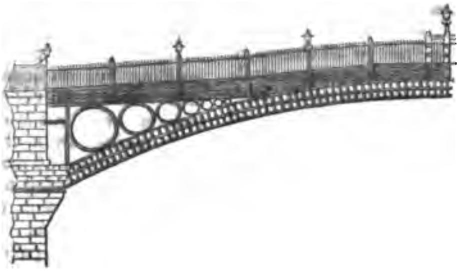


Fig. 103.



BISHOPWEARMOUTH BRIDGE.

fairly to each abutment; the idea of the structure is elegant, and appearances correspond; all being small pieces, there could be little difficulty in constructing, besides reducing any chance of open joints—an accident not unlikely with larger pieces, either from improper shape or settlement when together. Cross-struts instead of rings in the haunches would have been far superior as a support; when any pressure acts on these circles, four nearly equidistant points of each circle are greatly strained. Fault has been found for the unusual reason that too little weight of metal was employed; 250 tons of iron was supposed to supply too little stability; but all experience and science demonstrate that it is not the heaping together of ton upon ton of material, but its proper distribution, that constitutes stability in a structure.

Since the preceding examples were built, various attempts have been made to extend the use of iron in forming bridges with curved pieces, imitating the stone arch in form. In some cases imitation extends only to outward view, each support being a girder, and not acting similar to a number of wedges sustained by mutual resistance to compression; yet this condition is the very best for cast iron, for which its strength is most effectively and safely exercised; not unfrequently it has occurred that a considerable mass of abutment has been erected for iron bridges, which, from the character of the castings, are almost entirely girder-supported, giving no horizontal thrust, and requiring little abutment.

A proposal was made by Mr. Telford for a large iron bridge over the Thames in place of Old London Bridge; it was to have a span of 600 feet, with a rise of 65 feet, or one-ninth of the span; a series of wedge-pieces were to form a rib, and cross struts at the haunches filled in the great space arising from so great a rise and span. Much discussion arose about the practicability of such an enormous arch, and the matter was referred to a number of persons supposed to possess judgment in these matters, but so much disagreement of opinion prevailed, that nothing was elicited except a possibility of its standing: for some reason this idea was dropped, and the present magnificent stone structure preferred; certainly if Telford's bridge would have presented such an appearance as Southwark iron bridge, it is fortunate for the river view that the stone building was chosen. Southwark Bridge is the largest of cast-iron bridges; its middle arch extends about 246 feet, or nearly ten feet more than the beautiful bridge at Bishopwearmouth, with a rise of 23½ feet; the two side arches are each 210 feet span, and 18 feet 10 inches rise; eight ribs lie side by side to support the roadway, made up of plates or wedges, but instead of these being light frames, as in *fig.* 102, they are solid, and as portions of a girder—a top and bottom 4¼ inches thick, while the intermediate plates are 2½ inches thick; as a matter of sectional strength, no advantage arises from an extra thickness of top or bottom, as a comparison with the girder cannot be instituted in a rib of this kind, and the line of pressure appears to travel nearly through the middle of each rib, at no part varying above two inches from this line. The ribs are in fifteen blocks, 6 feet deep at the crown, and 8 feet at each end, attached together by means of dovetailed sockets and wedges of cast iron: cross and diagonal ties unite all the ribs of each arch. The spandrils or spaces between ribs and roadway are filled up with a cast-iron framing of diagonal struts, bearing cast-iron plates, on which lies the roadway. In the middle arch there are 1665 tons of iron, and 2920 tons in the side arches. The laying of each rib was commenced at the crown or middle, and ended at the springing, where bed-plates fastened to the stone-work received every terminating block; when this operation had been concluded, the centres were let free by lifting the whole iron arch with long wedges driven in at the springings. This is a fine iron bridge, although a profusion of material has been heaped upon it, neither does it present so light an appearance as might have been given, and which marks Bishopwearmouth Bridge—an equal in width and span within a few feet, yet six and a half times less weight of material was employed! Southwark Bridge was built by Rennie, between 1814 and 1819, in the spring of which year it was opened for traffic.

There are many other cast-iron arched bridges of great merit erected in this country. At Tewkesbury Telford constructed a fine light arch, of 170 feet, across the Severn, with a rise of only 17 feet; there are six ribs, about 3¼ feet deep, each in eight pieces, iron-bar cross-work filling the spaces between ribs and roadway. A very flat arch was thrown over the Witham, at Boston, in Lincolnshire, by Rennie, the rise being no more than four feet, with a span of one hundred. Another of Telford's iron arches is a pretty bridge over the Gloucester and Birmingham Canal, at Galton; it reaches 180 feet across, and rises 18 feet: the appearance of solid ribs is here avoided, each piece or block of a rib being a rectangular frame with two diagonal bars joining the corners.

(95.) Iron bridges have been built under the specific title of suspension, not having any scientific claim to such a name; they consist of a huge and by no means elegant iron rib, abutting considerably below the roadway, although stretching far above it in the middle, from which bars or rods are hung to carry a roadway: the latter is thus literally suspended, although a moment's consideration shows that the title of suspension bridge cannot justly be applied, since all pressure conveyed by these vertical rods is resisted by an anti-compression power of the great ribs, which are inflexible masses, not requiring any of those nice considerations and adjustments necessary in the structures commonly called suspension bridges. There are two bridges near Leeds framed after this method.

CHAPTER X.—*Suspension Bridges.—Historical Notice.—Character of the Catenary Curve; Calculation of its Form and Tensions.—Variation of Chain-Section.—Oscillations and Vibrations produced in a Chain Bridge by passing Loads and Winds.—Of the Chains: figure of least tension; effect of temperature; length of links; strength of bolts.—Wire Cables.—Stability of the Piers.—Of the Chain Fastenings.—Descriptions of the Freiburg, Menai, Hungerford Market, Dredge's, &c., Bridges.*

NEXT to a common log or beam, the most easy and simple contrivance for establishing a constant communication from bank to bank of a river, or between projecting portions of an intervening gap, is that of a rope or flexible line; indeed, necessity must have given birth to the idea*, since, in many parts of South America and Central Asia, the mountain roads between important places are crossed by chasms and torrents which are either too wide for a level plank, or permitting of no means at hand in early or even the present time of making the passage. Journals of travel afford abundant evidence of the use and antiquity of these trembling highways in those regions. The Himalaya have their *jhoola*†, or rope, stretched across a rapid waterfall or chasm between two legs, which are held by upright strong stakes driven in the banks; this rope is passed over from six to ten times, for the sake of security, and carries a block of wood or kind of pulley,

---

[*] The invention is certainly one prompted by high instinct merely, as we see the spider swinging to attach a web for future passage through the air, or as the prehensile-tailed monkeys (*Ateles*), on arriving at a stream in the course of their journey, will choose the branch of a tree, and then lash themselves one to another, forming a chain, which commences swinging until the extreme monkey catches a tree on the opposite shore, when the suspension line is completed and the troop passes over.

[†] In Sanskrit *jhūlā* means a swing.

with an eye through the middle, and from which depends a loop of ropes, constituting a questionable seat for the traveller, who holds fast by the sides of this loop, while he is hauled across by means of a loose rope attached to the running block. Mr. Frazer mentions a jhoola at Rampore as being nearly 300 feet long from stake to stake. In the mountain roads of South America a similar bridge is formed under the title of *tarabita*, sometimes consisting of two ropes, each carrying a basket, which is easily pulled across—as the ropes are inclined, one towards the nearer, and the other towards the further, bank. All these hanging ways are of exceedingly simple material—animal hide, vegetable fibre,—canes, osiers, and such like, affording abundant strength for their simple purpose. Capt. Basil Hall mentions the use of hide in a bridge of a somewhat more refined character than the tarabita; two lines of these ropes, with a roadway suspended between them by vertical ropes, stretched across the river Maypù (in Santiago, a Chilian province), with a span of 123 feet. Humboldt, in his travels, speaks of a rope bridge in which the tough fibres of the aloe (*Agave Americana*) are twisted into a bundle or rope, four inches thick, and stretched over timber frames on each bank; the roadway lies on them, and is therefore curved. This bridge is stated to be 131 feet span. Mr. Miers* describes an admirable example of this kind in the valley of Aconcagua (Chili), having a railing of suspension-ropes with vertical ties. We learn of two instances in which iron chains were long since used for suspension; over the Tchin-tchien, a river running through the mountain country of Bhootan, north-east of Hindustan, there is one called the Chuka-chazum, formed of five iron main chains, with links a foot long, on which a floor is laid; two additional chains are suspended nine feet above the former, one on each side; these are connected to the footway by vertical cords, affording both assistance in suspension and a parapet for safety. No date has been found for the erection of this bridge; its antiquity appears to be so great as to associate the period with fable-history. A fellow iron-chain bridge over the same river, but distant a day's journey, bears comparison in form with modern structures of this kind; from two main chains stretched over stone piers eight feet high, fastened at each end to the rocky banks, depend bands made of roots, which hold up a plank for a pathway. These are probably among the earliest efforts in constructing bridges; instances also occur of similar erections, probably as early in date, but more traceable in origin, in the iron-chain bridges in Yun-nan province: one of them at Quay-cheu is thrown across the Panho, a torrent of no great width, but with precipitous sides.

Few suspension bridges appear to have been built in Europe previous to the last century; one or two only in Alpine countries are recorded, and these were not calculated to inspire interest in this kind of structure. A beginning was made in this country with one over the Tees, near Middleton, in Yorkshire, about 1741; it was equally rude and unpromising with its ancient precedents, being nothing more than a footway laid on chains stretched nearly tight; this unsteady support is said to have been nearly seventy feet long, only two feet wide, and swung sixty feet above a dashing torrent. In addition to ignorance respecting the nature of a suspended chain, the want of a proper material for substantial and durable erections was a grand difficulty; ropes of woody fibre and animal hide might be used in parts of the world where a traveller is an unusual object, and mules carry the only loads, but in districts where commerce is active, and industrial products freely circulate, such bridges could appear but as toys. Improvement in iron manufacture awakened new interest in suspension bridges; their peculiar advantages were canvassed, their scientific character studied, their economy admitted; and since the commencement of the present century many admirable examples have been registered in the history of social enterprise.

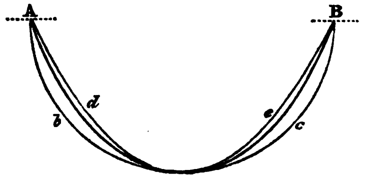
(96.) The general appearance of a proper suspension bridge is that of a roadway level, or slightly rising in the middle, attached to a curved line by vertical rods, the curved line or chain depending from two towers on each side of the opening or river, over which it appears to pass and descend into the ground. This curved line, being the source of all support in the bridge, is consequently the most important part, and attracts the greatest attention, since its peculiar character and properties must be known, to some extent, before any judgment can be attempted in reference to the strength it may be advisable to give to a bridge, besides its general arrangements.

---

* Travels in Chili and La Plata, vol. i., p. 334.

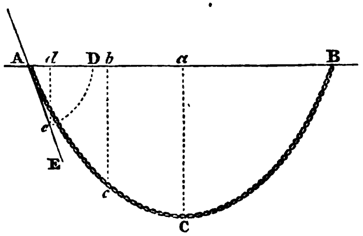
When a chain, consisting of a number of small equally heavy links is suspended at each end, and the middle part allowed to hang downwards to a considerable extent, we may observe plainly that the curve formed is not part of a common circle; it has a peculiar disposition, which has thence been very significantly termed the catenary or chain curve; a comparison with the circle A *b c* B and parabola A *d e* B will exhibit its characteristics; it bulges a little more than the parabolic and much less than the circular curve; but for small arcs, or where there is little deflection, scarcely any difference can be observed between the three curves: when the deflection begins to be about a twelfth of the span, one may be adopted for the other without serious error, except when the length of chain is very great.

Fig. 104.



The chain, although apparently configuring itself so variously, possesses certain constant principles, by which we may predict the curve taken when the weight and size are known, or determine the tension of each part of the hanging chain. At the lowest part the tension acts horizontally, and at the point of suspension in the first direction taken by the chain, or the tangent to the curve at that point: all the tension arising from gravity, that is, the weight of the chain, which acts vertically, we can proceed to calculate what must be the effect of a vertical force acting equally on all parts of the chain while the opposing forces are the chain's resistance to rupture and the tangential support at each extremity. Such a calculation reveals to us all that we require to know about the curve—namely, the nature of the tension or pull at different parts, from highest to lowest point; and the depth or deflection *a* C occurring with any given length of chain supported by points at a known horizontal distance from each other*.

Fig. 105.



We learn that the tension at C is least, and at A, the point of suspension, greatest; at the former point it varies at the measure A *d* (or cosine) of the angle D A E, made by the primitive direction of the chain and an horizontal line; it follows from this that as the chain is drawn more tightly, and its deflection decreases, the cosine A *d* will increase,—the tension at C will become greater, until, if the chain were drawn up to the horizontal line, A *d* would be equal to A D, and the tension at C would be equal to that at A: on the other hand, if the chain be further let down A *d* decreases, until, when the

---

[*] The following equations will give the tensions, ordinates, &c., when other quantities are known. T = tension of the chain at the piers; t = tension at any other point of the curve; l = length of chain; S = span or distance between piers; w = weight of a foot of the chain; x = any absciss, or part of S; y = any ordinate or deflection; z = any arc of the curve; and θ = the angle B A E. Then

$$x = \frac{\mathrm{T} \cos \theta}{w} w \times$$

$$\log. \left\{ \frac{\mathrm{T} - w y \mp \sqrt{\{(\mathrm{T} - w y^2)\, \mathrm{T}^2 \cos^2 \theta}}{\mathrm{T}\,(1 - \sin \theta)} \right\}.$$

$$y = \frac{\mathrm{T}\,(1 - \cos \theta)}{w}. \qquad \mathrm{T} = \frac{w\, l}{2 \sin \theta}.$$

$$z = \frac{\mathrm{T} \sin \theta}{w} \mp \frac{\sqrt{(\mathrm{T} - w y^2)} - \mathrm{T}^2 \cos^2 \theta}{w}.$$

$$t = \sqrt{(\mathrm{T}^2 - 2\, \mathrm{T} w z \,.\, \sin \theta + w^2 z^2)},$$

or, at the lowest point,

$$t = \mathrm{T} \cos \theta.$$

To find θ we have $\frac{\mathrm{S}}{l.} = \frac{\cos \theta}{\sin \theta} w \,.\log. \frac{\cos \theta}{1 - \sin \theta}$, which must be treated by approximation.

chain hangs vertically, A *d* would vanish, or the tension be equal to nothing, which we know must be the case. These facts show the advisability of allowing a considerable fall of chain in a suspension bridge, since, by drawing the chains too tightly, the tension becomes very great. At the point of suspension A the tension will of course vary with the weight and length of chain; but supposing that constant, or that we are speaking of the same chain in different positions, the tension varies inversely as *d e*, or the sine of the angle D A E; that is, as *d e* decreases the tension increases; thus, if the chain were horizontal *d e* would be least, or the tension greatest; if the chain hang vertically, *d e* would be greatest, and the tension least—equal only to the direct weight of chain.

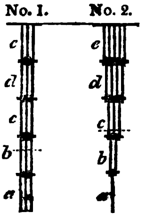
The deflection of a chain is, in practice, generally fore-appointed as respects the middle point C; that is, it is determined to allow a certain amount of deflection *a* C; when that is done, it is still necessary to have some means of computing the deflections at each point, as *b c*, where it is purposed to attach a suspending rod, that the latter may be made of a proper length. This may be approximately done by taking the difference between half the span, A *a*, and the distance A *b* of the place from the pier, squaring the result, multiplying by the middle deflection *a* C, and dividing the whole by the square of half the span. The quantity thus easily obtained must then be subtracted from the middle deflection, and we have *b c*, or any other ordinate, as may be required.

These must be considered as approximate statements; although an equal chain figures as a catenary, yet so soon as the slightest variation of weight occurs, there is a change of shape in the whole curve . in actual suspension bridges there are suspending rods, and a heavy roadway hanging on the chains, allowing them no longer a catenarian curve; they appear then to figure nearly as the parabola, between which and the catenary there is considerable resemblance.

(97.) A question arises from the fact that at the lowest point C, there is least tension, whether it is not proper to give less material there than at higher points; there can be no doubt as to the propriety of so doing, both because of economy and the advisability of relieving the whole chain of all unnecessary weight; for it is certain that much of the middle portion of chain in bridges, as generally constructed, is perfectly useless, and acts most injuriously in augmenting the effect of all oscillations from winds or passing loads.

The inutility of providing equal thickness or sectional area of chain is obvious without referring to any difficult calculation, if we suppose a chain to be suspended vertically, and of five links *a*, *b*, *c*, *d*, *e*; the highest row *e* will have to support 3 × 4 = 12 links; the next, *d*, keeps up nine links; the subsequent, *c*, six links; *b*, three links; and *a*, none at all; yet the series *a* is made as strong as those at *e*, which have to do all the work; if, then, the three top links can support the weight of twelve links, or four to each link, the row *b*, having but one to each link, has but one-fourth of its strength called into action. By the disposition shown in the adjoining figure (No. 2), something of improvement is evident, for there are only the same number of links, but the top row has five links, while there are ten only to be supported instead of twelve, as in the present case the *e* links require to be strong enough to sustain only two instead of four links each; still the lower parts are stronger than they need be. Then, again, we may consider what would be the effect on each chain of any vibration, such as may arise from wind. Supposing the chain (No. 1) to be fine and equal, its centre of percussion or oscillation (in this case nearly analogous) would be at about *b*, or two-thirds of its length from the point of suspension (see DYNAMICS, art. 87, p. 28); whereas in No. 2 it would be nearly at *c*, and much greater leverage is given to the disturbing force by the chain No. 1 to that by No. 2. Thus the latter is far more secure.

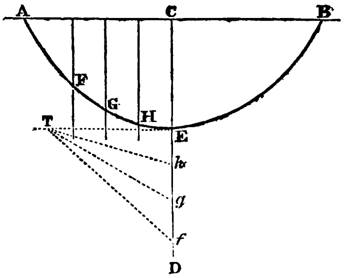
Fig. 106.



These remarks apply generally to the

chain when hanging in the curve, except that the lowest link a is not without some tension; but, according to the deflection, there is more or less change of tension from the middle to each point of suspension, for it is evident that the tension acts in the direction of the tangent to each point; thus, A B (*fig.* 107), being tangent to the curve at E, it will express the direction of the strain at E; then measure on the vertical, C D, as many units of length, Ef, as there are pounds weight in the portion E F of chain, and from f draw fT, parallel to the tangent at F, to form a triangle fTE; which, according to the property of triangles formed by lines having directions parallel to those of the forces, expresses also the relative amount or quantity of those forces (see art. 3, p. 117, of the present treatise); thus, while through the weight of chain measured by Ef there arises a tension equal to E T at the lowest point, a greater tension, equal to fT, bears on the point F. We might, by a similar procedure, setting off Eg, Eh, and other lines to express the weights of chain in E G, E H, and so on, find tangents gT, hT, all showing a variation of tension, finally becoming equal to E T.

Fig. 107.



The curve formed by such a modified chain is not a catenary proper, but approaches much more nearly to the circle, since the superior weight of the upper parts bends those portions more, and flattens the curve at its lowest point.

A bridge supported on such flexible lines cannot preserve its form, even under the influence of very small forces; the presence of an individual or light cart will give a new form to the chain, which, through the suspending rods, affects the roadway; if it be near one extremity, that part will be drawn downwards, to the elevation of the opposite portion; this, in general, will be small, since any weights which pass over the bridge are very inconsiderable in proportional weight, and a weight equal to one-twentieth that of the chain will not cause an increase of deflection greater than a fortieth at the middle, when the middle deflection is one-fifteenth of the span; it is, however, felt much more towards the towers of a bridge. A slight variation occurs in the tensions at different parts; the point of least tension is brought from the middle nearer to the weight; the angle of inclination D A E becomes greater, while at the farther tower it decreases, thus throwing a pressure on the nearer tower tending to overturn it in the direction D A: these are the actual tendencies of the structure, although it would be most improbable that such extreme effects, in all their injurious consequences, should follow, since the chainwork of a suspension bridge is, in most cases, so ponderous that any conceivable load commonly passing bridges could be but a minute fraction of its weight. These deflectional movements appear to increase somewhat less than as the weight increases; it decreases nearly as the original deflection is less, or as the span is greater; it would seem to be advisable to give as little depth and as great a stretch of curve as possible, but as the curve is brought nearer to an horizontal line there is great increase of strain on the chain; a medium depth of curve must, therefore, be chosen, to combine the desiderata of expense, convenience, and security.

(98.) Another matter of important inquiry in suspension bridges is, the variety of oscillatory and tremulous movements to which such structures are subject; they arise constantly from the passage of persons and vehicles, and frequently from currents of air. If foot passengers should happen to step in a time equal to that taken by the chain in making one, two, or more (whole) vibrations from pier to pier, a waving motion will commence and increase while the disturbent is present; if the successive impulses be irregular, there will be an interference of the undulations, partially or wholly destroying each other. The latter generally occurs on the passage of horses; both are, however, very injurious; a bridge at Broughton, near Manchester, fell, in consequence of sixty soldiers marching regularly over it; it was certainly not well built, but such a strain was thrown on the bolts, through longitudinal vibration, that one gave way; a similar accident, attended with a fearful loss of life, happened to a suspension bridge at Angers, in France, over which a number of condemned troops were proceeding on their way to exile in Algiers. While the bridge was covered from end to end, one of the chains gave way, and the unfortunate men were cast into a river swollen by heavy rains. There is no reason to suppose, in either case, that the soldiers' steps were synchronal with the undulations of these chains,—the disorderly interfering vibrations being enough, with any predisposing causes of rupture, to effect a downfall. The effect of these undulations is to carry the middle or lowest point backward and forward, which must cause a sudden strain and shock on the upper parts at each tower in succession; an additional strain arises from the roadway resisting this undulatory motion through the suspension rods. All these mixed movements give severe jerks to the bolts, especially at the points of suspension, where every succeeding wave must be stopped. Calculation and experiment show that the motion of the middle point increases with the deflection; it has been stated as the square root of the deflection, which makes the augmentation with increased depth of curve but small in comparison; it decreases as the span is greater, but in different bridges, where a deflection is given in constant ratio to the span, the motion may be reckoned nearly as inversely to the square root of the span; the velocity of the undulations increases as the span increases, permitting, as a consequence, less depth of wave.

The oscillations arising from winds are far more serious, giving a swinging motion to the whole chain, to which there is nothing capable of affording certain resistance. A hanging chain, like a pendulum, will oscillate through any laterally impinging force with a rapidity inversely as the square root of its middle deflection nearly: when the deflection is very great, a chain will oscillate somewhat quicker than a simple pendulum of a length equal to the deflection,—more so as it approaches a vertical line, for if both halves of the chain were brought together, it would vibrate in the same time as a simple pendulum of two-thirds its length; but as the curve becomes flatter this difference diminishes. The following figures explain a few cases:—

$\frac{\text{Deflection}}{\text{Span}}$	$\frac{\text{Pendulum}}{\text{Deflection}}$
$\frac{1}{1}$	$\frac{3}{4}$
$\frac{1}{2}$	$\frac{3}{4}$ nearly.
$\frac{1}{4}$	$\frac{5}{6}$
$\frac{1}{12}$	$\frac{6}{7}$
$\frac{1}{15}$	$\frac{7}{8}$

These approximate values show that when the ratio of deflection to span is as one to one, or both are equal, the catenary will oscillate in the same time as a pendulum which is in length to the deflection as 3 to 4, or 3-4ths of the latter; but when the deflection is one-fifteenth there is a difference of one-eighth only between the depth of curve and a pendulum beating synchronically. We learn from these facts the importance of reducing, as far as practicable, the depth of curve in a chain bridge, since its vibrations will be very much less, and consequently less velocity and momentum will be attained by the swinging chains. As this destructive movement cannot be prevented altogether, and the available preventives are only two—variation of quantity of chain, and flatness of curve, it is necessary to compensate and provide for any possible disturbance. The two preventives mentioned prove also a relief to the parts most ill-used by horizontal oscillations; if the chain be one of equal strength, that is, varying in section (art. 97) from the middle to the towers, not only will the vibrations be less because its centre of oscillation (*fig.* 106 No. 2) is raised, but having less mass than an ordinary chain of similar total strength, and that mass moving with less velocity, it cannot but greatly reduce the danger of vibration, and the roadway can better check its motion: the effect of reducing deflection we have just considered, but in so doing we subject the chain to far greater tension at the points of suspension with the same force of wind. Little care is manifested in most suspension bridges to render the oscillations as harmless as possible; should the swing be regular, there is a fearful wrenching of the bolts at the towers, which are also endangered, but this is not the case with a chain tied to a roadway, incapable of vibrating in the same manner; there must be an horizontal undulation combined with the general oscillation, producing a strain most trying to the bolts and eyes of the links; this effect has been much increased in some bridges by making the links very long, so that the leverage of these undulating forces is very great. The above remarks apply only to chains; many bridges are suspended from wire cables, when the only bad effect of such motions is a transverse strain at different parts of the cable, especially at the towers, but not nearly so destructive as in the chain-work. It is the curious property of some winds to move by starts or fits, often occurring with regular intervals, and to these are the violent oscillations to be attributed. Two remarkable instances are on record. The fine chain bridge over the Menai Strait was visited by a tremendous storm of this character thirteen years after its erection; the chains beat against each other, to the serious injury of several bolts, and some parts of the roadway were torn up through the suspending rods, many of which were broken; this occurred in one night, and on the following morning the bridge was found to be impassable. Another accident of this nature happened to the Brighton suspension pier, on the 30th of November, 1836; a storm harassed the pier, causing the four 255 feet spans of which it is composed to oscillate and undulate for several hours, until the third of them began to yield; its side-rail first fell into the sea, when the roadway was so torn up by the great longitudinal undulations of the chains, that shortly afterwards it gave way also; in this instance, as probably in the former, destruction was caused principally by undulations.

(99.) So long as the preceding properties of the chain curve are borne in mind, various dispositions may be adopted for suspension bridges. The most common practice is to erect two piers at each side of the opening to be spanned, and by the chains over them, each extremity of chain being firmly attached to masonry or rock underground. This is probably the simplest proceeding, and forms an object of great beauty. If it be desirable to reduce the tension, it may readily be done by removing one tower; for in order to have the least tension it is necessary that the tangent at one end be horizontal, as it is at E, last *fig.* This plan has been adopted in a modified manner, in the Bourbon Bridge (see p. 226) and others; its advantage does not consist only in causing least tension, it directly prevents the oscillations and vibrations to which one whole curve of equal span would be liable; the expense of one pier is saved, as also a portion of chain. The principal objection is the difficulty arising in building a pier in deep water, or the middle of a stream.

There is no practical limit to the span of a catenarian bridge if the points of suspension or towers may be raised as high as necessary, and that with any particular or given section or power of chain; for, knowing by calculation what will be the tension with any span and deflection, we may assign such a value for the deflection as shall bring out any desired amount of tension. No bridge at present known approaches the limit of span; for if we take the weight which an iron rod will sustain before any destructive extension of its parts occurs, the limit is about 3000 feet when the deflection is 1-15th of the span*, an extent more than three times that of the longest suspension bridge.

Variations of the temperature will affect large bridges to some extent; as the chain-ends are fixed, any lengthening of it must depress the middle portion, and with it suspension-rods and roadway: this will happen to a considerable amount with most structures, since but a small deflection is allowed, when, as is common, it is 1-15th, an increase of length of chain to the extent of 1-1000th of its whole length will produce a depression in the middle of 1-25th of the deflection. Thus, if a chain 600 feet long suffered an extension so great as 1-1000th of its length, and its deflection were in the above proportion, or 40 feet, the latter would be increased to $41\frac{2}{3}$ feet. A considerable variation occurs in the Menai suspension bridge, from the change of temperature between winter and summer, since the metal expands 1-3040th of its length for 50° increase of temperature. We learn from this fact the necessity of allowing a little space for movements arising from this and other causes: if the fabric be made rigid it must suffer, but should it be constructed so as passively to adapt itself, like the chain, to meet any change of circumstances, it will, like the latter, escape injury: alterations have sometimes been made subsequent to erection, in order to allow of more play.

---

* Navier, Mémoire sur les Ponts Suspendus, p. 175.

(100.) Chains, as they are called, are made for this purpose of long links or bars of wrought iron, with holes or eyes at each end, through which stout bolts are threaded, and kept in by a nut screwed on at one end, and a head at the other; such a chain forms a polygon, through each angle of which the catenary curve passes. Various lengths have been given to the bars or links; in the Isle of Bourbon Bridge they are 4½ feet long, in Hammersmith Bridge 8 feet 10 inches, in the Menai 10 feet, in the Union Bridge (over the Tweed) 15 feet, and in that at Hungerford Market, over the Thames, 24 feet long. The only reason assigned for making long links appears to be that of Capt. Brown, on whose model the Tweed Bridge was erected,—that the iron links are weak in the eyes, so that it is desirable to have as few eyes as possible; otherwise the long links are very prejudicial,—the line of pressure or tension does not pass through them, except at the bolts or points of junction between each set of links; there must be a constant unequal strain on the parts where this line cuts the edge of the bars. Again, in vertical undulations arising from passing loads, a much more harassing strain is communicated through the rods to the roadway; when horizontal oscillations and undulations occur, the evil of long rods is still more striking, the twisting and wrenching at each joint or bolt gaining greater effect as the bolt is longer, since a force applied at one end has the leverage of nearly the whole length of the bar, when acting on the eyes of its companion bars at the opposite extremity; Hungerford Bridge stands remarkable for these unfavourable qualities.

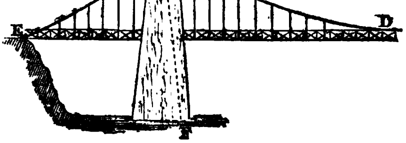
The bolts are an important element in the structure of a chain; although there may be abundance of strength in the links, unless there is similar strength in the bolts, safety is uncertain in the bridge. A few pages back Broughton suspension bridge was mentioned as having given way at a bolt; in the Menai Bridge it appears that the bolts are not so strong as the chains. Their strength is as the cube of their diameter, and inversely as their length; whence we make a formula for calculating their power in any case, multiplying the quantity by the strength of (or weight borne by) a bar one foot long, and one inch in diameter, which is taken as a unit; this is 800 lbs; then the rule stands

$$\text{strength} = \frac{\text{diameter}^3 \quad 800}{\text{length}}.$$

A number of bridges are supported by ropes made of wires in place of chains; the largest suspension bridge hitherto erected, that at Freiburg, is of wire rope. Much has been said of the inferiority of wires, in comparison with chains, but most of those evil effects from vertical and horizontal motions which apply to chains vanish in a wire rope: there are no joints to rupture, the transverse strains which may happen in undulations are very small, and the undulations more regular; and there is no weakness arising from eyes and bolts. In addition to these lesser bad qualities, iron wire is stronger than bars, and a rope can be made much more easily than a chain of bars, as there is no trouble of adjustment similar to the latter: the reasons advanced against the use of wire are on the supposition of bad workmanship, and, therefore, not of moment; when variation of section, however, is considered, the rope is unsuitable in its plain form, which makes of equal diameter throughout; but where this important improvement is not attended to, the objections are irrelevant.

(101.) The points of support require less careful consideration than in ordinary arched bridges, since, if the chains be properly placed, there is a plain vertical pressure on them; any more intricate consideration must arise from improperly hanging the chains or horizontal oscillations of the chains in high, and, therefore, rarely-occurring storms. We may determine the pressure exerted by the chain according to art. 44; the

Fig. 108.



horizontal pull of the central span C D will be proportional to C *a*, or the cosine of the angle A or *a* C *b* of suspension, and the opposite pull of the back-stays C E is proportional to *c* C, or the cosine of the angle B; also a vertical pressure arises from both curves proportional respectively to *a b* (the sine of the angle A of suspension) and *c d* (the sine of B). So long as these angles A and B are equal there is no pressure, except the combined downward pressures *c d* and *a b;* but should the chains be brought down too rapidly to their fastenings, so as to give a smaller angle at A than B, or *vice versâ*, there must arise a strain tending to drag the pier inland or into the water equal to the difference of the cosines; that is, C *c* – C *d*, or C *d* – C *c;* otherwise it may be stated that the diagonal or resultant C F of the two forces acting in the direction of the tangent to the curve, as A E (see *fig.* 105, *ante*), will not be in a vertical direction, but more or less approaching the edge of the piers at their base, and endangering their stability: the conditions of their equilibrium under such circumstances may be learned from the principles stated in art. 44 of the present treatise. Instances of chains so placed occur in the Union Bridge over the Tweed, in Hammersmith, and Hungerford Market. The error in these cases permits the chains to pull the piers towards the middle of the stream; in the Menai Bridge the error is on the reverse side; there the chains have a slight tendency to carry the tower summits towards the land. Practically, no destructive result happens in consequence of this force, or those arising from horizontal oscillations, because a spreading base is generally allowed to the piers.

(102.) Next to providing sufficient strength of chain, its secure fastening at each extremity is of greatest moment; inattention in this respect must prove most inimical to the structure. Where rock constitutes the foundation on which a bridge is erected, a natural abutment is at once offered; the chains need then only to be carried down through a tunnel cut in the rock, and attached to a cross-bolt or saddle, catching against shoulders cut in the rock; such a method of fixing the chains occurs in the Menai and Freiburg Bridges. In general, however, no natural provision can be expected; and masonry must supply its place; no dependence can be placed on the light ground surrounding, neither should there be any allowance for the assistance it may be supposed to offer to the chain-tension,—a deep flood, or accidental overflowing of any neighbouring collection of water, may show the untrustworthy character of the foundation, by their giving the chains full liberty to go where they please. Such an accident occurred in a suspension bridge, built in 1823 over the Seine, at Paris, by M. Navier; although he had well investigated the principles on which abutment-equilibrium rested, his bridge was doomed to demonstrate that the abutments were unsound; when the chains were suspended, the masonry exhibited a few chinks, which increased to fissures two inches wide when the work was completed. In a little time a water-pipe in the vicinity burst, and softened the ground about the masonry; in consequence, the chains pulled up the stonework, ornamental lions and all, and, with the nearer suspension-piers, quietly slid into the river. It seems plain, that in this example the weight of masonry was not equal to the pull of the chains, and that it stood, before the water-pipe burst, merely by the weight and cohesion of the surrounding and incumbent earth. The latter material should certainly be employed as far as possible, but only as accessory to the general security. In all cases, where possible, the chains should be carried in a vertical direction through the masonry, and act upon it in that direction, for the pull is resisted by all the weight of stone-work; if it be merely slanting, the only force to resist rupture is the adhesion of the mass of abutment. Also the chain should be fixed at the base of the masonry, as all below the attachment is nearly useless. In M. Navier's bridge this fault was present, and much of his abutment did no work.

(103.) The principles of the catenary curve were known many years since, Leibnitz, Maclaurin, and others, having investigated it by means of a new method of calculation which had not long been discovered. Its character, as then published, did not invite the engineer to turn it to use, and not until by slow degrees materials were found capable of carrying out the idea, and men were forced to find some new means for the onward pressure of social improvement, did any one think of a catenarian bridge as a means of general transport. Mr.

Finlay, in America, stands first in practical attempts to utilize a suspended chain, and built several bridges of considerable span on the method which he patented; one was over the cataract of Schuylkill, 306 feet long, of two chains, deflecting one-seventh of the span; at the middle the roadway rested on the chains. In Europe, M. Belu appears to have commenced the subject by proposing a suspension bridge over the Rhine, at Wesel, having a span of 820 feet; this was followed in 1814 by a proposal still more extensive on the part of Mr. Telford, for a bridge over the mouth of the river Mersey, at a place called Runcorn Gap; this structure was to be 1000 feet span between the towers, with a deflection of 50 feet; the whole supporting work being four parallel lines of chains.

Shortly before this time Capt. S. Brown had brought before public notice a plan for erecting chain bridges, in which long round or flat bars, with welded eyes or drilled holes, formed the links, and he undertook a number of experiments to determine the strength of such links in composition, and to what extent they might be strained without injury; these experiments relate not simply to the ultimate strength of iron, but to the strength shown in a modified state, and under certain practical restrictions; they are, therefore, valuable as specially concerning suspension bridges. Mr. Telford made another series of experiments, with a similar object, when he proposed the great Runcorn Bridge; and his results, together with those of the former and other experimenters, are very instructive. Nine experiments were made on plain iron bars, by stretching them with an hydrostatic press, and elicited the following results:—

			tons.	cwt.
No. 1.	Welsh iron;	bore	29	6
No. 2.	Ditto	,,	29	16
No. 3.	Staffordshire	,,	27	3
No. 4.	Ditto	,,	27	10
No. 5.	Welsh	,,	29	0
No. 6.	Swedish	,,	29	0
No. 7.	Faggotted	,,	29	0
No. 8.	Staffordshire	,,	31	0
No. 9.	[not stated]	,,	31	16
			9)263	11
	Mean strength		29	5⅔

These are Mr. Telford's results, and give a greater strength than those of Capt. Brown, whose mean is 25 tons; both cases are supposed to be somewhat beyond the truth, as the captain's testing machine was calculated to show less, while the hydrostatic press showed greater, than the actual amount; the mean of the two, or 27 tons per square inch, is, therefore, considered to be nearer the true strength. When the iron was bent to form chain links an area of 3½ square inches bore 76 tons, which gives $\frac{76}{3\frac{1}{2}} = 21\frac{1}{2}$ tons per square inch, showing that through bending the bar at each end to form an eye, 5½ tons of its strength were lost.

When 27 tons is stated as the strength it is not to be supposed that such a strain could be safely allowed in any practical case; long before this ultimate number was obtained the bars under trial stretched considerably; in experiment No. 9 the bar, which was 2 inches in diameter, bore 100 tons before breaking, but under a pull of 45 tons it began to stretch so much as a tenth of an inch per foot of length, and recovered only one-fortieth of an inch when the straining force was removed; subsequent experiments prove that ten tons acting on a square inch will injure the elasticity of a bar, but nine tons can be constantly borne with safety. Thus, from 27 we are reduced to 9 available tons per square inch, or one-third, as safely permissible in a suspension bridge

Steel has been advocated for chain-bridges, on account of its great strength, Damascus steel once refined bearing 36 tons, and twice refined, 45 tons before breaking; its scarcity in comparison with common iron appears to be the chief obstacle to its use.

Iron wire has also been tried with reference to its ability as a suspension material. Mr. Telford proved its strength when suspended vertically, and also when hanging in a catenarian curve over props, with weights attached at different parts. The results obtained in the former position of the wire were as follows:—

No. of Experiment.	Diameter of Wire. in.	Breaking weight. lbs.	Breaking weight per square inch. tons.
1	·0857	531	35·7
2	·1000	738	42·0
3	·0600	277	42·9
4	·0479	157	38·1
5	·1000	630	35·8
6	·1000	634	36·0
		Mean . .	38·4

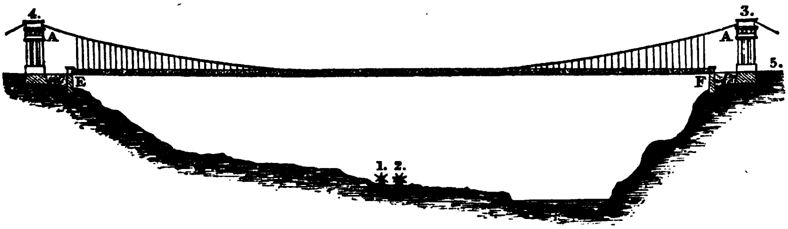
An average strength of 38½ tons nearly was thus shown by iron wires of different descriptions: this speaks most favourably for its ability as a bridge-building material.

These wires were further tried while hanging over a support near one end, and attached to the wall of a house at the other; three weights were hung on the wire at equal distances from each other, and a greater weight at the movable end, to determine the tension. They exhibit great powers in iron wire, although, unfortunately, the experiments were conducted in a manner which renders them of little service. Colonel Dufour, of Geneva, made some similar experiments, previous to erecting a wire bridge at that place, and obtained a mean strength of 41·7 tons per square inch, being above 3 tons to the inch more than Mr. Telford's numbers; and this was exceeded by M. Vicat, who found some wires able to bear 47 tons per square inch; the least value—38½ tons—must, therefore, be considered as a very safe estimate.

1. **Freiburg Suspension Bridge.**—With respect to proportions, this is the bridge of the greatest span hitherto erected; it is suspended highest in the air, occupied a moderate time in erecting, and incurred but a small expense. It was built in consequence of the natural difficulties which prevented the town of Freiburg receiving many visits, commercial and otherwise, which it has done since this means of communication was formed. Freiburg stands on the sloping side of a hill, which is separated from its neighbouring height by an intervening valley between two and three hundred feet deep, and called, because of the little river which winds through it, the valley of the Sarine; across this valley lay the road connecting Freiburg with the German frontier, and the ascent of the hill was made by a twisting and difficult way, presenting some very steep inclines. In 1830 a French engineer, M. Chaley, proposed to erect a wire-cable bridge reaching from one hill to the other,—a distance of nearly 900 feet at the spot pointed out. After pecuniary agreements this engineer proceeded with the work on the plan he offered, which was to erect a bridge capable of bearing passenger and goods traffic; two towers were to be built on each hill side, at a distance from each other of 870 feet, between which four wire cables,—two on each side of the roadway,—were to be stretched, giving a curve 63 feet deep in the middle; suspending the roadway 167 feet above the level of the river.

The general appearance of the bridge may be conjectured from the accompanying figure, and the details of its construction will receive illustration from the partial diagrams; a description of such a structure must necessarily be somewhat long, but it is very instructive, since the bridge is a model of its kind; to give it as clearly as we may, it will be advisable to divide the explana-

Fig. 109.



tion according to the several parts constituting the bridge:—1st, the main cables and the manufacture;—2ndly, the mooring cables;—3rdly, the masonry above and below ground;—4thly, raising and fixing the cables, and arrangement of the roadway.

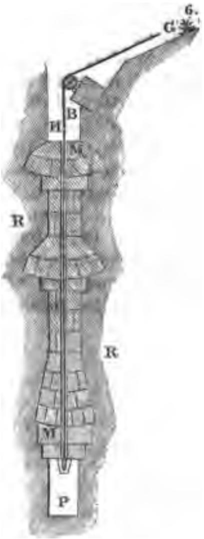
1. *The Main Suspension Cables.*—These most important elements of the bridge are composed of iron wire 0·12, or little more than one-tenth of an inch in diameter, a lineal yard of which weighs nearly two ounces. Each cable (made up of 1056 lines of wire), is 5½ inches in diameter, and 1228 feet long, being bound up into a cylindrical form by iron wire at every second foot of its length. The wire was supplied in coils of 18 or 20 lbs. weight each, and if found to be good, passed through an introductory process of boiling in a mixture of linseed oil, litharge, and soot; the wire was then hung up to dry. This was intended to preserve the wire from rusting. The lengths of wire, after the preceding treatment, were rolled on reels above a foot in diameter, and when one length had been wound on, the workman twisted the remaining end with the extremity of another length, tightly binding the two with annealed wire; so well was this junction effected that on testing the part it never gave way before some other part of the wire broke. On account of the great weight of a complete cable, and the difficulty of raising it to so great an elevation, it was made in parts, called strands, which could be separately raised: they were twenty in number for each cable, and themselves consisted,—twelve of them of 56 wires each, and eight of 48 wires. To manufacture one strand a walk or level line was chosen 614 feet long—for want of a longer—which being just half the length of the strand, the wires had to be carried twice along the walk. At one end were firmly fixed two blocks of oak, to which were hooked iron stirrups; an end of wire on the reel being fastened to one block was passed round the stirrup, and the reel carried to the other end of the walk, where the wire was tried by a weight of 220 lbs., and, sustaining the proof, it was passed round a semicylindrical block, also firmly fixed, and then borne to the point whence it started, but was placed round the second stirrup; this length was also tried by the test weight, and the movement continued until one strand, or 56 threads, had been unwound from the reels, when the end thus arrived at and the end at first fixed to the block were bound together; the whole bundle was also bound at each end, and every three or four feet of length. A coating of the preservative mixture before mentioned was now applied, and the prepared strand laid aside. Five of these strands were made in a week by as many workmen.

2. *Mooring Cables.*—These were intended to take the ends of each suspension cable when it passed through the sloping gallery G (*fig.* 110), and constitute the final attachment to the heavy masonry. They were made in a manner similar to the former, but thicker, and when taken off the stirrups on which they had been wound, in consequence of elastic force, they twisted and curled up like a corkscrew. To obviate this difficulty series of laths were bound round the cable before loosing it, and little of the former effect followed. Great care was taken to bring all the wires composing these cables into equal tension. They were each four inches in diameter, and composed of 528 wires.

3. *The Masonry.*—Each pier is founded upon the rock, is 66½ feet high above the road level, and gracefully arranged as a Doric portico. They present an arched opening 43 feet high to the passenger, each of the sides bearing three pilasters and an entablature. Jura limestone faces the basement courses, but the interior and upper parts are sandstone, finely dressed, so as to allow of no vacant spaces in the stonework; the latter material was readily obtained, as it constitutes the mountain masses on each side of the valley; when first quarried it was found to be easy in working, but afterwards dried and became hard; on proving its strength by an hydrostatic press, it bore 555 lbs. per square inch of surface without injury. In the upper part of the piers apparatus was placed for accommodating the chains; it consists of three rollers, giving as many points of support to the cable A, which is allowed to spread out and form a band at these points; every facility is afforded by these friction rollers for slight movements of the cables in consequence of changing temperature or similarly-acting agents, while, by their disposition, the cables are not damaged by sudden bends.

Those portions of the masonry which may be called underground, are the sloping galleries G, through which the cables pass, and are connected with the mooring chains, and the vertical mooring shafts H; the former were excavations six and a half feet square, roofed with limestone arches. The mooring shafts H commence at the lower extremity of the galleries, and are cut in the solid rock R, to a depth of 52½ feet; they are also 10 feet broad, 3 feet 3 inches wide, and four in number. In them was fitted the masonry M, intended to bear all the weight from the cable, and for this all-important purpose it was admirably arranged. Angular incisions were made in the rock, at three places, which were subsequently filled with stonework of a dovetail form, and having all the peculiar excellency of that description of joint; the mooring cables B, which were carried through this filling-in, had thus a most secure abutment at the lower part. A small passage P, about six feet high, extends under the cable fastenings, to facilitate their inspection, communicating with the ground surface by an horizontal gallery cut from the hill side—a distance of nearly 350 feet. Limestone supplied the material for these mooring shafts, and was obtained from the quarries of Neuville and Lengnau, situated on an inferior branch of the Jura mountain-chain, about 30 miles from Freiburg; on being tried similarly to the sandstone it was found to be capable of bearing 3307 lbs. on the superficial square inch without injury.

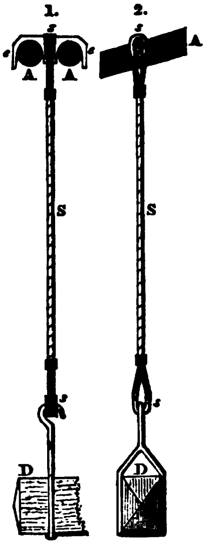
Fig. 110.



4. *Raising the Cables.*—When every preliminary arrangement had been made, and all the stonework secure, two windlasses were placed on each tower, one at each point; two midway between them down in the valley; and one in each sloping gallery. Round No. 5 a hempen rope, an inch in diameter, was wound, thence it passed twice round the axles of the windlasses No. 3 on the tower, and descended towards the valley, being joined to another smaller rope, which reached windlass No. 2; the same being done on the other side. A drum six and a half feet in diameter, bearing one strand, wound not as usual, but beginning from the middle, so as to leave both ends on the same side, was then placed between windlasses 1, 2, and an end of the strand attached to each rope; this was followed by a working of the windlasses 3, 4, at both piers, and the small cable gradually unwound and rose in the air towards its intended location; when the ends had reached the towers, one side ceased to work, while the workmen at the other side drew their end over temporary rollers on the top of the pier, and attached it to a rope from windlass No. 6 in the sloping gallery, which, on being worked, drew the strand over permanent friction rollers on the pier, and brought its end into the sloping gallery. Workmen at the other end of the bridge wrought similarly, and brought up this first strand to a proper curvature. To determine this properly two logs or benchmarks had been fixed at each side against the piers, the line of sight between which formed the appointed level for the roadway, and as it was intended that the cables should descend to this level, it was determined, with tolerable accuracy, by drawing the strand until the lowest point of its curve touched this line of sight. Mooring cables were now brought down the galleries, and drawn down the shaft, through its small opening, by a windlass-worked rope. When this cable was secured in the manner shown in *fig.* 110, it was ready to take hold of the sus-

pended strand; but the latter was not attached until another strand was raised, when one of them was placed on each side of the mooring cable, and a connecting bolt passed through the stirrups attached to them. This process was continued until forty strands were drawn up on each side of the bridge, forming a pair of bands each above two and a half feet broad, and about 30½ feet apart; they were divided each into two bundles A A (*fig*. 111), of twenty strands, and bound up in a cylindrical shape by iron wire, leaving those parts which rest on the pier friction rollers as a band. The work of attaching the suspension cords S now followed; they are composed of 30 wires, making a diameter of one inch; their greatest length is about 54 feet, and least 6 inches; each end is bent round a stirrup *s*, *s*, to afford means for suspension, in a manner shown by the diagram; a hook loop catches the lower stirrup, and takes in its loop one end of a road beam D; the upper stirrup embraces a saddle *e*, which also embraces the pair of cables A A. A distance of 4 feet 11 inches separates the suspension ropes.

Fig. 111.



When these had been distributed on each side of the bridge, and the suspended beams placed between them, longitudinal planking was laid on the cross-beams D, with another layer of cross planks, which form the carriage way. The footpaths are raised about 7 inches above the carriage way, and are very narrow, being not more than 2 feet 9½ inches wide, while the carriage way takes up 15 feet 5 inches, making the total width between railings 21 feet. The total deflection of the bridge, when completed, was 63¼ feet, and its length 870⅓ feet, which gives the proportion for the deflection 1-13¾ths greater than the most advantageous ratio, which has been stated to be about 1-15th. The platform hangs 167⅓ feet above the river Sarine, immediately below it. The following is a summary of the dimensions of its principal parts :—

	ft.	in.
Length of suspended roadway (E F, *fig*. 109)	807	0
Deflection of the main cables	63	3
Height of roadway above the river Sarine	167	4
Rise of platform in middle	1	8
Main cables, of 1056 threads each, diameter		5·5
Iron wire, diameter		0·12
Width of roadway	21	0
Total suspended weight	296	tons.

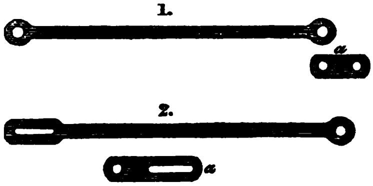
Altogether Freiburg Suspension Bridge must be considered as a noble specimen of its kind; its elegant simplicity accords with the locality and scenery; its great span and comparative lightness are most striking to the eye and reason, and its scientific disposition is very instructive. The cost of this bridge was 24,000*l*., or one-fifth the expense of the Menai Bridge, which is 300 feet less in span.

2. Menai Suspension Bridge.—This bridge, although now not the greatest span, by a hundred feet, of similar structures in this country, is, in other respects, the most remarkable of our chain bridges; and previous to the erection of its grand companion, the tubular bridge, it stood without an equal. In describing the tubular bridges abundant reference has been made to the character of the Menai Strait; it interfered, in the present instance, with a main road, made at the country's expense, through North Wales to Holyhead, which is the nearest

station for steam communication with Ireland. A ferry was the only means of transition between the Welsh shore and that of Anglesea, and although but a few hundred yards, it proved, on account of the current, an unsafe conveyance. In 1818, after previous entertainments and abandonments of the idea, Parliament came to a decided opinion, and Mr. Telford, who had been engaged in laying out the main road, selected a convenient site for the intended bridge; it was at a point where both banks approach to within 906 yards at high, and 160 yards at low water, with a rock appearing at low water near the Anglesea shore, called Ynys y Moch (*the pig rock*). Over this place, and between two high rocky shores, a bridge was to be built so as to leave clear waterway between the submarine rock and Carnarvon shore, which are distant nearly six hundred feet, for large vessels to sail under it. From his experiments made to gather data for a proposed large suspension bridge at Runcorn, Telford had obtained some acquaintance with bar-chains, and the strength of iron in that condition; and as the only available means of crossing so formidable a spot he proposed a suspension bridge. The work was commenced by the erection of two towers of a pyramidal form, one on the Moch rock, and the other also on a rock forming the Carnarvon shore, but to reach which a slight excavation of soft earth was necessary. The towers were constructed of Anglesea marble, and carried up partially hollow, as far as the intended level of roadway, which is 102 feet above high water: at the base these towers have a width (transverse to the roadway) of 70 feet, decreasing to 45 feet at their summit; in the direction of the roadway their thickness at base is 50 feet, at the road level 29 feet, and at the top 11 feet. Between these pyramids and the Anglesea shore are four arches, each 52½ feet span, resting on very tall piers (65 feet high); on the Carnarvonshire side there were three similar arches between the pyramid and elevated shore. If this masonry merits attention on account of its bulk and height,—for the piers are 153 feet high,—it cannot be said to do so on account of any tasteful disposition; as a piece of architecture it contrasts strikingly with the simple and elegant stonework of Freiburg Bridge.

While progress was made in this part of the bridge, preparations for the chain-work were quickly proceeding; the parts composing each chain were links of wrought iron 10 feet long, 3¼ inches broad, and 1 inch thick; at both ends an eye or hole was bored to admit a bolt three inches diameter, and weighing 56 lbs.; with this a series of links were united to the next. Five of these bars, side by side, form one link of a chain, and are connected with the next row or link by coupling plates or small links *a*, No. 1, 16 inches long; when a chain was completed it consisted of 935 bars, was 23½ tons in weight, and 1710 feet in total length. Every bar was dipped in linseed oil and dried before it was used for the chains, to protect against oxydation. In erecting the chains a platform was laid between each tower-top and the chain tunnels in the rocky shores, and on it was placed a proper series of links, of five bars each, to form a chain, which were thus constructed as far as the towers, but on the Carnarvonshire side sufficient chain was made to reach over the tower and hang down to the water surface, and was there met by a large float, bearing a length of chain sufficient to reach across; this being bolted to the hanging end, was then drawn over to the Anglesea tower, and the remaining extremity of chain gradually lifted up until it reached the portion on the Anglesea platform, when, being properly joined, the whole was drawn with sufficient force to bring up the great curve to its proper elevation. A similarly simple operation was adopted for the remaining chains, which, after passing over the towers and reaching the road level, disappear in tunnels cut through the rock; their fastenings can be seen on the Anglesea side by walking along a small tunnel also cut through the rock for a considerable distance; they are very simple, since the rock affords abundant natural masonry to resist the pull of the chains; the last link, which is large, carries in its

Fig. 112.



eye a very stout and long bolt, the ends of which rest on an iron saddle abutting against the rock; that this is an effective fastening several severe storms have borne witness. The chains do not rest immediately on the towers, there being cast-iron saddles, movable on small rollers, for a short distance, to and fro, which rest on a cast-iron cap or table fixed to the masonry; thus all expansions and saddle movements are transmitted without affecting the towers. The suspension chains are sixteen in number, and divide the whole width of the bridge into three lines; the central, which is for foot passengers, being four feet wide; and the outer, each twelve feet wide, for general traffic; thus the whole working width of the bridge is 28 feet. Four chains constitute one row, having one line of suspension-rods; the latter are bars an inch square; placed 4 feet 11 inches apart, which gives in one row 199, or in the four, 796 suspending rods; their lengths vary from 56 to about 7 feet, for the chains do not descend to the roadway at the middle. To adjust these sixteen chains four adjusting links were put in each chain, two between the towers, and one in each backstay or land curve; they were merely bars with long eyes, and having coupling plates *a*, No. 2 (*fig.* 112), with a similar eye, wedges might be driven to effect any little adjustment of length. To prevent undulation and oscillation transverse and diagonal ties were added between the chains; the former were cast-iron tubes reaching from one row of chain to another, and placed at intervals, while between them wrought-iron straps tied up the corners. On estimating the chainwork it appears there are 5 × 16 = 80 bars in a cross section, which present a total surface of 80 × $3\frac{1}{4}$ = 260 square inches. From Telford's experiments (see p. 216) we learn that a square inch of section will bear a strain of 27 tons without breaking, but only 9 tons without at all injuring the powers of the iron; taking the latter as a safe working value, the strength is 9 × 260 = 2340 tons: this far exceeds both the constant load of the chains and roadway, and any possible passing weights. According to Mr. Rhodes, who made some experiments to find the tension on the chains with a deflection equal to that of the bridge, this tension will be 1·7 times the load, which is estimated as follows:—Sixteen main chains = 394·25 tons; transverse ties = 3·83 tons; and suspending-rods, with roadway, &c., = 245·68 tons; making in all 643·76 tons of constant load; this being multiplied by 1·7, we obtain 1094·4 tons for the actual strain; so that there is a free available power of 2340 — 1094·4 = 1245·6 tons, which being divided by the former ratio of increased tension, through the flatness of the curve (1·7), shows that 732 tons might be safely placed upon it.

This bridge took six years and a half in building, and seven days after it had been opened a storm broke several of the suspending-rods and iron roadway bearers; again in January, 1836, just ten years after its completion, a severe gale so racked the bridge as to make the roadway rise and fall in successive undulations to the terrific amount of sixteen feet: some of the roadway planks were jammed against the Anglesea tower, and rods broken. This was trifling in comparison with the trial it underwent in January, 1839. Mr. Provis thus relates the circumstance:—"The storm, according to the account of those who observed it at the bridge, commenced on the evening of the 6th, with a strong but unsteady wind veering a few points on both sides of S.W. The gale increased during the night so much that between two and four o'clock of the following morning it became a hurricane, and all approach to the suspended part of the bridge was impracticable. The fury of the storm having abated after four o'clock A.M., the bridge keeper made an attempt to examine the chains and roadways; it was with difficulty he could make his way to the platform, and it was only by watching the lulls of the wind and holding fast by the iron work that he was enabled to reach the suspended part of the bridge. The general darkness was too great and the tempest too violent to allow him to make any close or accurate observation, but he ascertained, during occasional gleams of moonlight, that the roadways were broken through, and that it was necessary to take immediate measures for preventing any attempt to cross the bridge."* Daylight revealed the damage; one-third of the suspending rods had been torn asunder; the roadway on the south side was torn up in one place, and on the north side in two places, so that a piece 175 feet long, and 12 wide, was hanging towards the water and swinging about in the wind; much of the parapet railing had been blown into the straits; also the ties and tubes between the chains had been destroyed, for although the main chains had admirably borne this fierce assault, it appears that they oscillated most violently, since two or three of the powerful screw bolts connecting the chain links had their heads knocked off by collision with the neighbouring chain. Only three common bars and one adjusting link had been injured, by breaking at the eyes; had these links been so large and long as those in the Hungerford Market Bridge, much more damage might have happened in this way. In restoring the injured parts, care was taken to modify the arrangements in some places, according to the experience gained through the accident. Previously the suspending rods had but one joint at the extremity, namely, where it took hold of the pair of iron bars forming one road bearer, and this joint was such that it permitted of motion in a transverse direction only; this allowed for any horizontal oscillation of the chains, but offered a positive resistance to any great vertical undulation; thus many of the rods had been broken at the roadway; to obviate this in future, another joint, allowing of longitudinal motion, was made immediately above the roadway. Again, the platform bearers were constructed most curiously by Mr. Telford; they were made of two lengths in each bar, one of 16 and the other of 12 feet, and one of 12 was placed opposite another of 16 feet; any motion, transverse or in the direction of their length, could thus only tend to break them asunder at the joints: these were replaced by opposite joints, which permit each roadway to move freely of the other. A layer of planking, three inches thick, was added in the platform, and a longitudinal beam underneath, to afford greater stiffness.

---

* Transactions of the Institution of Civil Engineers, vol. iii.

Considering the early date of its erection, the Menai Bridge is a remarkable object; its great dimensions well accord with the locality, and the curve of the chains, at all times a beautiful object to the eye, lends to it a grace which makes it an ornament to the noble strait and its charming woody banks; to this effect, however, the side arches add nothing, as, like the pyramids, they are by no means pleasing or well formed. As might be supposed, this bridge is affected by changes of temperature; in winter the roadway is nearly a foot higher than in summer, causing the saddles on each pier-top to recede from each other for a distance of about an inch and a half; this must produce some variation in the angle of deflection, which, it appears, is not equal on both sides of the piers at any time, for Mr. Hodgkinson* remarks that on taking the inclination of the first link each way from the saddle, he found the angle on the water side to be 16° 10′, and that on the land side 18° 3′; this difference, he has estimated, must produce an horizontal force of 68 tons tending to push the towers inland; to meet this, however, they have their own enormous bulk, and the help of three or four side arches.

The following is a summary of the principal dimensions of the Menai Bridge:—

			ft.	in.
Length of suspended roadway			551	0
Deflection of the main chains			43	0
Angles of deflection at towers	of middle span	16° 10′		
	of side spans	18° 3′		
Width of roadway			28	0
Height above high water			102	0
Chain links—length			10	0
„ section				3¼
Total suspended weight		643 tons	15½ cwt.	

Another suspension bridge, on a similar plan, was erected by the same engineer on the same line of main road between Chester and Holyhead, at Conway; there was a ferry previously, being a most inconvenient mode of crossing the river Conway, which falls into the sea a little northward of the town. This bridge is 327 feet span, 22½ feet deflection, and supports a roadway 15 feet above high water; corresponding to its lesser size, there are but eight chains. It stands so near the splendid Conway Castle as to give the impression of being a drawbridge on a distant view, especially as its masonry is architecturally designed to accord with that of the castle; this also proved a great ornament to the delightful spot in which it is situated, before the railway obtruded with its gigantic tubular bridge.

---

* Transactions of the Literary and Philosophical Society of Manchester, vol. v., 2nd series.

3. Hungerford Market Suspension Bridge.—This is the longest bridge of the kind erected in this country, and the greatest span: it is confined to foot passengers, although with a better disposition of the iron at present used, it would have been strong enough for all purposes. It is 1352½ feet long, and the central span 676½ feet, or 106½ feet greater than the Menai Bridge. Its two suspension towers are built out in the stream, and the backstays or land curves have to support the roadway between the towers and shore; they are 80 feet high, and allow 50 feet deflection to the chains, or about 1-13·5th of the span. There are four ponderous lines of chains, each consisting alternately of ten and eleven links or bars, except near the towers, where they become eleven and twelve in a row; the links are no less than 24 feet long, 7 inches broad, and 1 inch thick, and weigh 5½ cwt. each; about 2600 of these links are employed in the whole bridge, weighing 715 tons. The pair of chains on each side of the footway are placed one above another, and rest on saddles, which move on 50 friction rollers, with a play of eighteen inches each way. An ingenious and simple contrivance was adopted to throw the chains across from tower to tower: as the river at this point is alive with traffic, any scaffolding would have been a great impediment; the engineer (Mr. Rennie) therefore laid two pairs of wire ropes over one side of the piers, each pair at such a distance apart that the chain bolts, which connect one row of links to the next, should rest on them by their ends; two more ropes were then hoisted in a similar way above the former, to carry a kind of open box containing two men and a windlass, by which instrument the men drew up links from the boats below, and having joined them by bolts, they constructed a line of single links from one tower to the other, and, when fixed, no longer required the wire-rope support; the elementary chain thus formed became a support in drawing up and adding other series of links until a complete chain was made; a similar operation on the other side finished this part of the work. Suspending rods descend from the chains at every twelve feet; at their upper extremity they have several joints, the first being that of the rod with a solid iron triangle; the rod is jointed to one angle, while from the other angles two pairs of bars reach upwards—one having an eye, through which the main chain bolt passes, the other pair join above with the second pair of bars, which are screwed at their upper extremity to a flat bar, laying on the upper chain; each suspending rod is therefore hung on both chains: at its lower end it passes between the transverse pathway supports, and through the main longitudinal beam, under which it is keyed. No transverse play appears to be allowed for in this construction,—a circumstance which in the Menai Bridge proved the destruction of many suspending rods, with part of the roadway.

This bridge is certainly an ornament to the river, but a closer observation of its parts will cause a little surprise either at its curious arrangement or unscientific character; it appears to be, in several respects, a product of mere empiricism. It is difficult to conceive to what end is an alternate succession of ten and eleven links in the main chains; thus, while there are eleven links to sustain the tension at one point, at the next above it there are ten links to sustain a greater tension; if the ten be sufficient, as undoubtedly they are, the other is an extra, serving no purpose except that of loading the bridge, and assisting every injurious movement. Its superfluity of strength is another striking feature; at the piers it is stated to have a section of 312 square inches, which, taking 9 tons on the square inch as the constant load which can be borne without the least injury to the texture of the metal, gives a strength of 2808 tons; but on a calculated estimate the whole weight of the span, chains included, cannot exceed 530 tons, and, allowing this extreme amount, it appears that with a deflection of 1-13·5th, which is that of this bridge, a strain of 1·83 time the load must be allowed for, so that the tension on the upper links is 530 × 1·83 = 966½, or, we may say, 970 tons; this leaves a surplus of 2808 — 970 = 1838 tons for the few foot passengers which, at a push, might be crowded on a way only fourteen feet wide, whereas the Menai Bridge, made for all kinds of traffic, and in a far more trying position, has but 1245 tons free strength for both foot passengers and heavy merchandise. This bridge surpasses all others in the length of its links:—they are 10 feet in the Menai, 15 feet in the Union Bridge, near Berwick, and 15 feet 8 inches in the Pont des Invalides, at Paris; but the Hungerford are above eight feet longer than the latter. It is difficult to conceive wherefore they are made so gigantic, but it is certain that their great length enables them to strain the bolts and suspending rods much more than usual, while it obstructs them in vibrating, during heavy gales, in that steady manner which prevents many ill effects, although the undulations may be, as they can be observed, irregular both as to time and place: this is a consideration, especially with such large bars; for an awkwardly-moving lump of five and a half hundredweight—which each of them weighs—must be calculated to distress some part of the work. How much better it would have been to have given a variable section according to mechanical principles, to the saving of expense, and lightenment of the structure. The fanciful style of the towers cannot be praised, any more than acanthus leaves or other niceties about the piers of solemn arches: the Doric suspension piers of Freiburg must excite more admiration than the airy Italian brickwork of Hungerford Market; but we are to allow, in this particular case, for the character of the locality, as it must be admitted a prominent specimen of noble architecture would be a singularity in comparison with the bank scenery.

The following table supplies the numerical account of its parts:—

	Feet.	
Middle span	676½	
Deflection of chain	50	
Length between abutments	1352½	
Height of roadway in the middle	32½	
,, ,, at the piers	28½	
Width of platform	14	
Piers in height	80	
Number of links	2600	
Weight of ditto	715	tons.
Sections of chains at lowest point	296	square inches.
,, ,, piers	312	,,

Descriptions of many excellent suspension bridges might be added if space permitted; they have rapidly increased in numbers within the last thirty years, though little has been generally done to improve them on scientific principles. Below are a few short notices of several of the common form.

4. **Niagara Bridge.**—In 1847 a spot was selected for this structure two miles below the celebrated Falls, where the rushing stream is 400 feet wide, its banks being 230 feet high, and 800 feet apart at the top. Over this terrific chasm a kite was flown which carried a small string; by the latter a cord was drawn over, then large ropes, and finally wire cables an inch and a half in diameter; these are hung over towers of wood, on each bank, 57 feet high, and pass thence into the solid rock, where they are fixed. This bridge forms a communication between Canada and the United States; it is of great social interest, and no less remarkable as a daring enterprise.

5. **Pesth Bridge.**—This fine specimen was begun in 1840, over the Danube, and connects Pesth with Buda (or Ofen). Great difficulty was encountered in its erection, on account of the deep water and ponderous icebergs which may sometimes be seen of a mile in length floating down this noble river after the cold season; the piers were founded in enormous coffer-dams—so large as to accommodate 5000 persons on laying the first stone; they are 200 feet high from their foundations, the lower portion being of granite; they stand out in the stream, giving the suspended roadway a span of 670 feet in the middle, and two shore

spans of 297 feet; it is also 42 feet wide for foot and carriageways, and hangs 50 feet above low water. The chains have a section of 520 square inches, which must supply a superfluity of strength, and weigh 1300 tons. This bridge was opened with a melancholy procession—if such it may be called—the hasty retreat of the unfortunate Hungarians from their Austrian pursuers. Afterwards the bridge was well nigh being destroyed by the contending parties during the same war, in which the inhuman conduct of the Austrians has earned for them an unenviable notoriety in the history of the age. On one occasion 60 lbs. of gunpowder was placed on it as a destructive charge, and although the person who officiated was blown to pieces, the bridge sustained little injury. The engineer was Mr. Tierney Clark.

6. THE UNION BRIDGE over the Tweed, near Berwick, was an early specimen of Capt. Brown's patent system of suspension-bridge building. Round rods, 2 inches in diameter, and 15 feet long, form the chains, which are twelve in number, and arranged in three rows. The chord line of suspension is 449 feet, and the deflection 30 feet, or nearly 1-15th of the chord; but the roadway is suspended 387 feet, the remaining portion resting on a projecting abutment. The roadway is only 18 feet wide. It was opened in 1820.

7. HAMMERSMITH BRIDGE is an elegant structure, with a middle span of 422¼ feet, and two side waterways of 143 and 145½ feet respectively. The chains are of links 8 feet 10 inches long, and in four series—two on each side of the whole roadway, with three links in a row; and two dividing the carriage and foot ways, of six links in a row: suspending rods depend from them to transverse iron girders, on which a longitudinal flooring is laid. The roadway is 30 feet wide, of which a carriageway occupies 20 feet, leaving 5 feet on each side for a footpath. The chains have a deflection of 29½ feet, or 1-14·3, which brings them in the middle to the level of the roadway. Admirable stone piers, in the Tuscan style, with entrance arches, bear the chains on a carriage, with cast-iron friction rollers, from which they descend in an abutment of 2160 tons of masonry, which, however, cannot be called the resisting power of the abutment, since the weight does not act vertically against the chains. This bridge was an early work of Mr. Tierney Clark, and was opened in 1827.

8. THE PONT D'ARCOLE at Paris and a bridge in the Island of Bourbon are of a class which possesses considerable scientific advantage. Instead of two suspension piers, as usual, one only is erected midway between the river banks, and from this pier the chain descends each way to a level, or nearly so, with the roadway: not only is there less tension, but much oscillation is prevented; this, in the Island of Bourbon, is of great consequence, as it is a notable locality for violent hurricanes; to assist the bridge against them, the engineer, Mr. Brunel, attached four inverted catenaries under the roadway, and when any upward rush of air occurs they will exercise a salvatory influence over the framework. Each span of the Bourbon Bridge is 131¾ feet, and the central tower 25 feet above the roadway; the Pont d'Arcole has a great deflection, with a span of 125 feet.

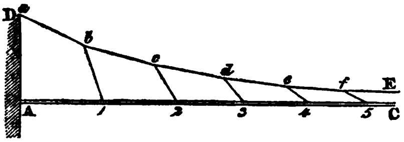
9. MONTROSE BRIDGE, over the South Esk, in Forfarshire, is a pretty structure of 432 feet in span, 42 feet deflection of the chains, and 26 feet wide. There are two chains, one above another, on each side of the passage-way, each of which is four bars wide; the bars being 5 inches wide, 1 inch thick, and 10 feet long. It was built by Capt. S. Brown in 1829; and in October, 1838, during a heavy storm, it suffered the loss of a part of the roadway, at the middle of the bridge, apparently from the want of joints at the extremity of the suspending-rods, in addition to inefficient stiffening of the roadway. The former cause is similar to that which broke the roadway of the Menai Strait Suspension Bridge (see p. 222).

A dangerous proposal was made some time since for a bridge over the Haslar Lake, Portsmouth, to be suspended by cast-iron chains; they were to have a section of 256 inches, which, with a span of 300 feet, and a deflection of about 33 feet, it was estimated would have to support, as the greatest possible load, 991·5 tons, and leaving a free power of 800 tons, when the strength is taken at 7 tons per square inch. To show the utter absence of all science and safety in this or any such structure, it is sufficient to mention that 7 tons is the ultimate strength, or that at which the material is crushed; and supposing we might venture to take one-half of this amount as a safe working-strength, we should find that the strength would not be equal to that required for the greatest load. Besides, in a locality where the regular tread of soldiery is frequent, the structure must be in imminent danger were it much stronger than the proposer imagined: we have now a sad collection of evidence to show the peril of that which depends for safety on the tensile strength of cast iron.

10. There are several interesting examples of wire bridges. Tournon Bridge, over the Rhone, is in two spans of 278¾ feet each; there are six cables of 112 wires each, the wires being 1-9th inch diameter; the curves have a deflection of 1-11th of the span, or about 23½ feet. As the platform is only 13¾ feet wide the cables were proved with a weight of 200 tons, but they were shown to be capable of bearing 443¾ tons before breaking.—The bridge at Argentât, over the Dordogne, a river watering a department of that name in France, is 328 feet in span; the platform is supported by six cables on each side, composed of from 186 to 216 wires each; each cable has a different deflection, the deepest being 26½ feet, and the flattest 22 feet.—At Pittsburgh, over the Alleghany River, U.S., a light aqueduct was lately erected 1140 feet long, in seven spans of 160 feet each; it was built to hold a depth of four feet of water, or, altogether, 2100 tons of water continually passing over it; to support this there are two cables, 7 inches in diameter, one on each side of the aqueduct, having a total section of 53 inches: this is small, but affords abundant strength, as the deflection is 14½ feet, or 1-11th of the span.

(104.) Many bridges have been classed under the title of suspension bridges, although presenting more or less difference from the simple structure of a roadway held up by a swinging chain. Probably the most conspicuous of these modifications is that of Mr. Dredge, who, some years ago, proposed a new arrangement for a bridge at Bath, which was adopted; and, since the erection of that specimen, several of small span have been thrown across rivers and ornamental waters. The consequential characteristics claimed by Mr. Dredge, are lightness of chainwork and roadway, to the saving of expense, and the readiness with which bridges on his plan can be put up in their place. As some dispute has arisen concerning the mechanical features of the system, owing, in most instances, to an apparent want of information on the part of the disputants, it would be desirable to give Mr. Dredge's own explanation of his ideas; but they are unfortunately clothed in mathematical language, which of itself would be unacceptable to the general reader, were it not rendered useless as it is by its confused statement. The three specific characteristics of this bridge are—rapidly tapering chains, inclined suspending rods, and a roadway made stronger than usual; it presents somewhat of the following figure, which shows that it is similar to a bracket; for, if we suppose *f* 5 to be farther inclined, so as to form a prolongation of the link *e f*, A C would become a bracket sustained by three forces, that of gravity in the direction D A, a second in the direction 5 *a*, and a third in the direction of the platform, A 5; consequently, while the chain D E bears part of the load, a portion is thrown on the platform, tending to crush it against the towers; if, however, a number of oblique rods, as *b* 1, *c* 2, *d* 3, be attached to the platform and chain, they will reduce the amount of thrust in the roadway at the farther end, C, of the supposed bracket, although no diminution of pressure is effected at the piers. But another consequence arises from this oblique disposition of the suspending rods, to a far greater extent than with a vertical arrangement: while they throw a force into the roadway, to be resisted at the pier, they also take it away from that part of the chain which lies beyond it; thus, D *a* has to sustain

Fig. 113.



the tension arising from the pull in *a e* and *b* 1, but *a b* is free from the latter force; similarly, *d e* has to sustain a strain from *e* 4 and *e f*, and were the latter identical with *e* 4 that would be the whole strain, but in practice the angle formed by *e f*, *e* 4, is bisected by the direction of *d e*, so that there is equal strain in both directions. The nearer, then, *e* 4 lies in the prolongation of *d e*, the less use is there for any sustaining bar, *e f;* and, as the tension diminishes so rapidly through the adoption of oblique rods, the chains, as they are called, diminish also very rapidly, there being but a single bar, *f e*, at the middle. The opposite bracket, acting similarly to A C, prevents the thrust against the towers, which would occur if one stood alone; as it is, this force consists of a strain tending to tear the platform asunder at the middle, C. Mr. Dredge concludes one of the expositions of his system* as follows:—

"The principal feature of my bridge consists in the oblique suspending-rods, which, by their action, divide the horizontal force from the tension of the chain†, causing the platform to bear the former, whilst the chain has only the latter to resist. The effect of this is—

"1st. To reduce the weight of the platform to a very great extent, for the bulk of the material in the roadway of the old suspension bridge is placed there to produce rigidity; but, on my plan, the action of the horizontal force is far more effectual for that purpose than any system of trussing that can be devised, whilst the weight of the platform to resist horizontal force need not be increased above what is absolutely necessary to resist the transit loads.

"2nd. The tension increases progressively from the centre, where it has been shown to be comparatively nothing, to the base of the chain [at the towers], and the material in the chains varies in the same proportion.

"3rd. Not only is the material above shown saved, but also that quantity which is necessary to support it; then, if in a bridge 500 tons were saved, not only would this quantity be saved, but also perhaps 250 tons more, which were necessary to hold the 500 tons in suspension."

This form of bridge is very simple, although not so simple as the proper suspension bridge, neither is it to be considered as such, for part of the force or load is sustained by compression of the roadway; it thus presents a feature totally distinct from the true suspension bridge, and comparisons between the two can be only partial; Mr. Dredge, however, has abundantly compared his structure with the common form, and, as in the above summary, by a few errors and oversights, discovers the great superiority of his plan. The only alteration which is due to Mr. Dredge is the oblique placing of the suspending-rods; the tapering of the chain to suit any variation of tension in its length, had been plainly set forth long before. By means of these oblique rods, he approaches so nearly to the bracket bridge as to pretend he could cut through the middle of the span and neither half fall: this is not true, since a part of the tension is sustained by the middle link, unless he brought the last oblique rod, *e* 5, into the direction of the link, *d e*, of chain, which, indeed, he allows may be done; but then the bridge is no new invention, as it would be simply the old-fashioned bracket bridge: as it is, the bridges hitherto erected after this plan, exhibit a combination of the bracket and suspension principles.

Several bridges on Mr. Dredge's plan have been erected within the last few years; most of them are small, the largest, as far as information has been afforded, being 250 feet span. The first of the kind was thrown across the Avon, at Bath; a bracket bridge was at first proposed by Mr. Motley, but was finally relinquished for one then, it appears, just conceived by Mr. Dredge, who was an inhabitant of the town, and concerned in the erecting of a bridge at this place. The Victoria Bridge, as it is named, is 150 feet in span. Another bridge at Balloch Ferry, in Dumbartonshire, is 200 feet in span, having 13 bars in each chain at the towers, and one bar at the middle‡; to meet the horizontal strain thrown on the roadway, iron beams of sufficient strength receive the oblique rods on each side the roadway. One of 150 feet span reaches over the ornamental water in the Regent's Park. A bridge built over the Ballee Khal, near Calcutta, was of 250 feet span, but, by some means, as soon as finished, it broke in the middle, and, in despite of Mr. Dredge's decision that his bridges could be cut through the middle without injury, it fell into the water: a somewhat similar accident occurred with another of his bridges in India, which gave way through a sudden rush of a number of people, and fell also into the water; in this case, the roadway appears to have given way, an accident which could scarcely have happened to a proper suspension bridge, and certainly would have been followed by no destructive consequences.

---

* Practical Mechanic and Engineer's Magazine, vol. iv., p. 179.

† This is a confusion of terms; there is no horizontal force, as such, acting on the chains, but the more a catenary is loaded the greater is the tension, part of which, according to Mr. Dredge's plan, is transferred to the platform.

‡ This bridge is reported to have fallen during the transit of some sheep, by a fracture at or near the middle.

A question naturally arises in contemplating this description of iron bridge, as to the advisability of allowing any pressure to be sustained by the roadway, or whether it is well to venture the fate of a bridge on a thin straight line, when, at the same time, that line is connected to a moveable system of jointed rods: Mr. Dredge evidently considers that much stiffness of the platform must arise from the horizontal pressure; it is not so, however, since several specimens of his bridges are very moveable, even when urged by a small force, and were they thus stiffened it would be a dangerous stability, which the slightest wave of the chains—a thing quite unavoidable —would most likely upset; it is in any case a dangerous attempt to associate highly mobile and rigid frames together, especially when the chance of destruction depends on the rigid frame keeping its form; this was felt so strongly by Mr. Fairbairn, as to lead him constantly to oppose Mr. Stephenson's idea of adding chains to the tubes for the tubular bridges (see p. 184). In small specimens, such as those hitherto executed, the ill effects of the conjoint system are not so striking, but much apprehension might justly be entertained if it were applied to such a bridge as that over the Menai Strait; and when we contemplate the effects of the storm, in 1839, on that fine suspension-way (see p. 222), when a great portion of the roadway was broken up, it must create great alarm to suppose one of Mr. Dredge's bridges in that situation, and similarly tried: nothing could save it from following the example of those over the Ballee Khal and at Junguruchy.

For small spans, not in dangerous situations, this form of bridge is very well calculated, on account of its cheapness, lightness, and readiness of construction; but it appears not to be safe for those places where the true suspension bridge is found to be of eminent value.

The following **Table** exhibits the Measurements, as far as authorities supply, of the principal Bridges in the World.

	No. of Arches.	Length.	Span.		Rise.		Width.		Thickness at Crown.		Form of Arch.	Date.
STONE BRIDGES.		ft.	ft.	in.	ft.	in.	ft.	in.	ft.	in.		
Aberdeen, over the Denburn	...	...	130	0	29	0	...		3	0	arc.	1804
Alexandria, over the Tanaro	10	...	95	2	...		24	0	...		ellipt.	1487
Allah-Veirdy-Khan, over the Zendeh Rud, at Ispahan	29	2378†	64	0	...		52	6	...		arc.	...
Ashiestrel...............................	...	...	131	6	26	0	16	0	4	0	ellipt.	1848
*Blackfriars, Thames, London......	9	930	100	0	43	0	42	0	5	0	ellipt.	1771
Pont de la Boucherie, Pregnitz, Nuremberg	...	...	96	10	12	8	39	0	4	0	arc.	1599
‡Brioude, Allier	...	...	150	0	...		...		...		...	...
Castel Vecchio, Adige, Verona ...	...	...	159	10	53	3	...		...		ellipt.	1354
Ceret, Tech, near Perpignan	...	...	147	7	73	9	...		5	3	semic.	1336
Claix, Drac, near Grenoble	...	...	157	6	...		20	0	...		arc.	1611
*Chester (new), Dee	...	...	200	0	42	0	33	0	4	0	arc.	1832
Cestius, Tiber, Rome	...	...	78	9	...		48	7	...		arc.	380
De la Concorde, Seine, Paris	5	...	101	3	9	9	50	0	3	9	arc.	1791
Dunkeld, Tay	7	557½	90	0	30	0	27	6	3	2	arc.	1809
Dean, Water of Leith, Edinburgh	4	447	90	0	30	0	39	0	...		arc.	1831
Emilius (*Ponte Molle*), Tiber, near Rome	7	414	77	6	...		28	9	...		semic.	B.C. 100
Du St. Esprit, Rhône	25	2680	107	0	...		14	6	...		arc.	1305
Elbe, Dresden	16	1490	...		...		...		...		...	...
Fabricius, Tiber, Rome	2	...	82	0	...		48	7	...		arc.	1680
§Fo-ken, over the Min, China......	100	26,033	127	0	...		52	6	...		semic.	...
Gignac, l'Erault	3	...	159	8	53	1	...		6	3	ellipt.	1793
*Gloucester, over the Severn	...	...	150	0	35	0	25	0	4	6	ellipt.	1828
Guillotière, Rhône, Lyons	18	1870	105	0	...		...		...		arc.	1245
High Level, Tyne (railway).........	6	...	125	0	...		...		...		arc.	...
Lavaur, Agout	...	...	159	10	64	8	38	0	9	6	ellipt.	1775
*London, Thames	5	785	152	6	37	10	56	0	5	0½	ellipt.	1831
§Loyang, over an arm of the sea, at Fo-kien, China..................	300	28,876	80	0	...		72	0	...		flat.	...
Mantes, Seine	3	429	127	0	37	3	34	0	6	3	ellipt.	1765
Martorell, Spain	sev^1	...	133	0	...		...		...		gothic.	...
St. Maxence, Oise	3	...	76	8	6	5	41	0	4	9	arc.	1785
*Neuilly, Seine, near Paris	5	690	127	0	31	6	65	0	5	3	ellipt.	1774
Nogent, Seine	2	...	96	0	28	9	...		4	3	ellipt.	1769
Neuville, Ain	2	...	96	2	...		...		...		ellipt.	1775
*Orleans, Loire	9	1108	106	6	34	9	48	0	6	0	ellipt.	1760
Pont Neuf, Seine, Paris	12	918	63	4	...		68	0	...		semic.	1607
*Pont y Prŷd, Taaf, Glamorganshire	...	...	140	0	35	0	...		3	0	arc.	1755
Prague, Moldau	16	1705	76	6	...		35	6	...		arc.	1638
Runcorn, Mersey	...	...	150	0	20	0	...		5	0	arc.	1831
Rialto, Great Canal, Venice.........	...	...	97	0	20	7	66	0	...		arc.	1578
Salara, Arno..........................	...	...	95	10	...		...		...		arc.	...
Senatorius (*Ponte Rotto*), Tiber, Rome	...	...	80	0	...		...		...		...	...
Tournon, Doux........................	...	...	156	9	65	0	16	0	...		arc.	1545
*Turin, Dora	...	...	147	8	18	3	40	0	4	11	arc.	...

* Those with an asterisk prefixed are more or less described in the preceding chapters.

† Chardin gives the length at 2177 feet, but there are various measures extant.

‡ Formerly a very large arch occupied the site of this bridge; it was 189 feet in span, and was built in 1454; it fell a few years since.

§ These colossal specimens of Chinese perseverance allow of vessels passing under them; that at Fo-ken is 127 feet above the water, with piers almost as thick as the arches are in span. Loyang Bridge, nearly 5½ miles long, took eighteen years constructing, and employed 25,000 men.

	No. of Arches.	Length.	Span.		Rise.		Width.		Thickness at Crown.		Form of Arch.	Date.
		ft.	ft.	in.	ft.	in.	ft.	in.	ft.	in.		
*Most Holy Trinity, Arno, Florence	3	...	95	9	15	2	32	8	3	2	ellipt.	1569
Têtes, Durance, near Têtes	...	...	112	6	...		15	0	...		semic.	1732
Tours, Loire	15	1460	80	0	26	0	45	0	...		ellipt.	1762
Tongueland, Dee	...	...	118	0	38	0	...		3	6	arc.	1806
Verona, Adige	3	...	160	0	...		...		...		ellipt.	...
Vicenza, Bacchiglione	...	...	100	0	29	10	...		...		arc.	...
Vizile, Romanche (road from Grenoble to Briançon)	...	...	137	0	38	4	...		5	5	ellipt.	...
*Waterloo, Thames, London	9	1240	120	0	35	0	42	0	4	6	ellipt.	1816
BRICK.												
Carbonne, Garonne	3	...	101	4	39	3	25	6	4	2	ellipt.	1770
*Maidenhead (Great Western Railway Bridge), Thames	2	...	128	0	24	3	...		5	3	ellipt.	...
Toulouse, Garonne	7	...	111	9	42	9	35	0	3	0	ellipt.	1632
WOOD.												
Bamberg, Regnitz	...	...	208	0	17	0	30	0	...		rib.	1809
*Bassano, Cismone	...	...	108	0	...		...		...		polyg.	...
Bredon, near (Birmingham and Gloucester Railway)	...	...	117	0	...		17	6	...		lattice.	...
Pont de la Cité, Seine, Paris	2	...	101	0	5	5	29	0	...		rib.	1802
Elysville, Patapsco River, U.S. (Baltimore and Ohio Railroad)	...	...	150	0	...		...		...		tie-bm.	1838
*Kandel, over the, in Berne	...	...	166	0	...		...		...		polyg.	1764
*Ladykirk and Norham, Tweed	...	...	190	0	17	0	18	0	1	6	rib.	1807
Mellingen, Reuss	...	...	157	6	...		...		...		rib.	1794
*Kandy, near Mahavillaganga, Ceylon	...	...	205	0	25	0	...		...		rib.	183-
Mulatière, Saône, Lyons	11	...	55	9	...		...		...		polyg.	...
*Philadelphia, Schuylkill, U.S.	...	...	340	0	20	0	...		...		rib.	1813
Do. do. Market Street	3	...	194	0	12	0	42	0	...		rib.	1805
Portsmouth, near, Piscatagua, U.S.	...	...	250	0	...		...		...		rib.	1794
*Richmond, James River (railway bridge), U.S.	19	2900	153	0	...		...		20	0	lattice.	1838
Scharding, Rott	...	...	190	0	18	7	25	0	...		rib.	1809
Susquehannah, over the, U.S.	10	2200	220	0	...		...		...		lattice.	...
Tournus, Saône	5	...	89	6	...		...		...		rib.	1801
*Trenton, Delaware, U.S.	5	...	200	0	...		...		1	6	rib.	1804
Ulm, Danube	...	...	181	0	22	3	...		6	1	rib.	1806
Vilshofen, Vils	...	...	177	0	11	0	26	0	...		rib.	1809
Wirtemberg, Rhine	...	...	64	4	...		...		...		rib.	...
Zurich	...	...	127	0	...		...		...		rib.	...
IRON.												
Austerlitz, Seine, Paris	5	...	106	0	10	6	...		4	10	arc.	1806
Bonar, Dornoch Firth	3	...	150	0	20	0	...		3	0	arc.	1812
Bristol, Avon	2	...	100	0	12	6	30	0	...		...	...
*Bishopwearmouth, Wear, near Sunderland	...	...	236	0	34	0	...		5	0	arc.	1796
†Bordeaux, over the Garonne	17	1588	86	0	28	9	47	0	...		arc.	1819
*Buildwas, Severn, Shropshire	...	...	130	0	...		...		...		arc.	1777
Boston, Witham	...	...	100	0	4	0	...		...		arc.	...
Carrousel, du, Seine, Paris	...	558	187	0	16	6	35	0	...		arc.	...
*Colebrook Dale, Severn, Shropshire	...	...	100	6	...		...		...		semic.	1777
Craigellachie, Spey, Invernesshire	...	...	150	0	20	0	...		3	9	arc.	...
Crown Point, Aire, Leeds	...	...	120	0	12	0	22	0	...		arc.	...
Galton, over Birmingham and Gloucester Canal	...	...	180	0	18	0	...		...		arc.	...

† This bridge was anciently of wood, afterwards stone piers were supplied, and in 1819 the wooden were supplanted by iron arches.

	No. of Arches.	Length.	Span.		Rise.		Width.		Thickness at Crown.		Form of Arch.	Date.
		ft.	ft.	in.	ft.	in.	ft.	in.	ft.	in.		
Hunslet, Aire, near Leeds	...	...	152	0	35	0	38	0	...		arc.	...
Sawley, near, Trent, Derbyshire (railway)	3	...	100	0	10	0	27	0	...		arc.	...
*Southwark, Thames, London	3	...	246	0	23	6	...		6	0	arc.	1819
*Tewkesbury, Severn	...	...	170	0	17	0	24	0	3	3	arc.	...
*Britannia, Menai Strait, near Bangor	4	1523	460	0	...		...		...		tubular.	1850
*Conway, Conway River	...	...	400	0	...		...		...		tubular.	1848

SUSPENSION BRIDGES.	Central Span.		Deflection of Chains.		Width.		No. of Chains or Cables.	Section.	Date.
	ft.	in.	ft.	in.	ft.	in.			
Argentat, over the Dordogne (cable)	328	0	24	3	13	9	6+6	28·5	1829
*Balloch Ferry, Leven, Dumbartonshire	200	0	21	0	20	0	1+1	...	...
*Conway, River Conway, N. Wales	327	0	22	4	...		1+1 1+1 1+1 1+1	130·0	1826
Dryburgh, over Tweed	260	0	$\frac{1}{10\cdot3}$		...		1+1 1+1	...	1818
*Freiburg, Sarine, Switzerland (cable)	807	0	65	0	...		2+2	...	1834
*Hammersmith, Thames, near London	422	8	29	6	30	0	1+1 1+1 1+1 1+1	180·0	1827
*Hungerford Market, Thames, London †	676	6	50	0	14	0	1+1 1+1	...	1845
*Invalides, Seine, Paris	221	0	26	4	25	8	4+4	41·6	1829
*Jarnac, Charente	229	6	19	8	...		6+6	39·5	1827
*Menai, Menai Straits, N. Wales...	579	10½	43	0	28	0	1+1 1+1 1+1 1+1 1+1 1+1 1+1 1+1	260·0	1826
*Montrose, South Esk	432	0	...		26	0	1+1 1+1	...	1829
Norfolk, New Shoreham	284	0	20	2	28	6	3+3	84·0	...
*Pesth, Danube, Hungary	670	0	...		42	0	...	520·0	1840
Tournon, Rhône (cable)	278	9	$\frac{1}{11}$		13	9	6+6	...	1825
Union, Tweed, near Berwick ...	387	0	30	0	18	0	2+2 2+2 2+2	37·68	1820
Vienna, Danube	324	0	21	5	11	10½	1+1	15·5	1825

Note.—The figures in the fifth column are placed so as to represent the disposition of the chains and cables; the separate figures, when added together, giving the number of chains or cables in the bridge.

---

† This is also called Charing-Cross Bridge.

THE
STRENGTH OF MATERIALS.

CHAPTER I.—*General Notice.—Cohesion.—Elasticity, and its Modulus.*

No subject in practical mechanics can claim so eminent a position in point of every-day value as the passive ability, relative and absolute, of materials used in constructions to meet those various forces, exerted in many ways, which they are required to sustain. Its value is fundamental: no working view of any matter connected with machinery or architecture can have much worth except a knowledge of the natural qualifications possessed by the substance concerned be tolerably well comprehended; and generally in proportion to the time devoted to understanding the comparative power of materials, is the safety of a structure increased, and the cost of labour and materials diminished.

(1.) In the times when experiment was rare, and dogmatic hypotheses usurped the place of fact, little light can be expected to have fallen on a point so obviously experimental as the strength of materials; the mass of facts now known, although by no means equal to the formation of a system, is principally due to the indefatigable labours of late years. Years since, Galileo, Mariotte, Leibnitz, Jas. Bernoulli, and Euler, gave their theoretical views on the strength of beams and columns, under different kinds of strain; while Musschenbroek, Buffon, Emerson, Robinson, and many others, presented a mass of experimental results, marked, however, by singular disagreements. Later experiment and theory have been pursued with greater consistency by Messrs. Rennie, Brunel, Barlow, Hodgkinson, Fairbairn, and others. Their conclusions, and the application of them to practical mechanics, will constitute the present treatise.

(2.) Unfortunately the theorist has little ability in handling the direct question of strength in general cases, as the important power of matter—cohesion—the groundwork of such an inquiry—is yet somewhat a mystery in physics. Modified developments of this force present the common phenomena of elasticity and plasticity, hardness and softness, brittleness and ductility, all dependent on the attraction and repulsion existing between the atoms of matter or centres of force: these attractions and repulsions do not appear to be amenable to any ascertained law, though it is generally considered that the cohesion is proportional to the straining force; but as bodies are imperfectly elastic, and very small strains are found to impair their elastic power, it is not strictly true in practical cases. To add to the difficulty of discovering any general law whereby the strength of a substance may be predicated, we find the force of cohesion to be affected by heat and electricity*, so that the only certain mode of learning the effect of forces on different materials is to experiment upon them, and so deduce data for subsequent calculation and application.

(3.) *Elasticity.*—All bodies appear to possess some amount of elastic force while the temporary displacement of the particles does not exceed a certain extent†; when that extent is over-

---

* Mr. Faraday mentions a curious and significant instance of electric disturbance of cohesion in mercury; Exper. Res. in Electr., vol. ii. pp. 156–7.

† From Mr. Hodgkinson's experiments on balls of the following substances, treated in the manner explained at page 9, text and note, he found the following proportional elasticities: perfect elasticity being considered = 1:—

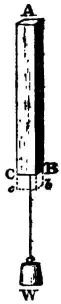
Bell metal . .	·67	Glass . . .	·94
Brass	·41	Iron, cast . .	·70
Clay, baked .	·89	Ivory . . .	·81
—— soft . .	·17	Lead	·20
Cork	·65	Limestone . .	·79
Elm	·60	Steel, hardened .	·79

—(Brit. Ass. for Advancement of Science Reports.)

reached, either rupture takes place, or the particles have no longer the ability to return towards their original position. The limit is soon passed, as might be supposed, since this property is a direct consequence of the particles of matter possessing minute circles of cohesive attraction and repulsion, the former of which may be easily exceeded, and the latter, through an alteration in the relative position of some of the particles, prevents a return to the original shape. It is highly probable that no body can be compressed or extended in any degree without causing some change in the relative places of its particles, or producing the effect called defect of elasticity. The experiments made by Mr. Hodgkinson on cast-iron beams, deflected by small weights, testify in favour of the supposition: so that the common method of estimating a limit of elasticity for the limit of working strength is unnatural.

(4.) For the purpose of calculation it is convenient to have a measure of the elastic power of different bodies, expressed in terms of its own substance or in absolute weight. This measure is called the *modulus** *of elasticity*, each different kind of substance having its modulus. It is estimated by supposing the material to present a square unit of surface, and by any weight or force to be extended to double, or compressed into one-half the original length: such a weight will, of course, represent the modulus. The utility of a modulus is, that it serves as a standard, for no common substance will permit of so great a change in its length as here supposed. When a modulus is thus obtained by calculation it can be used for future calculations of elastic power,—the extension of a body being considered as proportional to the tensile force, by the following consideration:—

Fig. 114.



If the surface of section presented by the body A B C be = 1, or unity, it will require a force (which we may call ϵ) to extend it to twice its length A B; but if it require any weight W to extend it to the length of B b, this proportion of the weights and extensions is at once established:—

$$\epsilon : W :: 2AB : Bb,$$

$$\text{or, } W = \frac{Bb}{2AB} \times \epsilon.$$

Also the extension B b, due to any weight W, is

$$= \frac{W}{\epsilon} \times 2\,AB.$$

The symbol ϵ thus represents the modulus, and may therefore be found from any experiment of extension, since

$$\epsilon = \frac{2\,AB}{Bb} \times W.$$

(5.) The following table presents the late estimates of the modulus of elasticity for various useful substances, expressed in pounds weight, and also in feet, of their own material:—

	lbs.	feet.
Ash	...	4,970,000
Beech	...	4,600,000
Brass	10,440,000	2,460,000
Elm	1,340,000	5,680,000
Fir, red and yellow .	2,016,000	8,330,000
—— white	1,830,000	8,970,000
Glass (window) . .	8,580,000	...
Gun metal . . .	9,873,000	2,790,000
Iron, cast	18,400,000	5,750,000
—— wrought . .	24,920,000	7,550,000
Larch	10,740,000	4,415,000
Lead (cast) . . .	720,000	746,000
Mahogany . . .	1,596,000	6,570,000
Marble, white . .	2,520,000	2,150,000
Oak	1,700,000	4,730,000
Pine (Amer. yellow)	1,600,000	8,700,000
Portland Stone . .	1,533,000	1,672,000
Slate (Welsh) . .	15,800,000	3,240,000
Steel, shear, best, not hardened . . .	20,000,000	8,330,000
Zinc	13,680,000	4,480,000

Several other moduli, such as those of working strength and resilience, have also been named, but they more or less depend on the supposition of an elastic limit in the materials.

---

* *Modulus* (Lat.) signifies a measure of a small thing. The measure meant in the text is rather a *modus*.

(6.) There are four kinds of strain to which materials are commonly subjected in mechanical works: *tension*, *compression*, *detrusion*, and *torsion*; of which the two former comprise most cases occurring in ordinary practice; under them, as consisting of both, may be placed the transverse strain, or the case of a beam supported at one or each end, and pressed by any force between.

Chapter II.—Tension; *its laws; extension of bodies.—Strongest form of Section.—Tensile Strength of Timber: Musschenbroek and Barlow; causes of different results.—Tensile Strength of Cast Iron: early Estimates; Mr. Hodgkinson's Experiments on Hot and Cold Blast.—Minard and Desormes' Experiments.—Wrought Iron: Telford's, Brunel's, and G. Rennie's Experiments; Mr. Fairbairn's trials of Iron Plate.—Wires and Mixed Metals.—Practical reference to forming Joints.*

(7.) TENSION is the simplest form of strain which can affect materials, being opposed by direct cohesion in substances with a regular texture; but in most woods, and many other kinds of useful material, where the fibres are undulating, or twisted, the simplicity of this tensile strain cannot be true: probably owing to this, with many other contingencies, it happens that experiments on the tensile strength of bodies have shown so much disagreement.

(8.) Two facts may be affirmed of bodies subjected to a tensile strain:—

1. The strength or resistance to tearing asunder is as the area of section.

2. The extension of a body under tension is as the straining force.

The first law is somewhat modified by practical difficulties. According to the principle of cohesion it is evident that a rope two inches in diameter will resist a tensile force with four times as much ability as another of one inch in diameter (since the areas are as the squares of the diameters); but that this whole strength may be shown, every fibre in the rope must be stretched, and equally stretched; if the strain be greatest along any side, that side must sustain the whole force or break. This consideration is of great practical moment in estimating the value of all kinds of ties, as king and queen posts, and connecting rods in machinery; for it has been calculated by Mr. Tredgold that if the line of tension be removed from the axis or centre of the section to one-half the radius, only one-quarter of the whole strength will be available: this appears to be incorrect, from direct experiments by Mr. Hodgkinson, who found that similar bars of similar iron, when stretched along their axis or centre, broke with 7·65 tons, but when the strain was allowed to act along their side, they broke with 2·62 tons; which gives for the ratio $\frac{2{\cdot}62}{7{\cdot}65}=\frac{1}{2{\cdot}92}$, or a little above one-third. The consequence of inattention in practice to this fact is, that a framework is loaded with material which is doing no work.

The second law appears to be true under all ordinary strains, although the defect becomes practically evident under severe tension. Mr. Barlow subjected a bar of wrought iron to a strain of nine tons, and the successive elongations with each ton were in millionths of the whole length of the bar,—160, 150, 130, 120, 110, 120, 120, 120. Again he tried another bar of two square inches in section, and adding two tons at each trial, loaded it with 40 tons; estimating in millionth parts as before, the extension up to 36 tons was—180, 140, 110, 110, 110, 110, 100, 100, 100, 100, 95, 90, 95, 85, 75. Putting aside the two former results in each experiment, as the elastic power appears not to have been fully brought into action, there remains a tolerably regular series of extensions in all the trials until the strain exceeded ten or twelve tons on the square inch, when the measure changes. These facts are interesting, for it is commonly acknowledged from experiments that a strain of nine or ten tons per square inch is a limit for long-continued work, and by these trials extension is shown to be very equable as far as, and even beyond, that amount.

(9.) From these and several other experiments Mr. Barlow found that wrought-iron bars suffer an extension of 1-10,000th of their length for every ton strain per square inch. Vicat, from an experiment, has given the extension of iron wire for a strain of 1428 lbs. (or $\frac{1}{20}$th the breaking weight) at ·000057, or $\frac{1}{17579}$th of its length; this, however, must be regarded as a particular case, since the effect of drawing in wire-making is greatly to increase its tenacity.

(10.) In circumstances where a body is pendant, and has, therefore, to support its own weight as well as the load, it is not economical to give it an equal section from top to bottom; as the lowest part has nothing to sustain except the load, it should have no greater section than is requisite to meet that strain; and as the successive parts above must sustain all below them, the greater section of the lower parts are not only useless, but incumbrances. The upper section, according to this method, must exhibit a section sufficient to meet the strain arising from the weight + the load. In the note are given formulæ for estimating the sections*.

Numerous experiments have been made on woods and metals, to determine their tensile strength; and the facts developed by them are of great interest. This chapter will comprehend a summary of these experiments on 1, Wood; 2, Iron, cast and wrought; and some practical consequences of the general facts in the making of joints under tension.

WOOD.

(11.) Musschenbroek, Buffon, and Robison practically tried several kinds of wood, but unfortunately with very different results, which the loose manner of investigation may probably in part account for. The first-named philosopher took considerable care to obtain a mean strength of timber, by cutting rods from four sides of the same tree, and at several distances from the centre towards the circumference; each of these lengths presented an area of 1-25th of an inch. Reducing his results to the section of one square inch, they are as follow:—Beech and oak bore 17,300 lbs.; elm, 13,200; willow, 12,500; ash, 12,000; fir, 8,330; and cedar, 4,880. Subsequently lesser numbers were given for the strength of beech, oak, and some others. These woods were more satisfactorily examined by Mr. Barlow, who has detailed his experiments in a treatise on the strength of wood and other materials. He had pieces of the various woods, about a foot long, turned in the middle to a cylinder of 1-3rd or 1-4th of an inch in diameter, using the shoulders afforded by the large ends to catch between two beams at the upper end, and support a scale-pan for weights at the lower end. This appeared to allow a fair exercise of the strength. The greatest and least, with the mean values obtained from his experiments, are as follow:

Kind of Wood.	lbs.	Mean in lbs.
Ash	11,000 13,446	12,203
Beech	11,338 11,626	11,467
Box	19,595 20,348	19,89[illegible]
Fir	11,000 13,448	12,208
Mahogany	7,950 8,224	8,041
Oak	8,889 12,008	10,389
Teak	14,662 15,405	15,090

When the variable character of the materials under experiment is considered, in addition to some possible errors in operating, there can be no difficulty in understanding why such differences should appear in the conclusions. Musschenbroek, and Robison after him, agree in considering that the wood taken from the parts surrounding the pith or centre is the weakest. Gerard and Barlow agree in stating them to be the

---

* When the section Σ is uniform, the length of the rod in inches being $= L$, every cubic inch of the material in weight $= w$, its ascertained tenacity per square inch $= t$, and the weight or load attached $= W$. Then the topmost section will have to sustain a pressure of $\Sigma L w + W$. Supposing then that the strain altogether is a fraction (as $\frac{1}{s}$) of its strength, we have the tenacity of the whole

$$\Sigma t = s (L w \Sigma + W),$$

whence the section

$$\Sigma = \frac{s W}{t - s L w}.$$

The weight W_1 of the bar is also $\Sigma L w$, or

$$W_1 = \frac{s W L w}{t - s L w}.$$

Mr. Moseley has given for the improved section

$$\Sigma = \frac{s W}{t} \varepsilon^{\frac{sw}{t} x}.$$

where ε is the base of the hyperbolic logarithms, or 2·7182818 and x is the distance of the section under inquiry from the point where the weight W is attached. The weight W_2 of such a rod is

$$W_2 = W \left(\varepsilon^{\frac{sLw}{t}} - 1 \right)$$

and the saving of material over the equal section is found, since it is equal to $W_2 - W_1$.

strongest. Age, the time of the year when cut down, and the soil whence the wood has been taken, greatly influence the strength; a marshy position for a tree is not conducive to the excellence of its timber in respect of strength and solidity, for Mr. Barlow found the specific gravity of wood thus grown to be to that of another specimen from dry ground as 5 to 7, and their strength as 4 to 5. This observation would testify in favour of the view that the exterior are less strong than the interior portions of a tree, since the sap of exogenous vegetables (such as oak, fir, &c.), passes up in the outer portions; while the inner, yearly becoming compact and less fluid, possesses greater solidity. The few facts at present known to us show the necessity of observing all the circumstances attending any particular specimen under experiment before anything like a law of strength can be established.

IRON.

We are fortunate in possessing much more precise information concerning this universally useful material, owing to the labours of several experimenters. As in the case of woods, there is a different strength to each owing to the contingencies of manufacture.

(12.) *Cast Iron* is a substance of crystalline texture, and very variable in its qualities; the section of a fracture generally exhibits a beautiful lustre, apparently possessing more or less crystalline regularity of disposition, when the specimen is very hard and brittle; the more tenacious or rough kinds expose a surface of dark grey, yet lustrous, with a texture more resembling the fibrous: it would appear from analyses of cast iron that this toughness is due to the admixture of foreign matter in the smelting*.

(13.) The tensile strength of cast iron was long very much overrated. Mr. Tredgold, from experiments on the transverse fracture of several bars, most erroneously estimated it at 20 tons. Three direct experiments brought the estimate more nearly to the probable amount: Captain Brown finding it 7.26 tons; Mr. G. Rennie (Phil. Trans., 1818), in two trials, obtained 8·52 and 8·66 tons; a mean of all giving 8·14 tons. Mr. Barlow, nevertheless, conjectured from theoretical principles, that the strength must be at least 10 tons per square inch.

The most recent experiments on cast iron are those of Mr. Hodgkinson on the strength of that metal from furnaces supplied with a blast of air previously raised to the temperature of 600° or more, and comparing the specimens with those from common cold-blast furnaces. The castings in the middle were of the form +. This part was of course weaker than the ends, which were connected with the trying machine: with such a form of section the strain was more certain of passing through the centre, and so relieving the results from any uncertainty as to their value from unequal strain. The following table is a summary of Mr. Hodgkinson's trials on these castings:—

	Hot Blast.				Cold Blast.			
	lbs.	lbs.	t.	cwt.	lbs.	lbs.	t.	cwt.
Carron Iron, No. 2. .	13,892 12,993 13,629	Mean 13,505=	6	0½	16,772 16,594	Mean 16,683=	7	9
Carron Iron, No. 3. .	16,840 18,671	„ 17,755=	7	18½	13,984 14,417	„ 14,200=	6	7
Devon (Scot.) Iron, No. 3.	21,907	...	9	15½	...	...	...	
Buffery Iron, No. 1. .	13,434	...	6	0	17,466	...	7	16
Coed Talon (N. Wales) Iron, No. 2. . . .	16,279 17,074	„ 16,676=	7	9	19,610 18,100	„ 18,355=	8	4
		Mean .	7	4⅗		Mean .	7	14

---

* Dr. Thomson undertook, at the request of the British Association for the Advancement of Science, to make a few analyses of cast iron from hot and cold blast furnaces. The results are interesting. Three specimens of No. 1, Muirkirk, and one each of pyrites, Carron, and Clyde iron, of cold blast, were taken, and as many specimens from hot-blast furnaces, affording the following mean percentage of matters:—

	Cold Blast.	Hot Blast.	
Iron	91·154	95·584	With occasionally a small quantity of copper, sulphur, magnesia, or calcium.
Manganese	2·027	0·871	
Carbon	3·855	2·099	
Silica	1·177	1·086	
Aluminum	1·651	0·422	
	99·874	99·062	

The hot-blast iron is evidently purer than

The difference observable between hot and cold-blast iron appears to be small, if anything, in favour of the cold blast, but the general mean may be taken as a safe average tensile strength of cast iron; considerably less than the supposed value of Mr. Barlow, and greatly below Mr. Tredgold's estimate from transverse rupture *. A confirmatory view is given by Messrs. Minard and Desormes, who undertook to find what influence variation of temperature might have on the cohesion of cast iron; their final results (at the square inch), are as follow:—

Temperature.	Breaking Weight.	
—5°F . .	6·51 tons.	
+21·2 . .	6·52	
23 . .	6·50	
23 . .	5·41	
37·4 . .	8·29	
41 . .	7·39	Mean=7·189 tons.
41 . .	8·23	
41 . .	9·08	
41 . .	6·73	
140 . .	7·23	

Not only do these results accord with those of Mr. Hodgkinson, but we learn, that within more than the ordinary natural range of temperature no variation in the cohesion of cast iron is detectable. From these experiments, therefore, we must conclude that the ultimate tensile strength of cast iron may be safely estimated at 7 tons.

(14.) *Wrought Iron.*—Owing to the application of this material to cable-making, bridge and ship-building, and very many other important purposes, it has undergone several trials of strength. Mr. Telford subjected some iron bars, from 2¼ feet to 1 foot 7 inches long, to breaking strains, by means of an hydrostatic press, and made the following observations, which are reduced to the square inch of section:—

Kind of Iron.	Breaking Weight.	Extension per foot.
1. Welsh	29·30	1·83
2. "	29·80	1·42
3. Staffordshire .	27·15	4·30
4. " .	27·50	1·21
5. Welsh	29·00	2·75
6. Swedish . . .	29·00	0·37
7. Faggotted . .	29·00	2·37
8. Staffordshire .	31·00	1·00
9.	31·80	2·20
Mean . .	29·08	2·13

The elongations of 6, 7, 8, were taken before the breaking load was attained, No. 6 being measured at 27 tons; No. 7 at 28 tons; and No. 8 at 30 tons. The mean of the elongations comprise all but the Swedish specimen, but the quantity in each case is irregular; not so, however, the ultimate strength, the mean differing but little from the maximum or minimum.

Mr. Brunel obtained higher averages from three sets of experiments on hammered iron, his numbers being 30·4, 32·3, and 30·8 tons for the ultimate strength.

Mr. G. Rennie, in a series of experiments on different materials, found 24·93 to express the tensile strength of English iron; this accords with Captain Brown's trials, which afford a strength of 25 tons.

A mean strength of bar iron in general use is 27 tons, one-third of which, or 9 tons, is considered a safe load-strength.

(15.) Iron plate has received investigation at the hands of Mr. Fairbairn, in reference to iron ship-building. The following varieties of plate were taken, all being of uniform thickness, with extra plates riveted at each end to lay hold of the shackle or tension apparatus; one set being stretched across the fibre, and the other parallel with the fibre—

Kind of Iron.	Drawn across the fibre.	Drawn parallel with the fibre.
Yorkshire plates .	27·49	25·77
" .	26·04	22·76
Derbyshire " .	18·65	21·68
Shropshire " .	20·00	22·83
Staffordshire " .	21·01	19·56
Mean . .	23·04	22·52

No great difference appears to result from the direction of the force with respect to that of the fibre, but there is more regularity in the strength of the plates drawn in the direction of the fibre than in those drawn across it. The mean

---

the cold-blast, which accounts for its greater brittleness; this is said to be obviated completely by adding a small portion of Welsh *scrap* iron. As might be supposed, the density of hot-blast is greater than that of cold-blast iron, several trials with each kind giving, for the specific gravity of the former, 7·0, and for the latter, 6·7.—Reports, vol. 6, p. 117. (1838.)

* Two other trials were afterwards added of Low Moor (Yorkshire) iron, No. 3, which gave a strength of 6½ tons; and a mixture of iron, a mean of four experiments on which furnished 7 tons 7¾ cwt.

strength also is less than that of iron bars.

(16.) The processes of hammering and drawing greatly increase the tenacity of metals. Robison (Ency. Brit.) remarks, that the cohesion of lead, gold, silver, and brass, is in this way nearly tripled, and of copper and brass doubled. Mr. Telford made several experiments on wires of four different sizes, from ·100 to ·048 of an inch diameter, and obtained a mean result of 38½ tons per square inch as the ultimate strength (see page 217). Two continentals, Col. Dufour and M. Vicat, give a much higher estimate, the former stating 41·7 tons, and the latter 47 tons as the ultimate strength of iron wire.

(17.) The mixture of metals is said to increase the tenacity to a considerable extent. Musschenbroek states that 6 of copper and 1 of tin give a tensile power of 640, while the two constituents alone are respectively 370 and 65; 1 of lead, 1 of zinc, and 4 of tin, 130; separately, 8½, 26, 65.

(18.) A valuable practical lesson is derived from the principle of tension, concerning the making of joints; it is plain from the facts experimentally ascertained, that these estimates of strength can only be applied where the strain on any section is regular and even over that section; too commonly joints are made irrespective of the consequence that so much of the strength is lost by throwing the strain on one side, or not fairly bringing the whole material into work. In carpentry, the proper method of arranging joints is sufficiently evident, but the joining of iron work calls for more consideration. In forming eyes by welding, at the ends of iron bars, for chain-links, and other purposes, the bar is found to be weaker than in its plain form. In iron plate work the joints are made by riveting, on which the whole efficacy of the built-up plate work depends. We are indebted to Mr. Fairbairn for experiments made directly relating to this important subject, with a view to gather materials for scientific iron ship-building, and, more lately, to find the strongest mode of riveting for the plates forming the Britannia and Conway tubular bridge. By trying plates as before mentioned (art. 15), Mr. Fairbairn obtained a mean value of 22·78 tons as their breaking weight per square inch. Similar plates were then riveted, care being taken to expose as much surface (independently of the rivet holes) to tension as in the simple plates; one set of four, with a single row, and the other with a double row of rivets. He obtained the following results*:—

	Single riveted.	Double riveted.
	20,127 lbs.	22,699 lbs.
	16,107	23,371
	18,982	20,059
	19,147	22,902
Mean....	18,590	22,258

The manifest superiority of double riveting is very evident, and taking the mean of all his experiments, Mr. Fairbairn found the ratio of strength in the single and double-riveted plates to be as 742 : 1000. Thus, placing the strength of the whole plate and riveted plates in comparison we have—

For the strength of the plate . .	100
For that of the double-riveted plates	70
For that of the single-riveted plates	56

The weakening of jointed plates made Mr. Fairbairn fearful of the results in constructing the Britannia and Conway tubes, as so much depended on the riveting in the lower plates, which are under tensile strain (see page 196); he finally tried what he calls chain-riveting, that is, placing a row of rivets, one behind the other, in the tension, instead of spreading them about after the old fashion. This saved much weakening of the plates through hole-punching, since their strength is lessened by just so much as the sectional area of the part punched away. The trials made were with a single plate, having a top and bottom covering-plate over the joint; and a double plate, with a single covering-plate placed on the side of the joint (see fig. 99, p. 197). Half-inch rivets were used, and the following results obtained†:—

1. Single-plate.—24·41 tons (on the square inch); the plates were torn asunder through the rivet holes.

2. Double-plate.—18·73 tons (on the square inch); the plates broke asunder by shearing off the rivets close to the plate.

These trials showed a great strength for the first plate, but in the second case, the fault lay with the rivets; when, however, they were made of larger diameter, so as to be equal in area to the plates, similar strength was again realised. Now in the first experiment the ratio of the areas of the rivets and plates was ·785 to ·750, and in the latter, where the rivets were deficient, it was 0·785 to 1·500. These two experiments seem to point out a most important rule: —that to obtain the maximum strength in chain-riveting the area of the rivets should be nearly equal to that of the plates. .

---

* Brit. Ass. for the Adv. of Science, Reports, 1840, vol. 10.

† An Account, &c., of Britannia and Conway Tubular Bridges, p. 284.

Chapter III.—Compression; *physical laws of rupture by crushing.—Compression-power of Wood; Iron, cast and wrought; and Stone.—Strength of Pillars of Iron, Wood, and Stone.*

The principles governing the disintegration of a body by compression of its parts are more intricate and obscure than those of tension; the forces applied are so modified and resolved into others, by the peculiar constitution of any material or relative position of its particles, as almost to prevent any two experiments exhibiting similar phenomena of rupture, although the bodies may sustain nearly the same pressure; so far, therefore, as practice is concerned, the fact is of less importance, but it confuses theoretical views.

The following facts appear to be well established of materials under a crushing force :—

1. The strength is as the transverse area or section.

2. The plane of rupture in a crushed body is inclined at a constant angle to the base of the body.

3. The measure of compression-strength is constant only within certain proportions of the diameter and height in any specimen.

(19.) 1. The first law was not received as a natural verity some time since; indeed, several experiments appeared to decide on the contrary, but it is uncertain how many of them were conducted; others were prosecuted with insufficient accuracy, and as everything depends on the accuracy of an experiment in all its minute features, they must carry little weight. Mr. Hodgkinson, in his experiments on cast iron, discovered that his values pointed out this equality, though doubtfully; he therefore tried twelve cylinders of teak-wood of different diameters, which furnished these results :—

	½ in. diam.	1 in. diam.	2 in. diam.
Crushing weight	2439 lbs.	10,171 lbs.	40,304 lbs.
Proportion of weights . . .	1	4·17	16·5

The areas being as the squares of the diameters, an exact proportion would have been 1, 4, 16; but the above figures, particularly considered as the mean of twelve trials, are amply sufficient to justify the statement of proportionality of the strength and area; still there may be a disposition in some substances for the particles of one vertical layer to help those of another, and so increase somewhat the apparent strength.

(20.) 2. It was observed by Rondelet and others, that when a rod submitted to pressure was but three or four times its diameter in length, the parts were split about without any bending of the whole rod on one side; but that beyond this proportion—if the length be greater —it would fracture by bending at or near the middle. It is now a well-ascertained circumstance that the crushing strength of a body varies according to its relative height and breadth. Mr. Hodgkinson remarks in the detail of his experiments :—" When bodies are crushed, they give way by a wedge sliding off in an angle dependent on the nature of the material, and in cast iron the height of this wedge is about 1½ the diameter, or thickness of the base of the wedge. If the body to be crushed is shorter than would be sufficient to admit a wedge of the full length to slide off, then it would require more than its natural degree of force to crush it; because the wedge itself must either be crushed, or slide off in a direction of greater difficulty. If, on the other hand, the height of the body to be crushed be much greater than the length of the wedge, then the body will sustain some degree of flexure, and fracture will be facilitated in consequence."*

The following extract from one of his tables sufficiently illustrates these remarks, showing the superior strength exhibited by specimens as there is less room for a fair separation of their parts :—

---

* Philosophical Transactions, vol. cxxx., 1840, p. 419.

Height of specimens.	Cylinder ½ inch diameter, area of base ·1963.	Right prisms, bases, squares, ½ inch the side.
⅛ inch.	30,461 lbs.	
¼ ,,	26,983 ,,	
⅜ ,,	26,412 ,,	
½ ,,	24,210 ,,	25,721 lbs.
⅝ ,,		
¾ ,,	23,465 ,,	24,191 ,,
⅞ ,,		
1 ,,	22,867 ,,	23,950 ,,
1¼ ,,	24,177 ,,	
1½ ,,	23,453 ,,	
2 ,,	21,828 ,,	

Unless, therefore, the height be more than equal to the diameter, the strength found will not answer for taller specimens. With so important an observation now plainly before us, it is unfortunate that we possess no certain information on the proportions which exist in other materials beside cast iron; a great number of experiments on record are, by this discovery, shown to be unsafe for application except in cases under the same conditions. Calling the diameter of a specimen 1, we have for a fair crushing proportional height—

In stone (limestone?)	from	?	to	12
In cast iron	,,	1¼	,,	4
In wrought iron . . .	,,	?	,,	4
In wood	,,	½	,,	?

(21.) 3. When rupture occurs in a piece of sufficient length to afford a regular separation of its parts, there are conic portions, having for bases the upper and lower faces of the specimen, which appear to effect the breaking asunder of the other portions. The two following figures are instances, from the same experimenter, of a cylinder and prism crushed by the angular form of their parts. The first figure shows a cylinder being crushed, and forming two cones or wedges, which are pointed, and ready to slide past each other, driving the lateral portions outwards. This wedge or pyramid is exhibited plainly in the second figure, where its pointed apex *a* is remarkable; it has evidently been the instrument of pushing the other pieces asunder. From a number of experiments Mr. Hodgkinson found the form of fracture, in suitable specimens, to be of the form in *fig.* 117, the angle *a b c* (*fig.* 117) between the wedge-side and its base being tolerably constant for the same description of material. In cast iron the results were as follows:—

Fig. 115.

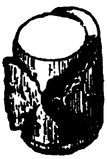


Fig. 116.

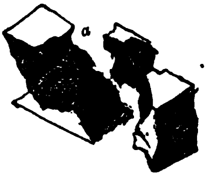
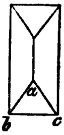


Fig. 117.



	Cylinders.	Rectangles.
Least angle .	49° 0′	51° 0′
Greatest angle	52 0	60 0
Mean angle	55 32	54 41

In wood the angle observed was about 26½°.

We possess a considerable number of experiments on the compression of materials, but most of them are unfortunately without that accompanying detail and variation of length in the pieces tried, which furnish all that is valuable for scientific purposes.

WOOD.

(22.) But few practical inquiries have been made on the crushing of timber, and those are not so concordant as to give satisfaction. M. Rondelet gives the power of oak and fir at the following values:—

Oak	6800 lbs.
Fir	8000 ,,

Several woods were tried by Mr. Rennie, who finds for English oak a value of 3860 lbs., and with a piece four inches high, 5147 lbs.; for elm, 1284 lbs.; American pine, 1606 lbs.; white deal, 1928 lbs.

(23.) Mr. Hodgkinson made some trials on short cylinders of different kinds of wood, when somewhat damp, as is commonly the case, and also after being kept in a warm place for two months. The pieces were, in the first series, two inches long by one inch diameter; in the second they were frequently one inch high only.

Kind of Wood.	Damp. lbs.	Dry. lbs.
Alder	6831	6,960
Ash	8683	9,363
Baywood	7518	7,518
Beech	7733	9,363
American Birch	...	11,663
English Birch	3297	6,402
Cedar	5674	5,863
Crab	6499	7,148
Red Deal	5748	6,586
White Deal	6781	7,293
Elder	7451	9,973
Elm	...	10,331
Fir (Spruce)	6499	6,819
Hornbeam	4533	7,289
Mahogany	8198	8,198
Oak (Dantzig, very dry) . .	...	7,731
„ (English)	6484	10,058
„ (Quebec)	4231	5,982
Pine (pitch)	6790	6,790
„ (yellow, full of turpentine)	5375	5,445
„ (red)	5395	7,518
Poplar	3107	5,124
Plum (wet)	3654	...
„ (dry)	8241	10,493
Sycamore	7082	...
Teak	...	12,101
Larch	3201	5,568
Walnut	6063	7.227
Willow	2898	6,128

The above table, from the experience in the fracture of material which the experimenter had obtained, must be considered as the most satisfactory in our possession, although it is highly probable that none of his values would agree with the most careful trial on any similar woods; it could not happen unless the structure of the piece should be altogether alike,—a chance almost impossible amongst the varied circumstances of formation in vegetable bodies.

IRON.

(24.) *Cast Iron.*—The most extravagant opinions were entertained of the power of cast iron to resist pressure, owing to dependence on a few ill-conducted experiments. Even those of Mr. Rennie, who exercised some care, and varied the length and breadth of his specimens, present as the lowest estimate a strength of 90,000 lbs., or above 40 tons per square inch.

The subject was taken up by Mr. Hodgkinson, a few years since, with a simple apparatus, but far more calculated to realize accurate results than any former method. It consisted of a small box of iron, having a hole bored at one end, to admit a small cylinder or plug capable of moving up or down without any side motion; the lower end of this plug was made accurately flat, so as to press evenly on the surface of any little piece placed in the box and under it for experiment; its upper extremity was rounded that a lever which pressed upon it might act along its centre. When the piece was to be crushed it was placed on a steel plate in the iron box, the plug pushed down upon it, and a weight carried along the lever above the plug; its varied behaviour until rupture commenced, and the manner of rupture, could be observed through the open side of the box.

The following table gives the means of several trials on each kind of iron, with the relative height and diameter of the pieces used:—

Description of Iron.	Form of Specimen.	No. of Experiments.	Mean strength per square inch. lbs.		tons.	cwt.
Devon (Scotch) iron, No. 3, hot blast.	Cylinder.	4	135,435		= 64	18½
Buffery (near Birmingham) iron, No. 1, hot blast	„	4	86,397		= 38	11¼
Do., cold blast	„	4	93,385		= 41	13½
Coed-Talon (Welsh) iron, No. 2, hot blast	„	4	82,734		= 36	18½
Do., cold blast	„	4	81,770		= 36	10
Carron (Scotch) iron, No. 2, hot blast.	Cylinders and prisms.	18	114,703		= 51	4
Do., cold blast	„	22	111,248		= 49	13¼
Carron iron, No. 3, hot blast . . .	Prisms.	3	133,440		= 59	11¼
Do., cold blast	„	4	115,442		= 51	10¾
Low Moor (Yorkshire) iron, No. 3, cold blast	Cylinder.	8	115,911	109,801	= 49	0⅛
	Rectangle.	2	103,692			
Mixture of iron	Cylinders.	6	100,049	110,908	= 49	10¼
	Rectangles.	3	121,767			
				Mean . . .	48	2

The general mean of 48 tons may be therefore allowed as a common estimate of ultimate strength.

(25.) We learn from a comparison of these experiments with those on the tensile strength of cast iron, that this kind of iron is very much more powerful in resisting a superincumbent load than in sustaining a pendant weight. Taking the least and greatest strength in the above table, and calling the tensile strength unity, we have this ratio :—

Least strength (Coed-Talon, No. 2) .	4·337 : 1	Mean 6·59 : 1.
Greatest strength (Carron, No. 3) . .	8·473 : 1	

The following experiments [*] will illustrate the behaviour of wrought iron under pressure, each piece being tried with a certain weight, and the consequential alterations measured by a gauge :—

Cylinders 2·5 in. long and ·62 in. diam.
Diameter to length as 1 : 4.

Exp.	Weight per sq. inch. lbs.	tons.	Compression.	Remarks.
1	20,610 =	9·2	·0 ?	No remarkable alteration.
2	24,319 =	10·9	·015-	
3	28,028 =	12·5	·02	Diameter increased.
4	41,218 =	18·4	·04	Diameter became ·63 inch, cylinder bent.
5	61,830 =	27·6	·16	Bent, diameter became ·65 inch.

(26.) *Wrought Iron.*—All our information on the compression strength of this material is, that M. Rondelet found that cubes of an inch bore 70,000 lbs. Mr. Hodgkinson's observations are not far different: 9 or 10 tons per square inch were found to flatten slightly the compressed specimens, and 27 or 30 tons per square inch permanently reduced them one-sixteenth of their length.

(27.) Mr. Fairbairn made an experiment to learn the ability of iron plates, when formed into a tube, to resist compression. The tube was constructed of plates half an inch thick, with angle iron at the corners; it was eight feet long and 1 foot 6 inches square, and probably, therefore, was somewhat beyond the fair proportion of length to diameter, yet the strength found was considerable; it presented an area of 50 square inches. When 615 tons pressure had been applied (by an hydrostatic press), there was a flexure on two sides—the one of ·75, or $\frac{3}{4}$ths of an inch, the other of $1\frac{3}{4}$ inch, and a compression of ·063 of an inch; a short time previous to 690 tons being laid on it was observed to yield, and three sides bent out; the ultimate strength of the tube was now attained, although not that of the material. The experimenter took 680 tons as the fair breaking weight, which gives for the specific strength of such a tube $\frac{680}{50} = 13·6$ tons per square inch.

STONE.

(28.) Our knowledge of the strength of stone is very limited, although it is the most generally useful of materials. The quality of the same kind of stone of necessity differs greatly in various localities—even in the same quarry much difference may be found, from the circumstances attending the formation of the stratum or bed; and this is increased by the common admixture, in stratified rocks, of more or less organic remains. Unfortunately those experimental results which we possess were obtained without attention to the fact that the specimens should be of a certain height (see art. 20) to show a proper compression strength. The bulk of the examples are with cubes; a fault excusable with those experimenters who made their work public before these peculiarities were well known, but the same cannot be said of the investigations conducted on various lime- and sand-stones, by a commission appointed to find out the best stone for the new Houses of Parliament: these experiments, executed with singular minuteness on some points, would have been most useful, from their variety and specification of the localities, but they were made on cubes, at a period when the laws of fracture were as public as at present, and are therefore of limited value. The following table comprises the principal facts known on the crushing strength of stone :—

---

[*] Mr. Hodgkinson on Strength of Pillars. Phil. Trans., 1840, p. 422.

	Specific gravity.	Cracking weight. lbs.	Crushing weight. lbs.	
Granite.				
Aberdeen (blue)	...	...	10,363	*B
———	2·625	...	10,914	R
Cornish	2·662	...	6,053	
Dartmoor	...	...	12,175	B
Heytor	...	...	13,865	B
Herm	...	...	14,873	B
Penrhyn	...	...	7,728	B
Peterhead (blue)	...	...	10,192	B
Peterhead (grey)	...	...	9,666	B
Peterhead	...	...	8,282	R
Limestone.				
(Not specified)	...	...	5,903	R
Marble (white)	...	...	9,580	
Ancaster	2·182	6,800	9,350	C
Barnack	2·090	4,538	7,083	C
Bolsover	2·316	19,831	30,147·5	C
Brodsworth	2·093	7,366·5	18,416·5	C
Cadeby	1·951	5,666·5	6,516·5	C
Chilmark (3 specimens)	2·410	10,285	25,500	C
Craigleith	2·452	...	5,480	
Hamhill	2·260	6,233	16,149	C
Haydor	2·040	4,533	7,083	C
Hildenly	2·098	17,565·5	19,266·5	C
Huddlestone	2·147	9,633	17,283	C
Jackdaw Craig	2·070	10,666·5	18,903	C
Ketton	2·645	6,233	10,285	C
Ketton Rag	2·490	14,166·5	35,983	C
Park Nook	2·138	7,366·5	17,283	C
Portland (Waycroft Quarry)	2·145	8,500	15,583	C
Portland	...	...	4,570	R
Purbeck	2·599	...	9,160	
Roche Abbey	2·134	6,800	15,583	C
Totternhoe	1·891	3,966	7,700	C
Sandstone.				
Bramley Fall	2·506	...	6,053	
Binnie	2·194	10,766·5	20,116·5	C
Box	1·839	5,100	5,950	C
Bramham Moor	2·008	10,666·5	23,649·7	C
Craigleith	2·266	17,000	31,449·5	C
Darley Dale, Stancliffe	2·628	26,014·5	28,333	C
Derby	...	...	3,110	
Dundee	...	...	6,490	
Giffneuch	2·230	13,698	19,266·5	C
Heddon	2·229	7,366·5	15,866	C
Hookstone	2·253	17,566·5	23,233	C
Kenton	2·247	13,698	19,831	C
Mansfield, or C. Lindley's red	2·338	8,038	20,397	C
Mansfield, or C. Lindley's white	2·277	10,285	20,963·5	C
Morley Moor	2·053	6,235	19,833	C
Park Spring	2·321	15,866	30,316	C
Redgate	2·239	15,383	23,649·7	C
Stanley	2·227	10,285	13,883	C
Brick, pale red	2·085	...	562	R
——— red	2·168	...	807	R
——— (Stourbridge, fire)	...	...	1,717	R

(29.) The subjoined remarks on the specific quality of sand and limestones are interesting :—

" With reference to sandstones, such as are usually employed for building purposes, and which are generally composed of either quartz or silicious grains cemented by silicious, argillaceous, calcareous, or other matter; their decomposition is effected according to the nature of the cemented substance, the grains being comparatively indestructible. With respect to limestones composed of carbonate of lime, or the carbonates of lime and magnesia, either nearly pure or mixed with variable proportions of foreign matter, their decomposition depends, other things being equal, upon the mode in which their component parts are aggregated, those which are most crystalline being found to be the most durable, while those which partake least of that character suffer most from exposure to atmospheric influences.

---

* To indicate the authorities, B stands for Bramah, R for Rennie, and C for the Parliament Houses Commission.

"The varieties of limestone termed oolites, being composed of oviform bodies cemented by calcareous matter of a varied character, will, of necessity, suffer unequal decomposition, unless such oviform bodies and the cement be equally coherent. Those limestones which are usually termed 'shelly,' from being chiefly formed of either broken or perfect fossil shells cemented by calcareous matter, suffer decomposition in an unequal manner, in consequence of the shells, which, for the most part crystalline, offer the greatest amount of resistance to the decomposing effects of the atmosphere.

"Sandstones, from the mode of their formation, are very frequently laminated, more especially when micaceous, the plates of mica being deposited in planes parallel to their beds. Hence if such stone be placed in buildings at a right angle to its natural bed, it will decompose in flakes, according to the thickness of the laminæ; whereas, if it be placed upon its natural bed, the amount of decomposition will be comparatively immaterial."*

PILLARS.

(30.) The strength of pillars, being equally an intricate and important inquiry, has, at various times, attracted the attention of scientific men. Euler published a very well-known paper on the subject, but treated it purely on theoretical grounds; his conclusions, however, are not so widely different from the truth as might be supposed from the irregular data with which he calculated. Divesting the formula of other quantities, he showed that the strength varied as the fourth power of the diameter, and inversely as the square of the length of a pillar.

(31.) Although a few laboured researches had been made on this matter, few consistent facts were made known until, by the advice and liberal assistance of Mr. Fairbairn, a course of experiments was undertaken by Mr. Hodgkinson, who has prosecuted a most useful research on the strength of cast and wrought iron, steel, and wood pillars. His investigations were extended to long and short pillars, varying in length from 2 to 121 times the diameter, and of the following forms:—

- Solid cylindrical pillars,
  - with rounded ends.
  - with flat ends.
  - with rounded and flat ends.
  - with discs at each end.
  - with enlarged middle.
- Solid square pillars.
- Hollow cylinders,
  - with rounded ends.
  - with flat ends.

The apparatus was analogous in kind to that mentioned in art. 24, for crushing small pieces of iron; but the box was, of course, much longer, and greater care was taken to preserve the direction of the force along the middle. The results of his numerous experiments are highly satisfactory for their mutual agreement: the results only can be given in these pages, owing to the extent of the original paper*.

Long Pillars,
(from 30 to 121 times as long as their diameter.)

(32.) Flat and rounded ends.—A comparison of the tabular results of the trials with these two forms, indicates the general mean ratio of strength of the two as 3·167 : 1, or long pillars with flat ends resist breaking by flexure with above three times the power of pillars with rounded ends, other dimensions being equal. In shorter pillars this ratio is not constant; and in a pillar 20 inches long, and ·76 inch diameter (or about 24 times the diameter), the ratio changed to 2·36 : 1, and so varied to small pillars in length 7½ times the diameter, which gave 1·63 : 1. There is, therefore, not any great difference between the two forms of ends in short pillars.

A comparison of the tables also indicates that a flat-end pillar has as much ability to resist flexure as a rounded-end pillar of similar diameter and half the length; and this also holds true if the flat-end pillar is supplied with discs whereby to fasten it at each end, the difference being some increase of strength. The following examples illustrate this fact:—

---

* Report of the Commissioners.

* Philosophical Transactions, 1840, p. 385.

With flat ends. Length 60½ in.		With discs on the flat ends. Length 60½ in.		With rounded ends. Length 30¼ in.	
Diameter.	Breaking weight.	Diameter.	Breaking weight.	Diameter.	Breaking weight.
·77 in.	2,456 lbs.	·775 in.	2,719 lbs.	·77 in.	2,726 lbs.
·996 ,,	6,238 ,,	1·00 ,,	6,830 ,,	·99 ,,	6,105 ,,
1·29 ,,	16,064 ,,	1·28 ,,	16,369 ,,	1·29 ,,	17,235 ,,
1·56 ,,	28,962 ,,	1·53 ,,	30,789 ,,	1·52 ,,	32,531 ,,

(33.) One end rounded and the other flat.—These were found to take a place in point of strength between the altogether flat and altogether rounded-end pillars: the strength of the latter being 1, these gave 2; while the flat ends, as we have seen, afforded 3; their strength is therefore an arithmetical mean between the flat and rounded-end pillars.

There are general properties, which Mr. Hodgkinson has stated to be common to wrought iron, steel, and wood.

(34.) It appeared from the experiments that long pillars break first near to or at the middle; this occurred in all cases, evincing that they were weakest in that part. Pillars were therefore tried, having a middle diameter of from 1½ to 2 inches, the ends being 1 inch; the strength was not increased according to the increase of middle diameter, but appeared to be from $\frac{1}{6{\cdot}62}$ to $\frac{1}{8{\cdot}05}$, or from one-seventh to one-eighth; they did not, however, fracture in the middle, as did those of uniform diameter.

(35.) Strength as dependent on the diameter.—By comparison of the breaking weights of pillars 60½ inches long, and ·5 and 1·765 inch in diameter, Mr. Hodgkinson commenced the calculation of the power of the diameter according to which the strength was found to increase; Euler had stated it to be as the fourth power. By the table* we see that the crushing weights were respectively 143 lbs. and 15,560 lbs., and the two diameters are to each other as $\frac{1{\cdot}765}{{\cdot}5} = \frac{3{\cdot}53}{1}$, or as 1 to 3·53; this proportion, then, furnished the index or power required—

$$1^n : 3{\cdot}53^n :: 143 : 15560,$$

or $3{\cdot}53^n = \frac{15560}{143}$, which is best resolved by taking the amount in logarithms†, and we have the resulting index 3·718: by treating the other experiments in a similar manner, a mean of 3·736 was obtained.

(36.) Strength as dependent on length.—By a similar proceeding to that mentioned in the last article, the lengths and breaking weights were compared, and a mean value of 1·7 was found; the highest being 1·916 and the lowest 1·424. The former approaches very closely to Euler's calculated rule, where the square of the length is given, and, as Mr. Hodgkinson remarks, would be the power to which the experimentally-deduced index would approximate if the body were incompressible.

(37.) Strength of long pillars from the preceding results.—The amounts obtained from the tables No. I. and II. furnished all that is requisite to obtain a formula for the strength of cast-iron pillars; according to the last article it appears that the strength was as the 1·7 power of the length, and increased as the 3·76 power of the diameter, that is, in a fraction, the comparative strength of the pillars tried was as $\frac{d^{3{\cdot}76}}{l^{1{\cdot}7}}$, d expressing the diameter and l the length of the pillar. To obtain from this a general rule the experiments were reduced to a unit of measure, thus:—A pillar one foot high and one inch in diameter will have a breaking weight (w), bearing the same proportion to the breaking weight (w_1) of any other sized pillar, as their respective diameters and lengths have one to another. Therefore

$$\frac{d^{3{\cdot}76}}{l^{1{\cdot}7}} : \frac{1^{3{\cdot}76}}{1^{1{\cdot}7}} :: w_1 : w.$$

---

* See Appendix.

† And the formula becomes

$$n = \frac{\log. \frac{15560}{143}}{\log. \ 3{\cdot}53} = 3{\cdot}718.$$

or $w = \frac{w_1 \, l^{1.7}}{d^{3.76}}$, for pillars with rounded ends. This value w, or particular coefficient, can thus be obtained from each experiment, since the other quantities are all given in the tables. From such a procedure Mr. Hodgkinson found a mean value of $w = 33{,}379$ lbs. $= 14{\cdot}901$ tons, for rounded pillars, and 98,922 lbs. $= 44{\cdot}16$ tons for pillars with flat ends: an error of not more than about one-eighth was the greatest which appeared when the formula was compared with the experiments. Collecting these amounts, since the breaking weight expresses also the ultimate strength, we have these formula for general application, S standing for the strength in tons:—

Pillars with rounded ends . $S = 14{\cdot}9 \times \frac{d^{3.76}}{l^{1.7}}$.

Pillars with flat ends . . . $S = 44{\cdot}16 \times \frac{d^{3.55}}{l^{1.7}}$.

(38.) Hollow pillars of cast iron.—A number of experiments were made on hollow cylindrical pillars, during which Mr. Hodgkinson noticed the highly-interesting fact that although the pillars were generally thicker on one side than the other, yet in bending the compressed was always the thinner side, and as cast iron resists compression with above six times the force with which it sustains tension, no danger resulted from this almost unavoidable difference of thickness. He obtained for hollow cylinders the following formula:—

Hollow pillars with rounded ends . . . . . . $S = 13{\cdot}0 \times \frac{d^{3.76} - d_1^{3.76}}{l^{1.7}}$.

Hollow pillars with flat ends and fixed by discs . $S = 44{\cdot}3 \times \frac{d^{3.55} - d_1^{3.55}}{l^{1.7}}$.

(39.) Strength of pillars are as their area.—The conclusions of Mr. Hodgkinson were, that had the material been incompressible the formula of Euler, that pillars vary as $\frac{d^4}{l^2}$, or as the fourth power of their diameter, and inversely as the square of their length, would have been attained by the present fractional powers; as there is but a small difference between them. For similar pillars, where the length is in a constant ratio to the diameter, we may call the length n times d, n always representing a constant number; then the proportion of strength according to the results obtained for the round and flat-end pillars, and the theoretically-deduced formula is—

$$\frac{d^{3.76}}{n^{1.7} \times d^{1.7}}, \quad \frac{d^{3.55}}{n^{1.7} \quad d} \quad \frac{d^4}{n^2 \times d^2};$$

and dividing by d, we have

$$\frac{d^{2.06}}{n^{1.7}}, \quad \frac{d^{1.85}}{n^{1.7}}, \quad \frac{d^2}{n^2}.$$

The first quantity exhibits the strength of round-end pillars as varying according to a power of their diameter rather greater than the square; the second quantity somewhat less than the square; while the theoretically-deduced value is as the square. If, therefore, the latter be taken, it will not, on the whole, be far away from the truth in either case, and shows that the strength of similar pillars increases as the square of their diameter; and, as the area is as the square of the diameter, we learn that the strength increases as the area of the pillar.

(40.) *Short pillars.*—Mr. Hodgkinson found, as the tables will show, that when the length was decreased to below 30 times the diameter in pillars with flat ends, or 15 times the diameter in pillars with rounded ends, the above formulæ do not apply: when the length is equal to 20 diameters, the value of w becomes 77817. Mr. Hodgkinson was led to the following reasoning in order to obtain an approximate rule for the strength of these pillars:—

"Considering the pillar as having two functions, one to support and the other to resist flexure, it follows that when the material is incompressible (supposing such to exist), or when the pressure necessary to break the pillar is very small, on account of the greatness of its length compared with its lateral dimensions, then the strength of the whole transverse section of the pillar will be employed in resisting flexure; when the breaking pressure is one-half of what would be required to crush the material, one-half only of the strength may be considered as available for resistance to flexure, whilst the other half is employed to resist crushing; and when, through the shortness of the pillar, the breaking pressure is so great as to be nearly equal to the crushing force, we may consider that no part of the strength of the pillar is applied to resist flexure."*

Thus he assumed that the real breaking weight would be equal to the breaking weight as obtained from arts. 37 and 38, multiplied by the force requisite to crush it without flexure; and divided by the same two quantities added together, minus the pressure which it would support as flexible, without being weakened by crushing†. By using this method of calculation Mr. Hodgkinson met with differences from the true breaking weight (as shown by the experiments) varying from $\frac{1}{42}$ to $\frac{1}{7 \cdot 3}$.

(41.) Wrought iron pillars.—Generally the results of Mr. Hodgkinson's experiments on wrought-iron pillars are similar to those on the cast iron, the powers of the length and diameter slightly differing; from trials on pillars from 7 feet 6¼ inches to 2 feet 6½ inches long, the following powers of the length were deduced:—

Rounded ends.	Flat ends.	Mean.
1·954	2·22	2·03
1·919	——	

The strength varies, therefore, inversely, as the square of the length. For the diameter a power of 3·75 was obtained for pillars with rounded ends, which is a little less than the quantity deduced from cast-iron pillars. Moreover, the value of *w* (see art. 37) was found to be for round-end pillars 95,848 lbs., and for flat-end pillars 299,617 lbs. The strength S of any pillar may therefore be thus calculated:—

Pillars with round ends . . $S = 44{\cdot}00 \times \frac{d^{3 \cdot 75}}{l^2}$.

Pillars with flat ends . . . $S = 130{\cdot}00 \times \frac{d^{3 \cdot 55}}{l^2}$.

These formulæ give the strength in tons, the length being taken in feet, and the diameter in inches.

(42.) Pillars of timber.—These pillars brought the formula of Euler remarkably near to the truth, several trials giving for the power of the diameter according to which the strength increases, as 3·9; it may, therefore, be safely taken as the fourth power. Mr. Hodgkinson did not try for the power of the length, but found the square suit the experiment better than any other; thus the strength of pillars of wood varies as $\frac{d^4}{l^2}$. For oak and deal the same experimenter has given the following formulæ to find the breaking weight or strength S:—

Dantzig oak . . . . $S = 24542 \times \frac{d^4}{l^2}$.

Red deal . . . . $S = 17511 \times \frac{d^4}{l^2}$.

(43.) Pillars of stone.—Small columns an inch to 1¾ inch square, and from 1 to 40 inches long, gave results in some respects analogous to the above: a great falling off in strength occurred when the height was greater than 12 times the side of the base. The following proportion expresses this variation where the length is from 12 times to 40 times the side of the base:—

---

[*] Philosophical Transactions, 1840, p. 404.

[†] The formula thus found for calculating the strength was $\frac{bc}{b+\frac{3c}{4}}$, *b* expressing the breaking weight of long pillars, and *c* the crushing force of the iron.

Length.	Strength.
12 times	138
15 ,,	little less.
24 ,,	96
30 ,,	75
40 ,,	52

Mr. Hodgkinson found that with pillars shorter than 30 times the thickness, fracture occurred by one of the ends failing, and as the longer columns deflected more than the shorter, they presented less of the base to resist the pressure, and therefore more readily gave way: thus the practical view from these experiments points out an increase of area at the ends as being most economical, and that in proportion to the middle as 13,766 to 9595 nearly.

"From the experiments it would appear that the Grecian columns, which seldom had their length more than about ten times the diameter, were nearly of the form capable of bearing the greatest weight when their shafts were uniform; and that columns, tapering from the bottom to the top, were only capable of bearing weights due to the smallest part of their section, though the larger end might serve to prevent lateral thrust. This last remark applies too to the Egyptian columns, the strength of the column being only that of the smallest part of the section."*

(44.) Relative strength of pillars of different materials.—The experiments on long pillars furnished the following proportional strength of pillars with rounded ends (except those of red deal):—

Steel, cast	2518
Iron, cast	1000
—— wrought	1745
Oak, Dantzig	108·8
Deal, red	78·5

(45.) Effect of long-continued pressure on pillars.—Several pillars of cast iron, 6 feet long and 1 inch in diameter, were subjected to a constant weight for a long period, with the following results:—

Exp.	Weight.	Time borne.	Deflexion.
1	4 cwt.	3 years.	·01 inch.
2	7 ,,	3 ,,	·025 ,,
3	10 ,,	3 ,,	·409 ,, increasing.
4	13 ,,	above 5 months.	broke.

The last experiment was made with $\frac{\cdot 97}{100}$ths of the breaking weight, and, with the preceding, show that the time occupied in the experiments did not affect the truth of the results given in the preceding pages.

In the Appendix are given several tables of the experiments on long and short pillars.

---

* British Assoc. for the Advancement of Science, 15th Report, 1845, p. 27.

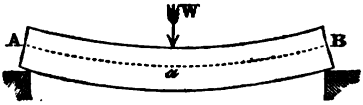
Chapter IV.—Transverse Strain.—*The Neutral Axis.—Length, depth, and breadth relations in a beam.—Rules for calculating the strength of beams.—Equated beams.—Deflection: its laws; defective elasticity.—Applications of the principle of a neutral axis.—Transverse strength of Wood, Iron, and Stone.*

The principles which govern a beam horizontally laid, supported at each end, and pressed down by a superincumbent weight, are of most extensive application and value; girders, joists, tie-beams, rafters, the main parts or sustaining elements of flat bridges and roofs, with many other specific works, are arranged and regulated by the laws of transverse strain.

(46.) Galileo supposed that if a beam were fixed at one end in a wall, and weighted at the other, the mechanical action of the fibres was so ordered, that the lower edge of the beam at any section is the fulcrum, or point of rest, while every line of fibres above it to the upper side is strained by the force, and resisted with equal power; thus, a fracture would commence at the upper side, and proceed downwards to the lower. The simplicity of the theory is pleasing, but its comparison with nature shows radical defects. He supposed the material to be inflexible, inelastic, and incompressible, which is incorrect, as a simple experiment of fracture proves a beam of any substance to be both elastic and compressible. The idea that every fibre exerts a tensile force is equally incorrect. Mariotti experimentally found its inaccuracy, and Liebnitz gave a fresh theory, founded still upon some of the old errors; but James Bernouilli afterwards studied this kind of strain, and noticed that when a beam broke a part of the fibres were stretched and a part compressed. This has been the foundation of modern improvement in the calculation of transverse strength.

(47.) *The Neutral Axis.*—If we suppose a beam to be supported at each extremity, and pressed by the weight W (fig. 118), it will be deflected, and the lower fibres extended, while the upper fibres are pushed together: these effects are distinctly shown in breaking by the hand a small stick of wood, as the fibres give way on the convex side, and those on the inner or concave side bulge out, indicating that they suffer compression; but since there are two opposite strains in the same beam, it is evident there will be a some line or point, *a*, in the depth which is labouring under neither the one nor the other: the tension, which is greatest at the convex edge, will decrease towards that point, and compression begin; this the experiment with the stick will, if carefully executed, point out. It cannot escape notice that the bulging part forms a wedge, or cone in bas-relief, the point or apex being nearest the extended side, while the base or wide and most bulging part is at the concave edge of the fractured stick. A like movement was observed by Mr. Hodgkinson, in breaking cast-iron beams: a wedge of a peculiar shape, its sides not being two straight lines, but curves, somewhat of the figure ⌣, broke out and exposed at the apex the place of rest; the wedges were from two to four times as deep, and proportionally longer as the depth of beam increased. In such a fracture, the point where the cone begins is the fulcrum, which, if the material were incompressible, would have been the upper or concave edge, as Galileo supposed; and about this line or fulcrum the halves of the stick turn; it is, therefore, very properly called the *neutral axis*, because it does no work, but passively stands between the tearing and crushing parts on either side.

Fig. 118.



A proposition was made some time since, to observe the position of the neutral axis in a beam, by means of polarized light[*]; it would detect it on this principle:—when a ray of light which has been polarized by reflection from a mirror at an angle of 54°, passes through a transparent substance whose particles are naturally or artificially in a state of irregular tension or compression, it will, on reflection from another mirror (or analyzing plate), present a variety of colours, lovely as the rainbow; but when the internal molecular forces are in equilibrium, no such decomposition of the ray is effected. This beautiful principle is thus applied:—if a bar of glass, well annealed, be placed in the line of the polarized ray, no colours appear; but if it be pressed by a force, as in the preceding figure, it will instantly interfere with the different colours, and so exhibit a succession of prismatic tints which vanish at the line where there is no unnatural force on the molecules: this line is of course the neutral axis, which, being so discovered, may be measured as to its distance from the edges. Unfortunately, transparent, and therefore the least useful materials, are alone treatable by this method, which is one of striking philosophical elegance.

---

[*] Inst. Civil Eng., Minutes of Proceedings, vol. iii. p. 248.

(48.) The position of this neutral axis has been the theme of many mathematical discussions, because, if found, it would prove the establishment of theory; but the great difficulty is, our want of knowledge of the ratio of compressibility and extensibility in any substance: were the compression equal to the extension, the neutral line would be in the centre of gravity of the beam, but experiments do not allow of such equality; moreover every substance has its peculiar ratio. Du Hamel cut beams one-third, one-half, and two-thirds through, and found the weights borne to be—by the uncut beam 45 lbs.; and by those cut, 51 lbs., 48 lbs., and 42 lbs. respectively, which would indicate that less than half the fibres were engaged in resisting extension, although it does not prove that two-thirds of the thickness contributed nothing to the strength, as Dr. Robison imagines. Barlow found that in a rectangular fir beam the neutral axis was about five-eighths of the depth, as shown by the section of fracture; this is quite accordant with Du Hamel's experiment. Mr. Hodgkinson, in experimenting on cast-iron beams, found the neutral line in sudden fractures from one-fifth to one-sixth of the depth; cast-iron, however, might anteriorly be supposed to present a high neutral axis, since its compression strength is so great.

Since the position of the neutral line is so uncertain, experimental evidence must be sought, which may correct theory, and with it establish rules for practical utility. From calculation the following laws are deduced, which apply to all cases of transverse pressure. The strength of beams in general is—

Directly as the breadth,
Directly as the square of the depth,
Inversely as the length;

or, placing the ratios together, it is as follows:—

$$\frac{\text{breadth} \times \text{depth}^2}{\text{length}} \; . \; . \; . \; . \; . \; . \; . \; (1).$$

This gives merely a general relation of strength in any beam whatever to its dimensions, but is no direct rule for application, since equal beams of all materials do not break by the application of an equal load: if the ratio of compressibility to extensibility were known for all materials, the element required in the above fracture might be supplied; in the absence of this knowledge, a certain supposed quantity must be used to express the specific strength of any material—a quantity which can be obtained only by experiment on the material; if we name this unknown and sought specific strength by S, and multiply it by the above quantities, we shall have the breaking weight of the beam, that is,

$$\frac{\text{breadth} \times \text{depth}^2 \times \text{S}}{\text{length}} = \text{breaking weight} \; . \; . \; . \; . \; . \; . \; (2).$$

With such a rule for calculation, all experiments can be reduced to give that value to S which expresses the distinctive strength-measure of every material tried; for, as in an experiment the breaking weight is found by observation, and the dimensions of the beam are all known, a simple transposition of the quantities immediately evolves the value of S thus:—

$$\frac{\text{length} \times \text{breaking weight}}{\text{breadth} \times \text{depth}^2} = \text{S} \; . \; . \; . \; (3).$$

This S may therefore be called a constant, since it comes out equal, or nearly so, in amount, in every experiment on the same sort of material. To obtain such a constant for woods, irons, and stones, have the labours of different experimentalists been directed, and their results will be subsequently stated in this chapter. The above simple view of the measurement of strength in transverse strains, appears to be generally accordant with experience: the length of a beam appears to have somewhat more influence on the strength than theory allows, as similar beams are rather more than twice as strong when of half the length; and the strength does not increase quite so rapidly as the square of the depth. To test this deduction, Mr. Hodgkinson took specimens of two kinds of iron, the bars being one inch broad, 4 feet long between the supports, and 1, 3, and 5 inches deep; the breaking weights were then divided by the squares of the depths, to reduce them to the standard of the one-inch bar, with the following results*:—

	Breaking weights.		
	1 inch deep. lbs.	2 inches deep. lbs.	5 inches deep. lbs.
1. With Carron iron, No. 2, hot blast	452	427	402
2. Ditto, ditto, cold blast	453	417	414
3. With Devon iron, No. 3, hot blast	537	576	617
4. Ditto, ditto, cold blast	448	377	405
Mean	472	449	459

It will be observed, that the mean gives a greater value to the first, or inch-bar, than to the three-inch bar, while the five-inch bar, though less than the first, has a greater value than the second; this, however, is accounted for by the high amount afforded by experiment 3, on the hot blast Devon iron, which altogether is very anomalous in respect to the other specimens: omitting this, the means are—451, 407, 407; and experiments 1 and 2 exhibit a regular decrease of value, which accords with a later experiment by the same experimenter, in which he found iron bars, 1 inch, 2 inches, and 3 inches deep respectively, afford values of 447, 349, and 338†.

Again, the above general rule is somewhat affected by the condition of the ends of a beam; if it be merely lying loosely on its supports, as is now supposed, its ratio of strength to a similar beam *fixed* at each end will be as 2 to 3. The following little table, from experiments by Lieut. Denison, shows the ratio to be nearly if not quite as 1 to 2:—

Name of wood.	Value of ends fixed.	Value of ends loose.	Proportion.
Black ash	2176	861	2·5 : 1
Birch	2652	1387	1·9 : 1
Beech	2684	1380	2·0 : 1
Rock elm	3835	2311	1·66 : 1
White oak	3652	1809	2·0 : 1
Iron wood	3172	1800	1·76 : 1
Bitter nut Hickory	2478	1465	1·7 : 1

The weight or load in the last figure is supposed to be collected at the middle of the length, but where the same load is distributed over the length of the beam, it tends to break it with half the force of the central load; thus, a girder can support twice as much evenly spread over its length as when applied at the middle.

(49.) Another very useful formula is founded on two laws experimentally demonstrated by Mr. Hodgkinson, in his researches on the strongest form of cast-iron girder: one of them is, that the strength is as the area of the bottom flange of a girder, or the extended part; and the other is, that the strength is also as the depth of the girder; then, with the addition of a constant, c, having a signification similar to that of S, a simple formula for estimating the breaking weight is compiled:

$$\text{Breaking weight} = \frac{\text{area of section} \times \text{depth} \times c}{\text{length}} \quad \ldots\ldots (4).$$

This formula, when a value of c is found by experiment on any material, is only applicable to a beam or girder of a similar sectional form to that from which the value of *c* was derived, since this constant expresses the specific strength of that form of section.

---

* Brit. Ass., 7th Report, p. 365.

† Railway Structures, Commissioners' Report App. A., p. 111.

(50.) A third formula for estimating the strength of beams rests on the knowledge of the resistance (which we will call R) offered by any material, to fracture by a tensile or crushing force, and the depth *n* of this area in the beam, or the depth of the neutral axis and the power with which fibres resist the strain; the latter of course cannot be calculated except from a previous experiment: the rule is, therefore,

$$\text{Breaking weight} = \frac{\text{R} \times \text{breadth} \times \text{depth}^2}{n \times \text{length}} \; \ldots\ldots \; (5).$$

Here *n* can be readily obtained from experiments; were the extension and compression of the material equal by the action of equal forces, the neutral line would be in the middle, as already stated, and *n* would be equal to 6. According to Galileo's theory, that the fibres or particles are incompressible, we have $n = 3$, and if the power of the fibres be considered the same for all extensions, still allowing incompressibility, *n* would be equal to 2. For cast-iron, as will be observed further on, the value of *n* has been found to be between the two latter amounts, or 2·63.

(51.) In practice, the whole strength of any beam found from formula 2 can never be used, since it is the pressure at which the beam gives way: one-third of the breaking weight is commonly taken as a safe limit, although this should not be applied in all cases, for with some materials, such as wrought-iron, more than one-third can be safely allowed, as the material does not suddenly give way in breaking, from its extensile capability; whereas, cast-iron beams are less able to bear much of their breaking weight, on account of the dangerous suddenness of fracture, and its inability to sustain tensile strain with safety. The work for which a beam is intended should also guide in this estimation; if the load be a dead weight the girder may be loaded with much more, and that safely, as the article on cast-iron beams will show, than if the load be variable or moveable. It is a practice to try cast-iron girders by the hydrostatic press, or other means, after their dimensions have been estimated according to the rule; this proves them with a pressure far greater than they are likely to meet in their permanent beds, and ought never to be omitted in any case with cast-iron beams, since they are liable to be less strong than calculated, from irregularity in the casting and cooling.

(52.) We have been hitherto speaking of beams, without reference to the economy of the material in sustaining loads differently arranged, or their manner of fixture or rest. Calculation affords the following forms of beams, as able to do the most work with the least expenditure of substance:—

Beams supported at one end.

1. If the load be terminal and the depth constant, the figure of the beam in breadth should be wedge-form, the breadth increasing as the length of the beam (the latter being measured from the loaded end).

If the breadth be constant, the square of the depth must vary as the length, or the vertical section will be a parabola.

When both breadth and depth vary, the section should present a cubical parabola.

2. When a beam supports only its own weight it should be a double parabola, that is, the upper as well as the lower surface should be of a parabolic form, the depth being as the square of the length.

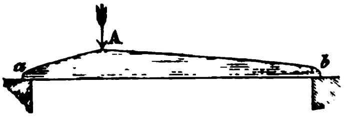
3. When a beam is loaded evenly along its surface, the upper surface being horizontal, the lower should be a straight line meeting the upper surface at the outer end, and forming a triangular vertical section; the depth at the point of support being determined by the length of the beam and the load to be sustained.

If an additional terminal load be added to such a beam, the under surface should be of an hyperbolic curvature; and in a flanged beam the lower flange should describe a parabolic curve, as in the last figure.

Beams supported at both ends.

1. A beam loaded at any point, as A (fig. 119), should have a parabolic vertical section *a* A, A *b*, each way from the loaded point. Such, therefore, should be the form of scale-beams, steam-engine beams, and all straight levers. In flanged beams the lines A *a*, A *b*, may

Fig. 119.



be nearly straight, and approach the straight line more as the flanges are thinner.

2. A beam loaded uniformly along the whole of its length, should have an elliptic outline for the upper surface *a*A*b*, the lower being straight; this form applies to girders used for bridges and other purposes where the load may be spread. With thin flanges, a beam so circumstanced should be of a parabolic figure.

3. If a flanged beam have its upper and lower sides level, and be loaded uniformly from end to end, the sides of the lower flange should have a parabolic curvature.

Deflection.—The phenomena of deflection are more intricate than those of rupture, and there is even more disagreement amongst theoretical opinions and practical results than on fracture. A few general laws, which have been calculated and are sustained more or less by natural evidence, will, however, be useful.

(53.) Within moderate pressures the deflection of a beam is as the weight, or twice the weight will produce twice as much deflection; this, however, is not the case when a load is so great as to produce much set among the fibres, that is to say, to make them move into another relative position, and when the weight is removed be unable to recover nearly their original order: this set becomes evident with the most trifling weights, although the ratio of deflection to the weight remains the same much longer in some materials than others, as the sections on cast and wrought iron will evidence. The dimensions of the beam thus affect its deflection:—It is

Directly as the cube of the length,
Inversely as the cube of the depth,
Inversely as the breadth.

With the above ratios we learn the deflection of a loaded beam in any case will be equal to the

$$\frac{\text{load} \times \text{length}^3}{\text{breadth} \times \text{depth}^3}.$$

This applies to beams which are simply resting on their ends; when these are firmly fixed, there will be but *one-fifth* of the deflection arising in the former condition.

(54.) A more general rule has been given for the estimation of deflection, but not more simple in its terms, although it dispenses with knowledge of the breadth and depth of a beam; it is—

$$\frac{\text{load} \times \text{length}^3}{48 \times \epsilon \times \text{I}} = \text{deflection};$$

ϵ signifying the modulus of elasticity of the substance (see art. 5.), and I the moment of inertia of the section of rupture, or of the beam, if it be of the same proportions throughout: this latter quantity is not easily obtained for any form of body, but as beams and girders are generally symmetrical, and the moment of inertia of symmetrical bodies is known, it is in some cases sufficiently practical (see pp. 27 and 42, DYNAMICS). The following are the moments of inertia of four forms of section, *b* standing for breadth, *d* for depth, and *r* for radius or semidiameter:—

A beam with a rectangular section,

$$\text{I} = \tfrac{1}{2} \times b \times d^3.$$

A beam with a triangular section,

$$\text{I} = \frac{1}{12} \times b \times d \times (\tfrac{1}{4} b^2 + \tfrac{1}{3} d^2).$$

A beam with a circular section,

$$\text{I} = \tfrac{1}{4} r^4 \times 3{\cdot}1416.$$

A beam hollow, of a circular section,

$$\text{I} = \tfrac{1}{4} \times 3{\cdot}1416 \times (r^4 - r_1^4).$$

In the latter case, the radius marked $_1$ means the inner, or radius of the hollow part.

(55.) It appears on a comparison of the formula when applied to similar beams, that the deflection is as the length*, but a greater deflection should

---

* That is, if δ inches be the deflection of beam of l feet long, another similar beam l_1 feet long will be deflected δ_1 inches, or,

$$l : \delta :: l_1 : \delta_1;$$

this latter amount is, therefore,

$$\delta_1 = \frac{l_1}{l} \delta.$$

This was almost physically demonstrated by Mr. Fairbairn, in the comparison of his experiment on the model tube with that on the permanent Conway tube, the former being only one-sixth the size of the latter.

be allowed for in practice than this deduction supposes.

In cylindrical forms, the breadth being equal to the depth, we may say the stiffness increases as the fourth power of the diameter. This fact points out the advantage of hollow over solid cylindrical beams, the same amount of material being made not only to afford more final strength, but very much more stiffness, which generally is a more desirable feature in girders.

A beam suffers much less deflection through a load spread over its length than from a centrally accumulated pressure, as is supposed in the above rules, the deflection in such a case being calculated at five-eighths of the beam loaded at its middle.

(56.) Beams have been said to bear considerable deflection without any injury to the elasticity of the material, and rules have been given founded on this hypothesis, for the safe permanent loading of beams; Buffon and Tredgold considered the elasticity to remain perfect until one-third of the breaking weight is laid on; but later experiments have proved this idea to be fallacious. Mr. Hodgkinson has practically shown that a very small weight injures the elasticity of material; he took an iron T shaped beam, and obtained the following results:—

Weight. lbs.	Deflection. in.	Deflection, load removed. in.
7	·015	visible.
14	·032	·001
21	·046	·002
28	·064	·004
56	·130	·005
112	·273	·020
224	·618	·058
336	1·030	·130
364	1·138 broke.	

While we may here notice, that for a considerable amount of pressure the deflections increased as the load, the third column sufficiently shows the early injury of the elastic power of the material, since 7 lbs., or 1-52nd part of the breaking weight, caused a visible set in the beam. A similar beam placed with the flange downwards thus ⊥, while it bore much greater weight, took a visible set with 1-80th of its breaking weight. The supposed limit of elasticity, therefore, cannot guide us in laying down a rule for a permanent load, since there is no such limit; and, moreover, by assuming this as a limit, much available strength has been lost, as the same experimenter found that cast-iron beams bore two-thirds, and even more of their breaking weight for long periods, without any indication of failing (see subsequent section on cast-iron). Mr. Hodgkinson found the set in cast-iron beams to be as the square of the load.

Impact and Motion on Beams.

These are subjects of especial interest in practice, as the estimation of real working strength and safety in beams, joists, girders, railway rails, and other parts of structures mechanically similar, involve the effects of sudden concussions which are very likely to occur, and of rapidly-moving weights which in railway bridges always occur, and which are so different from the effects of statical pressure, such as is applied to determine the strength of the material. A mass of most interesting information on these inquiries has been collected by a national commission on railway structures, which, though not so conclusive in the establishment of laws as could be desired, owing to the great difficulty of some parts of the inquiry, gives numerous valuable facts.

(57.) Mr. Hodgkinson, from experiments on iron bars, deduced several facts of an interesting nature, which are as follow:—

1. That cast-iron beams on being struck by heavy masses, or balls of metal of different kinds, were deflected through the same distance, whatever metal was used, provided that the weights of the masses were equal.
2. That the impinging masses rebounded after the stroke through the same distances, whatever was the kind of metal of which they were composed, provided the weights were equal.
3. That the effect of the masses of different metals striking an iron beam, were entirely independent of their elasticities, and were the same as they would give if the impinging masses were inelastic*.

(58.) The same experimenter, in gathering an Appendix for the Commissioners' Report on Railway Structures, adopted the mode of trial of allowing cast-iron balls, from 75½ lbs. to 603 lbs. weight, to fall suddenly from various heights on the bars to be tried; or the balls were suspended like a pendulum, and falling through any given arc, according to the velocity required, gave the bar an horizontal blow when at the lowest point in its oscillation.

---

* British Association for the Advancement of Science, 5th Report, 1835, p. 107.

The deflection of bars from impact was not found to be less whether the greater or less thickness of a bar was placed in the direction of the blow; the mass of the bar only, no matter whether disposed in breadth or depth, affected the deflection. Thus, a cast-iron bar 6 inches thick and 1½ inch wide, by 13½ feet long, was placed so that the broader side was parallel with the direction of impact, and struck by a ball 603 lbs. weight; when the work done by the ball was 766 (that is, the weight of the ball multiplied by the height it fell) it broke. Again, a similar bar was placed so that its narrow side was in the direction of impact, and it broke when the work done by the ball was 728. Similarly, a bar three inches square, which was of course equal to the other in area, was broken when the working force of the ball attained to 747. These numbers are certainly various, and show that the greater depth has somewhat more strength, while other experiments brought the results more nearly equal; but the deflection arising from equal impacts is greater in the beam of lesser depth. From a like principle, a cast-iron bar evenly loaded with extra weights, which in an ordinary trial would have the effect of reducing its available power, was found capable of more powerfully resisting impact, even to twice as much as the simple bar.

The manufacture of the iron by hot or cold blast, seems to have some, though an uncertain effect, on its power to resist impact: Mr. Fairbairn gives the following table* of different irons, in which the power of cold blast is taken at 1000, and that of hot blast given in ratio; while several irons appear to be less strong when hot blast made, others are stronger, and the Devon iron, hot blast, which generally is much improved by the hot blast, is above 2¾ times as strong as the same kind smelted with a cold blast.

	Power of cold blast.		Power of hot blast.
Carron iron, No. 2	1000	:	1005·1
Devon iron, No. 3	1000	:	2785·6
Buttery iron, No. 1	1000	:	962·1
Coed Talon iron, No. 2 . . .	1000	:	1234
Coed Talon iron, No. 3 . . .	1000	:	925
Elsicar and Milton iron . . .	1000	:	875
Carron iron, No. 3	1000	:	1201
Muirkirk iron, No. 1	1000	:	823

(59.) Beams of wrought-iron are deflected nearly in proportion to the velocity of the striking weight, but with cast-iron the proportion is greater, doubtless arising from the inferior elastic resistance of the latter.

(60.) The results of trial on the effect of long-continued impacts, or concussions, led to the conclusion that in practice it is scarcely safe to load beams constantly to one-third of their ultimate deflection; and that they ought not to be loaded with more than one-sixth of their breaking weight (as laid on rapidly). A wrought-iron tube 45 feet between supports was found greatly injured in the riveting, after many impacts from a mass weighing 2¾ tons, or nearly one-half the weight of the tube, although the deflection produced by the impacts was no-more than one-fifth that which would be required in a trial with dead weights to injure the tube.

(61.) The effects of running loads has been investigated both mathematically and experimentally, though there is great difficulty in treating the subject by the former method, since the curve given to any beam through the rapid passage of a four or six-wheeled carriage is different to that arising from the passage of a load touching in one point only: the latter is simple, and suited to mathematical inquiry, but the former is complex, and multiplies variable quantities in the calculation; still much that is useful has been disclosed, and important corrections made in opinions popularly held by practical men. A great number of experiments were made by royal commission, at Portsmouth, by Lieut. Galton and Capt. James, who arranged a small railway, curving up at each end to a height of nearly 40 feet, by which means the carriage intended to try the bars was impressed with velocities up to 43 feet per second, or about 30 miles per hour. In the horizontal part of this railway were placed two bars of equal size, carrying pencils, by which to mark on a vertical board covered with paper, the depth of deflection when the carriage passed over them. Mr. Willis also experimented in a similar way at Cambridge, but allowed the weight of the carriage to press on a single bar placed in the middle, in one point only: some beautiful operations were conducted with these instruments, from which it appeared that the effects of a running load was greater than those of the same load when at rest, and on the middle of the bar; also, that the deflection of the bar increased as the velocity of motion increased. Thus, in some experiments, with a trial railway carriage weighing in all 1120 lbs., running over bars 9 feet long and 4 inches broad, and 1½ inch deep, the deflection caused by this carriage at rest was six-tenths of an inch; on giving a velocity to the carriage of ten miles per hour, the deflection, as it passed, was eight-tenths of an inch; and a velocity of 30 miles an hour increased it to one inch and a half, or above double that arising from the load when quietly placed on the bar. Where 4150 lbs. quietly laid on broke the bars, 1778 lbs. effected the fracture when passing over them at a rate of 30 miles an hour. Two experiments were made on railway girder bridges, to verify these results, so far as regards the increase of deflection with increase of velocity; a pencil was attached to the flange of a girder, and marked the depression on a paper fixed on a scaffold; they were as follows:—

---

* Brit. Assoc., 7th Report, p. 415.

† Report of Commissioners on Application of Iron to Railway Structures. App. B.

Ewell Bridge (Epsom and Croydon Railway), 48 feet span.

Total weight of half the bridge . . . . 30 tons.
Weight of engine (25·2 tons), and tender (13·8 tons) . . 39 tons.

Velocity in feet per second.	Deflection.
0	·215
25	·215
30·9	·230
32·3	·225
53·7	·245
75	·235

Godstone Bridge (South Eastern Railway), 30 feet span.

Total weight of half the bridge . . . . 25 tons.
Weight of engine and tender . . . . 33 tons.

Velocity in feet per second.	Deflection.
0	·19
22	·23
40	·22
73	·25

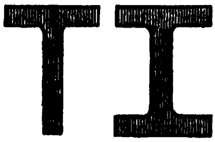
These deflections are irregular, but show a decided increase of deflection with a greater speed. Ewell Bridge bent one-seventh more when the engine ran over it at 75 feet per second, or about 51 miles an hour, the deflection being then equal to that produced by a quiet pressure of 45 tons.

It was found in the experiments, that the curve assumed by the bar during the transit of the carriage, was deepest near the further end of the bar, showing that the effects of the running load are cumulative; but little deflection occurs at first, but when three-quarters of the length of the bar is travelled over, the wave of force seems to gain its greatest power, and rapidly subsides at the other extremity: such a bending must greatly strain a bar. The experiments showed that fracture generally occurred beyond the centre, and sometimes in three, four, or five places.

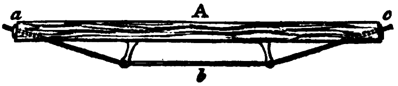
(62.) The natural principle of the neutral axis is full of practical interest and usefulness: it is the foundation of nearly all the improvements in beams and girders, and of the most gigantic attempts lately made to apply girder forms to the support of large bridges. As it appears that the upper and lower edges of a beam have to sustain the greatest strain, and must break first, the whole of the material of the beam would do most work, or afford its whole strength, if it could be placed in those parts. And again, if the farther the fibres are from the neutral line the more they resist deflection from the superincumbent load, it must be inferred that the material should be placed as far above and below the neutral line as other circumstances will allow, in order that they may be in a position to exercise the greatest power. The most simple and immediate application of these views is shown in Mr. Lavé's girder, which, while simple, appears to be a very effective contrivance. It is a beam cut nearly from end to end, and bound at each termination of the longitudinal cut with an iron strap; blocks, or little posts, are then driven in the cut, separating the severed pieces to several inches distance in the middle of the length, and thereby throwing the material farther above and below the neutral axis. A trial was made with a beam 40 feet long, and about 70 square inches in section, which deflected 5¼ inches with a load of 1700 lbs.; but when the upper and lower parts had been separated by the process 9¼ inches, it suffered a bending less than in the former condition by nearly three inches. To obtain the greatest gain of strength, the cut should be made so as to give a section to the upper and lower parts, proportional to the power of the material to resist tension and compression.

Arising from the preceding considerations, we have flanged, open, trussed, and tubular or hollow girders.

The flange beam is particularly applicable to iron, which can be cast or built up of rolled plates to any figure. It is simply an example of throwing the whole, or nearly the whole, of the material into the upper and lower edge, leaving but a thin plate or rib between to keep them at a proper distance asunder; this latter portion has little or nothing to do with the pressure directly, its principal function being to keep the two working beams or flanges in their place; the rib is, therefore, of great moment, although it can afford comparatively no resistance to either tension or compression, for except it were between the flanges no transmission of the forces could take place between them, and if it is injured, the beam is in a dangerous condition. A bridge at Chester fell some time since, while a train was passing over it, in consequence, it is said, of the engine running off the rails and breaking the thin web of one of the girders (see p. 180, *Equilibrium of Structures*). Proceeding on the same ideas, this rib has been replaced by simple upright struts, or diagonal braces between the flanges, which in cast-iron girders are one casting with the flanges, but experience has proved this not altogether politic, particularly in cast iron; Mr. Hodgkinson remarked, that such beams were weaker than those with a solid rib. These improved beams not only gain much strength for the structure supported by them on account of their advantageous form, but also a great deal of available or free strength which would otherwise have been required to sustain the material here dispensed with between the flanges: there is somewhat less strength in a flanged beam than in one solid of equal depth and breadth, but not at all proportionate to the weight of material avoided. A further improvement is the giving of so much material to the upper and lower flanges as may be pointed out to be a proper proportion by the ratio of strength of tension and compression exhibited by the material. As iron resists fracture about six times more powerfully under compression than under tension, it is useless to give as much area of material in the upper or compressed, as in the lower or extended flange of a cast-iron beam; attention to this fact has produced a cast-iron flanged girder, nearly half as strong again as an equal flanged girder, with the same amount of substance.



In generic association with the flanged girder, is the trussed girder, the difference being, that the rod or band *a b c* is generally of material different from the body A of the girder, and the rod has its main attachment at the ends



a, *c*; otherwise, the result sought and gained is the same—that of gaining distance for the working parts from the neutral axis without extra weight of material. The rod takes the principal part, or all of the tensile force, according as the beam A is left level, or by screwing at the end *a*, *c*, made to camber or curve middle upwards, before the load is superposed. This tension rod is useful in proportion to its distance from the beam (evidently within certain limits); if it be immediately under, or concealed, as in some cases within the under edge, it becomes nearly useless, especially if, as in a cast-iron beam with a wrought-iron rod, the beam is much less extensible than the rod; in such a case, the beam would break and fall before the rod has been brought into action*. The respective size, or sectional area of the tension rod and that of the beam should be regulated, as in the flanged beam, by the respective strength of the materials; it is useless to apply a rod capable of sustaining double the tensile force that the beam can of crushing force, and vice versâ; it is merely adding weight, and consequently diminishes the available strength of the girder.

Tubular beams are perfectly analogous to the flanged and trussed beams in their mechanical principle, but possess much practical superiority, owing to the distribution of the material. Mr. Fairbairn's interesting experimental researches on wrought-iron tubes, wherein he developed the Conway and Britannia tubular bridges, well illustrate the powers of hollow beams. A brief contemplation of the tables he compiled (see pp. 185–7, *Equilibrium of Structures*), although direct numerical comparison is wanting, will convince the judgment that the breaking weights of the tubes are surprisingly great for such thin plate and the little area presented by some of the tubes. In the first three trials on circular tubes, we find plates of 1–25th, 1–27th, and 1–9th of an inch thick, offering to the tensile and compressive forces areas of scarcely 1⅗ths, 1⅔ths, and 4⅖ths inches, yet they bore weights of 3040 lbs., 2704 lbs., and 11,440 lbs. respectively. As affording some comparison with the latter, may be mentioned an experiment on a double-flanged wrought-iron beam about two-thirds the length of the tube, and nearly 8½ inches deep, with an area of $6\frac{3}{10}$ths inches, which bore 12,955 lbs. before twisting so as to be useless; but making allowance for the extra length of the tube, which was one-third more than that of the girder, the ratio of the strength of the two is 20 to 17, while their areas are as 22 to 31½, so that though greater in strength, the tube had less surface of material by nearly one-third, to resist the breaking weight. Similarly, the elliptical and rectangular tubes afford examples of great strength, due to the disposition of the material in a tubular form. It is calculated that the greatest strength is gained with any given quantity of material when its thickness is three-twentieths of the diameter of the tube; the strength is then double that of a solid cylinder having the same quantity of material.

The physical student will be pleased to trace in natural structures abundant exemplifications of these great principles. In the animal kingdom the exo-skeletons, or animals with external skeletons, such as crabs and insects, have frequently a small total amount of osseous

---

* These remarks are well verified by an experiment made at the instance of Mr. Cubitt, on an equal-flanged iron girder 27 feet long, 10 inches deep, and 4 inches broad across the flanges; two frames were firmly fixed at each end, so that the ends *a*, *c*, of the tension-rod might be elevated at some height above the lower flange of the girder (which some have ridiculously practised); this height we will call *ah*; the horizontal portion *b* of the tension-rod was also kept by struts at different depths (which we may call *ad*) below the lower flange of the girder. Without any trussing the girder deflected 2½ inches with a weight of 4 tons, and when the rods one inch in diameter were added, weights were placed on in each trial until the girder deflected the same amount, namely, 2½ inches.

Trials.	*ah* in inches.	*ad* in inches.	Weight producing deflection.	
1	30	1	3 tons.	19 cwt.
2	24	1	4	0
3	18	1	4	1
4	10	1	4	11
5	24	6¼	4	8
6	10	6¼	6	1
7	5	6¼	5	12

This table shows that placing the ends of the rod above the beam makes it weaker than having no rod at all; and, comparing trials 4 and 6, where the fastening was made at 10 inches, or at the upper edge of the girder, we see that giving a distance to the rod of 6¼ inches instead of one inch from the bottom flange, gave an increased stiffness of above 1 ton.

substance, but by the disposal of it in a hollow form it obtains the greatest amount of leverage over any contrary force when the animal is in motion, and supplies a stiffness which is necessary for the well-being of the creature; were the flesh-inclosing skeleton of many of these animals, especially of the insects, collected into a solid internal frame, a gentle wave of the fluid they inhabit, or the slightest attempt to move, would inevitably effect the dislocation of their system. Internal, or endo-skeletons, also afford some illustrations of an analogous kind; thus, the long bones in the human subject are tubular, the compact osseous matter being thicker at the middle of the bone than at the ends, which spread out to form the joints; and it is worthy of observation, that as the bone increases in diameter by the external addition of fresh matter, so the tubular or hollow portion increases, preserving of course a proportion to the thickness of its bony sheath: this beautiful fact has been demonstrated by Duhamel and others, who, by fastening a silver ring round a growing bone, found it some time afterwards in the medulla or marrow which fills the interior. The thickness of the bony cylinder appears to be inconstant, and varies with age. In the stems of vegetable existences, similar laws appear to regulate their arrangement;—the grasses are a large order of plants whose flowers are borne in some cases at a great height above the soil by a simple tubular stem, containing a very small amount of material, yet possessing great strength, which it owes mainly to an external coating of flint, carried by the plant through its juicy vessels and thrown out over the stem. Mechanically considered, such a stem loaded by the flower at its extremity, and pressed by the aerial currents from top to bottom, is as a beam, fixed at one end in a wall, evenly loaded throughout, with an additional weight at its free end. This construction of the stem is more or less shown throughout the vegetable kingdom; in the oak and other exogenous trees, where the sap moves just within the bark, we find the heart-wood or most solid part taking a more interior space, while the central parts of the trunk are filled with medulla, or pith: here the heart-wood is the skeleton, as it forms the support of the whole system; and as the external parts of the tree extend, so does this tubular endo-skeleton increase in diameter, to afford the greater stiffness and strength necessary to sustain the greater strain. The larger endogens, such as palms and bananas, are not less characterised by this universal principle; the stem, or trunk, as compared with the exogens, is thin, and never increases in diameter, but owing to its development from the centre, the outer parts become compacted, and, therefore, the most powerful in resisting transverse pressures; in this sense such a stem may be compared to exo-skeleton animals. In the banana the stem is quite a thin cylinder, being made up of the sheaths of the leaves which surround the bases of the former flower-stalk. There is no doubt that if naturalists had observed their subjects closely, or with more generalizing views, we might in this portion of science have developed many points of connection between these leading principles and the order of nature, alike interesting to the physico-theologian and the student of general science.

Wood.

(63.) From the comparative facility of experimenting on this kind of strain in comparison with tension or compression, we are in possession of numerous trials on bars and beams of wood. Buffon made an extensive series of experiments, under the patronage of the French government. They were on oak, from 20 to 28 feet long, and from 4 to 8 inches square in section; the heart-wood, which was the densest, proved also the strongest, and the side on which the beam was laid also affected the strength: when the annual layers were horizontal, and the strength 7, the vertically laid layers gave a strength of 8; this probably arises from greater attachment between the fibres of a layer or annual ring of the trunk than between the several layers, as Mr. Barlow supposes. He found that the longer beams exhibited a falling off of strength, which increased in a ratio somewhat greater than their length.

Mr. Barlow experimented on several kinds of timber, observing the deflections and other behaviour of the beams during the trials; his results, with those of Messrs. Nelson, Moore, Denison*, and some others, are given in the following summary, which gives the mean of the whole: the numbers given are the values of S (see equation 3, art. 50), or the specific strength of each wood.

---

* See Papers of the Royal Engineers, vol. v.

Kind of wood.	Specific gravity.	Value of S.
Ash, English	·760	506
„ American	·626	450
Beech, English	·696	390
„ American white	·711	345
„ „ red	·775	435
Birch, common	·711	482
Cedar of Lebanon	·330	373
Elm, English	·579	200
Hiccory, American	·831	532
Oak, English	·829	424
„ African	·988	630
„ American white	·779	433
„ „ red	·952	422
„ Dantzig	·720	377
„ Memel	·727	416
Pine, American white	·432	307
„ „ red	·576	382
Pine, American yellow	·508	300
„ „ pitch	·740	432
„ Dantzig	·649	356
„ Memel	·601	334
„ Riga	·654	346
Fir, Spruce	·503	336
„ „ American	·772	260
„ Mar Forest	·698	308
Deal, Christiana	·689	400
Larch	·505	256
„ American (Tamarak)	·433	230
Mahogany, Nassau	·668	430
Teak	·729	527

The above table must be considered as giving only approximative values of the strength of timber; the great variety of circumstances affecting the strength of a beam of wood would require the mention of every particular, from the state of the soil on which it is developed to the position in which it is placed on its permanent bed, or the supports on which it is broken, in order to arrive at the exact values. None seem to have noticed these things more than M. Buffon, whose experiments at once destroy all implicit confidence in any trifling experiment. Speaking of the inferior power of a beam when it is laid so that the annual layers (of course if it be an exogen) are placed horizontally, and its superior power when those layers are vertical, and the like results when beams are cut from branches and trunk, he says, "These remarks convince us how little we should depend on the calculated tables or the formulæ different authors have given for estimating the strength of timber, which they had proved by pieces, of which the largest were one or two inches in thickness, and of which they give neither the number of woody layers that these bars contained, nor their position, nor the direction in which they are found when the piece broke,—circumstances, nevertheless, essential, as is shown by my experiments, and the care that I have taken to discover the effects of all these differences. Those who have made some experiments on the strength of timber, have paid no attention to these modifying conditions, but there are probably others still greater that they have neglected to foresee or prevent. Young wood is weaker than old; a piece taken from the foot of a tree sustains more than another from the summit of the same tree; a beam cut from the outer parts near the sap is less strong than an equal beam from the central portions: also, the degree of dryness adds much to the strength of timber; green wood breaks much more difficultly than dry. Lastly, the time employed in loading the pieces with their breaking weights should also be considered; because a rod which will sustain a certain weight during several minutes, may not support it for an hour, and I have found beams which had each supported, without breaking, a load of 9000 during one day, break at the end of five or six months with a weight of 6000; that is to say, they were unable to carry for six months, two-thirds of the weight they bore for one day. All this proves how imperfect are the experiments which have been made on this subject, and shows likewise that they are not very easily executed."

Iron.

(64.) On *Cast Iron* there is a vast accumulation of experimental facts, and the knowledge of transverse strain, in a practical respect, is considerable. In 1803 a few experiments were made by Mr. Banks on bars one inch square and three feet apart; a mean breaking weight in three trials was $971\frac{1}{3}$ lbs.; these give for the specific strength S 2914 lbs., which is much higher than later observations allow; it is, however, stated to have been very flexible iron, which would afford greater strength. In 1814 M. Rondelet published several experiments on bars nearly 4 feet long, and about one inch square: the gray irons afforded a value of S = 1780, and the soft irons S = 2971, the latter giving a value not much different from a similar kind of cast iron used by Mr. Banks. Mr. Tredgold made a number of experiments, but some of his results are inconsistent; he, moreover, supposed that a bar 34 inches long between its supports, and one inch square, would sustain 300 lbs. without injury to the elastic power of its particles, and that that part of the bar resisting tension would sustain 15,300 lbs. on the square inch, without injury to its elasticity*, a result wholly unsustained by later and more correct experiments.

The most extensive and valuable researches on the strength of cast-iron beams have been made by Messrs. Hodgkinson and Fairbairn, who have supplied this branch of mechanical science with ample data for practical application. Their labours have been directed to the general strength of irons from different foundries, both hot and cold blast; to the powers of different forms of beam or girder; to the supposed limit of elasticity; and to the effects of temperature and long-continued pressure on cast-iron beams. Under this classification we shall notice their experiments.

1. Mr. Fairbairn obtained pieces of iron from the principal iron works, and cast specimens from a model five feet long and one inch square; others of half the length were also broken, and, with the larger, fully developed for cast iron the elementary laws stated in the preceding paragraphs. They proved that the longer beams are weaker than the shorter in a greater proportion than their respective length; that the strength does not increase quite so rapidly as the square of the depth; that the deflection of a beam is proportional to the force or load; and that a set occurs with a small portion of the breaking weight. Fifty-nine irons, hot and cold blast, were subject to experiment, with the following extreme and mean results:—

	sp. gr.	Breaking weight.	Ultimate deflection.
Strongest, Ponkey, No. 3, cold blast	7·122	578 lbs.	1·74 hard
Weakest, Plaskynaston, No. 2, hot blast . .	6·916	357 „	1·36 soft
Mean value		440 „	

This mean value affords for the specific strength,

$$S = 1980 \text{ lbs.}\dagger,$$

or, if the constant c (in tons), art. 49, be used, we have

$$c = \cdot 884 \text{ tons.}$$

If the rule given in art. 50 be used, it is found, from a comparison of ten specimens, that

$$n = 2\cdot 63;$$

we have, therefore, for these different formula the following working values:—

$$\frac{\text{breadth} \times \text{depth}^2 \times 1980}{\text{length}} = \text{breaking weight.}$$

$$\frac{\text{area of section} \times \text{depth} \times \cdot 884}{\text{length}} = \text{breaking weight (in tons).}$$

$$\frac{\text{breadth} \times \text{depth}^2 \times 7}{2\cdot 63 \times \text{length}} = \text{breaking weight (do.).}$$

A table embodying the various results of Mr. Fairbairn's experiments, will be found in the Appendix.

A series of experiments were made to test the comparative transverse strength of iron from furnaces supplied with hot

---

* Tredgold, Essay on the Strength of Cast-Iron, 4th edit.

† Mr. Morris Stirling has considerably strengthened cast iron by adding a portion of malleable scrap iron, as the following values of S from four experiments by Hodgkinson (Railway Commiss. Rep., App. A., p. 82–3, 90–1) will show; No. 2 quality having 20 per cent., and No. 3 quality 15 per cent. of scrap iron:—

No. 2, bars 9 ft. long, 2 in. square . S = 2248
No. 3, bars 9 ft. long, 2 in. square . S = 1682
No. 2, bars 4 ft. 6 in. long, 1 in. sq. S = 2803
No. 3, „ „ S = 1996

Mr. Stirling's irons are also stronger under tension and compression; in the Reports just now quoted from, experiments show that the two qualities bear these forces as follow:—

	Tensile power.	Compression power.
No. 2 . .	11·50 tons.	54·62 tons.
No. 3 . .	10·47 „	64·41 „

and cold blasts, which, as we have noticed in considering tension and compression, slightly affect some kinds of the metal. The subjoined little table affords at one view all the results obtained by this proceeding.

Kind of iron.	Ratio of strength, that of the cold blast being represented by 1000.
Carron iron, No. 2 . . .	1000 : 990·9
Devon do., No. 3 . . .	1000 : 1416·9
Buffery do., No. 1 . . .	1000 : 930·7
Coed Talon do., No. 2 .	1000 : 1007
Coed Talon do., No. 3 .	1000 : 927
Elsicar and Milton do.	1000 : 818
Carron do., No. 3 . . .	1000 : 1181
Muirkirk do., No. 1 . .	1000 : 927
Mean	1000 : 1024·8

The internal appearance of two irons of hot and cold blast smelting, has been detailed by Mr. Fairbairn, and is of considerable interest*.

"The Carron, No. 2, cold blast iron, when viewed with the microscope, presents a dull grey colour, finely granulated, with an appearance of greater porosity in the centre than round the extreme edges of the fracture. It is a free working iron, easily cut with the turning tool, but indicates stiffness under the file.

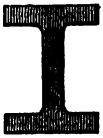
"Carron, No. 2, hot blast. This iron has nearly the same character in its working properties as the above; it files with rather more freedom, and possesses an appearance of greater fluidity than the cold blast. Colour, a greyish blue, accompanied with a greater degree of uniformity in its crystalline structure than the cold blast.

"Buffery, No. 1, cold blast, is finer grained than either of the Carron irons. It is chiefly composed of minute granules intermixed with small brown specks: it works with less freedom than the hot blast, and cuts with difficulty under the tool. In this respect it is much akin to the Milton iron, described in Mr. Fairbairn's paper.

"Buffery, No. 1, hot blast, has a similar appearance to the Carron, No. 2, cold blast; it has more lustre than Buffery, No. 1, cold blast; the crystals are widely separated in the centre, but more compact as they approach the outer edge of the bar.

"This appearance is nearly peculiar to the whole of the hot-blast irons."

2. Early in the use of iron girders, it was noticed that a simple rectangular form was not the most economic. It appears that Boulton and Watt in 1800 used a kind of girder very similar to the ⊥ form, except that the vertical rib was broader as it approached the bottom horizontal piece, or flange; and for a long period the ⊥ beam has been in general use. Mr. Tredgold, from his experiments on cast iron, or, rather, his hypothetical views in connection with them, proposed the figure in the annexed sectional view as most calculated to yield strength: he judged correctly, supposing his beam to be under the influence of a very moderate pressure, but the breaking weight of such a beam would be much less than that of one arranged according to the powers of the material to sustain tension and compression. Mr. Tredgold thought that until the elasticity became injured, the powers of resisting tensile and compressive forces were equal, and that a beam with equal upper and lower flanges was therefore strongest; but the idea of perfect elasticity under one-third of the breaking weight proved incorrect: it has no truth as regards the breaking weight of the beam. A convincing proof was obtained by Mr. Hodgkinson, at the outset of his research, that the tension and compression strength were unequal, by an experiment on a T shaped beam. Two bars, 4¼ feet long, and the flange 4 inches broad, the rib 1$\frac{1}{10}$th inch deep, with a thickness of metal about ¼th of an inch, were tried, one with the flange uppermost, and the other with the rib uppermost; they broke as follows:—



Breaking weight.

1. T, or flange at top . . . 2½ cwt.
2. ⊥, or flange at bottom . 9 ,,

The ratio of strength in the two was, therefore, as 10 : 36; or, laying the bar on its flange, it bore above three and a half times as much as in the former position.

Mr. Hodgkinson then proceeded to determine the best distribution of metal in a double-flanged beam, and after gradually varying the section in the lower flange he arrived at a ratio of section which gave him nearly half as much again of strength, in comparison with the equal-flanged beam. The subjoined condensed table tracks his progress*:—

---

* Brit. Ass. 7th Report, p. 415.

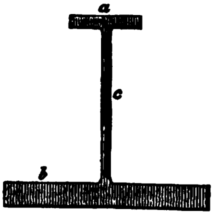
No. of exp.	Ratio of section of flanges.	Area of whole section in sq. inches.	Strength per sq. inch of section in lbs.
1	1 to 1	2·82	2368
2	1 to 2	2·87	2867
3	1 to 4	3·02	2737
4	1 to 4·5	3·37	3183
5	1 to 5·5	5·00	3346
6	1 to 6·1	6·40	4075

With the last-mentioned experiment (No. 6) the gain of strength became two-fifths, while the saving of metal amounted to three-tenths.

In this experiment the width and depth of the flanges were as follow:—top, 2·33 in. wide, 0·31 in. deep; bottom, 6·67 in. broad, 0·66 in. deep; the rib between was ·266 in. thick; and the total depth 5⅛ inches; the areas were then as follow:—

area of top flange = ·720 in.
area of bottom flange = 4·400 in.

The subjoined figure of this beam is from Mr. Hodgkinson's drawing.



The experimenter found that the strength in such a form of beam increased as the depth, and also as the area or size of the bottom flange; thus, the rule given in art. 49 applies here when the value of c is found from these experiments. It appeared to be 514 cwt., which is a mean between beams cast horizontally and erectly, the latter being somewhat stronger than the former; putting this value into tons, we have 26 tons as the worth of c, and the rule for all beams similar to the above figure is—

$$\text{Breaking weight} = \frac{26 \times \text{area bottom flange} \times \text{depth}}{\text{length}}.$$

All these measures of size are supposed to be in inches†.

3. There is now abundant evidence that the elasticity of a cast-iron beam is soon injured, and the deflection goes on increasing in relation to the weight. The set increases nearly as the square of the weight, and also as the square of the deflection. Mr. Hodgkinson tried some inch square bars, with bearings 4½ feet apart, and from 12 kinds of iron obtained the following results: the upper row of figures denoting the weights applied in half-hundreds, the middle row exhibiting the experimentally found amounts of sets under each half-hundred weight, and the lower row showing the computed set from a formula, showing that the square of the weight supplies a true calculative law of the relation of the weight to the set.

Weights.	1	2	3	4	5	6	7
Observed set	·0037	·0127	·028	·049	·076	·119	·149
Calculated set $= \frac{\text{weight}^2}{328}$.	·0030	·0122	·027	·049	·073	·110	·149

Subsequent investigations fully bear out the observation that the smallest weight causes a set, and that the deflection therefore increases more rapidly than the weight or load applied. A careful experiment lately recorded by Mr. Hodgkinson‡, on a bar 13½ feet long, 1¼ inch deep, and 3 inches broad, well exhibits this increase from the first load; by dividing the weights by the deflection we obtain the following numbers, which, as they decrease in value show the increase in the deflection above the simple proportion to the weight:—

---

* Moseley, Mech. Eng., p. 558.

† Tredgold's Essay on Cast Iron, 2nd part, p. 444.

‡ Reports of the Commissioners appointed to Inquire into the application of Iron to Railway Structures. Appendix A.

Weight.	Weight Deflection.
28 lbs.	154·7
56	149·2
112	148·7
168	141·9
224	137·3
280	133·0
336	129·0
392	123·7
448	119·3
504	114·5
560	111·2
616	106·6
672	102·4
728 lbs.	95·7
784	89·8
840 nearly broke.	85·0

4. To discover what effect different loads would have, when beams were retained under their pressure for long periods of time, Mr. Fairbairn subjected some beams, one inch square in section, and 4½ feet long between their supports, to from two-thirds to nearly their breaking weight for five years, and noticed the succeeding consequences*:—

Date of Observation.	Temperature of the air at time of observation.	Experiment 1. Hot-blast iron. Depth of bar, 1·030; breadth of bar, 1·010 in.	Experiment 3. Cold-blast iron. Depth of bar, 1·030; breadth, 1·020 in.	Experiment 8. Cold-blast iron. Depth of bar, 1·020; breadth, 1·030 in.
	Fahrenheit.	Deflexions. Load 280 lbs.	Deflexions. Load 336 lbs.	Deflexions. Load 448 lbs.
1837, March 6	...	...	1·267	1·410
„ „ 9	49°	·916	1·270	1·413
„ „ 11	...	·980	1·270	1·413
„ „ 17	...	...	...	1·413
„ April 15	47	·930	1·271	1·422
„ May 31	62	·932	1·274	1·424
„ Aug. 22	70	·937	1·288	1·438
„ Nov. 18	45	·942	1·285	1·431
1838, Jan. 8	38	·941	1·288	1·430
„ March 12	51	·945	1·298	1·439
„ June 23	78	·963	1·316	1·457
1839, Feb. 7	54	·950	1·293	1·433
„ July 5	72	·959	1·305	1·446
„ Nov. 7	50	·955	1·303	1·445
„ Dec. 9	39	·956	1·303	1·445
1840, Feb. 14	50	·955	1·305	1·446
„ April 27	63	·954	1·309	1·445
„ June 6	61	·951	1·303	1·445
„ Aug. 3	74	·953	1·305	1·447
„ Sept. 14	55	1·047	1·305	1·447
1841, Nov. 22	50	1·045	1·306	1·449
1842, April 17	58	...	1·308	1·449

After Aug. 3, 1840, it appears that a body must have fallen on the bar, exp. 1, increasing its deflection.

Exp. 9 was a hot-blast bar, 1·04 inch deep, and 1·01 inch broad; which broke down with 392 lbs. Other hot-blast bars also broke with 448 lbs.

Comparing the third, fourth, and fifth columns, there appears to be very little difference in the increase of deflection after the first superposition of the load, whether two-thirds or most of the breaking weight be imposed: taking the top row of figures, denoting the deflections, and subtracting them from those opposite August 3, 1840—giving a space of three years and five months—we obtain the increase of deflection:

With 280 lbs. ·953 — ·916 = ·037.
With 336 lbs. 1·305 — 1·267 = ·038.
With 448 lbs. 1·447 — 1·410 = ·037.

The increase of deflection during the whole time was, therefore, ·037, or nearly 1-3rd of an inch, and proves, in connection with the last article, the impropriety of adopting any rule founded on elastic limit, since it is evident that, while the elasticity of a bar is injured as soon as a weight is applied, the particles or fibres take up fresh positions, until the antagonistic forces in the beam are brought nearly to equality, when one-third or two-thirds of the breaking weight equally affect the subsequent deflection of the beam.

---

* Brit. Assoc. 7th Rep.

5. As temperature so mightily affects the density of bodies, or the relative closeness of its atoms, Mr. Fairbairn was led to try the effect of great changes in this particular on cast-iron beams. As the following table will show*, he subjected several bars of Coed Talon iron 2½ feet long and one inch square in section, to rather extreme conditions—some in ice, or surrounded with snow and salt, others in melted lead, or to a heat red in the daylight.

Temperature, Fahrenheit.	Breaking Weight.		Ratio of the strengths of the two irons.
	Cold blast.	Hot blast.	
	No. 2 Iron. lbs.	No. 2 Iron. lbs.	
16°		800·3	1000 : 967·2
26°	851·0	823·1	
32°	940·7 958·5 mean 949·6	933·4 906·0 mean 919·7	1000 : 977·6
190°	743·1	823·6	1000 : 1108·3
Red in the dark	723·1	829·7	
Perceptibly red in daylight .	663·3		
	No. 3 Iron.	No. 3 Iron.	
212°	905·0 943·6 mean 924·3	818·4	1000 : 885·4
600°	909·3 1157·0 mean 1033·1	834·1 917·5 mean 857·8	1000 : 847·7

It would hence appear that up to 600° the strength of iron remains practically perfect, but at a red heat its power fails. As in North America and elsewhere, beams are subject to a temperature of 40° below zero, it would have been additionally interesting if the table had included an experiment under such circumstances.

(65.) *Wrought Iron.*—The transverse strength of this material is not yet sufficiently known; beyond a few railway-bar experiments by Mr. Barlow, and some scattered observations, there has been little done to determine the specific strength of solid beams of wrought iron; of iron-plate or tubular beams we have, however, a great mass of information, owing to the study of them called forth by the proposal of the tubular bridges. As a safe material for supports of any kind, whether roofs or bridges, it is greatly preferable to cast iron, since it is so elastic, which it preserves under a great pressure, and affords such timely notice of fracture.

Mr. Fairbairn tried two girders of wrought iron of the double-flanged or I kind, with the following results†:—

	Area of top flange.	Area of bottom flange.	Height.	Rib. Thickness.	Length between supports.	Deflection.	Weight applied.
1.	1 × 2·5 in.	·380 × 4 in.	7 in.	·320 in.	11 ft.	·60	12,953
2.	1 × 2¾ ,,	·440 × 4·3 ,,	8 ,,	·350 ,,	10 ,,	·68	18,962

With these weights the experiments were discontinued, as the beams bent laterally, even to failing, in consequence; another beam acted similarly, showing that the common cast-iron girder form was not so well suited to the more ductile wrought iron. From these trials we obtain, taking the area of the bottom flange,

---

* Brit. Assoc. 7th Report, p. 473.

† An Account, &c., of the Tubular Bridges, p. 248.

1. $c = 4{\cdot}1$ tons.
2. $c = 4{\cdot}7$ „

Many girders are, however, made of the double-flanged kind, care being taken to stiffen the rib by small plates riveted at right angles to it on each side, and reaching from the upper to the lower rib.

A remarkable fact was discovered by Mr. Fairbairn in his experiments—that the top flange should have an area double that of the lower to give the strongest form of beam; whereas, in cast iron, the lower must be above 6 times as great as the upper flange to attain the same end.

In a former treatise (Equilib. of Struct. p. 188) will be found a series of calculations on Mr. Fairbairn's tubular experiments, the specific strength f being

	Tons.
From the circular tubes . .	$f = 13{\cdot}34$
From the elliptical tubes . .	$f = 16{\cdot}55$
From the rectangular tubes .	$f = 8{\cdot}26$

The latter value, however, does not fairly exhibit the strength of the rectangular tubes, which can be doubled in strength by allowing twice as much area in the upper as in the lower side; and this modification Mr. Hodgkinson's rules do not reach.

Wrought iron preserves its elasticity much better than cast iron; an experiment of Mr. Hodgkinson's, given in the Railway Commissioners' Reports, fully proves the superior elastic power of the material, showing little variation in the ratio of deflection to the load, while the latter was increased from one-quarter to six hundred weight. The bar was 13½ feet long, 1 inch deep, and 5 inches broad.

Weight.	Weight / Deflection.
28 lbs.	147·4
56	145·3
112	145·6
224	143·2
336	143·3
448	145·7
560	146·1
672	140·7
784	134·6
896	126·3
1008 nearly broke.	73·0

STONE.

(66.) Of the transverse strength of this material little is officially known, but we are perfectly aware of the danger of using any kind of stone for beams where there is much chance of serious or irregular pressure; its weakness in respect to this strain is manifest from all experimental evidence concerning it. Gauthey states the value of S for limestone to be as follows:—

Hard limestone . . S = 78 lbs.
Soft limestone . . S = 69 lbs.

Mr. Hodgkinson has thus compared the tensile, crushing, and transverse strength of several kinds of stone, the power of resisting a crushing force being = 1000.

	Tensile.	Transverse.
Black marble . .	143	10·1
Italian marble . .	85	10·6
Rochdale flagstone	104	9·9
Yorkshire flag . .	0	9·5
Mean . .	104	10·0

For common brick Mr. Barlow found S = 64 lbs.

(67.) **Detrusion.**—This term has been applied to denominate that kind of fracture which would occur in the use of shears if their edges were blunt, or which happens when a heavy wheel breaks its shaft close to the support or plummer-box, or when the punch of a punching machine makes a hole in a plate; it may be commonly termed the resistance to shearing across, and from its occurrence in machinery deserves investigation. Coulomb thought it was equal to the force of cohesion, but Robinson supposed it twice as much. Deal is stated to have a strength per square inch of section of 592 lbs., in the direction of the fibre; cast iron, 73,000 lbs., as deduced from experiments on the crushing of that material; wrought iron from 45,000 lbs. to 53,000 lbs.

GENERAL COMPARISON OF STRENGTH.

(68.) We may here add the ratio of crushing, tensile, and transverse strength in different kinds of materials, which has been deduced from all the experiments: estimating the resistance to crushing at 1000, we have for

	Compression.	Tension.	Transverse.
Timber. . .	1000	1900	85·1
Iron, cast . .	1000	158	19·8
Stone . . .	1000	100	10·0
Glass . . .	1000	123	10·0

CHAPTER V.—TORSION.—*Nature of the strain.—General Laws of Torsion; rules for calculating it.—Experiments on Torsion; Mr. Bevan's investigation, and moduli.—Comparative Torsion.*

(69.) THERE are frequently strains acting on such parts of machinery as axles, shafts, spindles, and all bodies subject to rotatory movement, which are not explained nor calculable by the principles of the tensile, compressing, and transverse strains; they are twisting forces, and tend by a curvilineal motion to force the longitudinal and straight fibres of any material into a spiral arrangement. Such a twist or torsion, or deflection, will be given to very ductile or plastic substances; while brittle and hard materials, resisting as long as possible, will at last suddenly break across, or otherwise split, at the weakest part. The strain of torsion is very peculiar; some have considered it as measurable on the same ground as we should estimate a blunt-edged tool by dead pressure, pushing forwards the fibres of a body and making a hole in it; it is, however, very unlike such a case.

(70.) Some calculations have indicated that the strength of an axle to resist torsion is as the cube of its diameter; the strength also increases as the torsion, as the transverse section of the shaft or axle, and inversely as the length of the axle. The deflection or angle of torsion of a fibre has been found to increase as the weight producing the torsion,—a law similar to that governing the deflection or bending of a beam under transverse pressure.

Hollow beams or tubes are said to be three times as powerful in resisting torsion as a solid cylinder containing the same quantity of matter and of a diameter to just fit in the tube.

M. Savart, from a number of experiments on different substances, has deduced the following laws, which must be regarded as naturally true, from the fact of their having been evolved from experimental results *:—

SAVART'S LAWS.

1. Whatever may be the figure of the transverse section of the rod, the arcs of torsion are directly proportional to the moment of the force and the length.

2. When the sections of the rods are similar, whether circular, triangular, square, or very long rectangles, the arcs of torsion are in the inverse ratio of the fourth power of the linear dimensions of the section.

3. When the sections are rectangular, and the axles possess a uniform elasticity in every direction, the arcs of torsion are in the inverse ratio of the product of the cubes of the transverse dimensions divided by the sum of their squares; whence it results, that if the width is considerable in comparison with the thickness, the arcs of torsion will be sensibly in the inverse ratio of the width and the cube of the thickness: these laws are also correct in circumstances where the elasticity is not the same in all directions.

(71.) These laws agree most satisfactorily with the calculations of MM. Poisson and Cauchy, from which rules have been given for estimating the deflection for torsion, and rupture of twisted bodies; they have been quoted by Mr. Hodgkinson †, as follows:—

Form of rod.	Resistance (F) to a twist, or angular flexure.	Resistance (T) to fracture.
Round	$F = w\lambda . \frac{2l}{\pi r^4 \theta}$.	$T = w\lambda . \frac{2}{\pi r^3}$.
Square	$F = w\lambda . \frac{6l}{d^4 \theta}$.	$T = w\lambda . \frac{6}{\sqrt{2}d^3}$.
Rectangular . .	$F = w\lambda . \frac{3(b^2 + d^2)l}{b^3 d^3 \theta}$.	$T = w\lambda . \frac{3\sqrt{b^2 + d^2}}{b^2 d^2}$.

In these formulæ, F signifies the specific resistance to turning round, and must, like S, be determined by experiment; T also requires an experiment to give it a numerical value, since it corresponds with S in transverse strength (see p. 251); the letter *w* expresses the weight or load acting on the rod or axle with any leverage λ, in inches; and l denotes the length of the rod, r its radius (when circular), b and d its breadth and depth, or thickness when rectangular. The angle through which the fibres twist is here shown by θ.

---

* Annales de Chimie et de Phys. vol. xxxi.

† Tredgold on Cast Iron, 2nd part, p. 496, 4th edit.

(72.) Several experimenters have afforded means for determining T and F. Mr. G. Rennie*, with cast-iron bars one inch square, and the weights producing torsion acting at the end of a lever three feet long, found one break with 191 lbs. and the other with 231 lbs. in the scale, the former being a vertical and the latter an horizontal casting. Then for the

$$\begin{array}{llll} & & & \text{Mean} \\ \text{Horizontal casting} \;\ldots & w = 191, \text{ and } T = 29172{\cdot}4 \text{ lbs.} & \left.\begin{array}{l} \\ \\ \end{array}\right\} & 32227 \text{ lbs.} \\ \text{Vertical casting} \;\ldots\ldots & w = 231, \text{ and } T = 35281{\cdot}8 & & \end{array}$$

Four experiments of Messrs. Bramah give T = 27,534 lbs., and a mean of all these gives T = 32,503 lbs.

The rules, then, applied to cast iron become as follows, all the dimensions being in inches:—

$$\text{For a cylinder} \ldots\ldots \quad W = \frac{51055 \,.\, r^3}{\lambda}.$$

$$\text{For a square rod} \ldots\ldots \quad W = \frac{7661\, d^3}{\lambda}.$$

$$\text{For a rectangular rod} \;\ldots \quad W = \frac{10834 \,.\, b^2 d^2}{\lambda \sqrt{b^2 + d^2}}.$$

(73.) Mr. Bevan† made experiments on a number of different kinds of timber, and several on cast and wrought iron and steel; whence he obtains what he calls a modulus T of torsion, and gives the following simple rule for finding the torsion in a square axle or rod:—

$$\text{Torsion} = \frac{\lambda^2 l w}{d^4 T}.$$

The value of T for several different kinds of timber and iron and steel, as given in his paper, is quoted in the subsequent table.

MODULI OF TORSION.

Kind of Wood.	Sp. grav.	Modulus in lbs.	Remarks.
Acacia	·795	28,293	Not quite dry.
Alder	·55	16,221	Cross-grained.
Apple	·726	20,397	
Ash	...	20,300	Of my own planting.
Ash, mountain	·449	13,933	
Beech	...	21,243	
Birch	...	17,250	
Box	·99	30,000	Old and very dry.
Brazil wood	1·05	37,800	Old and very dry.
Cane	...	21,500	Influenced by the hard surfaces.
Cedar, scented	...	12,500	
Cherry	·71	22,800	
Chestnut, sweet	...	18,360	
Chestnut, horse	·615	22,205	
Crab	·763	22,738	
Damson	...	23,500	
Deal, Christiana	·38	11,220	
Elder	·755	22,285	
Elm	...	13,500	
Fir, Scotch	...	13,700	
Hazel	·83	26,325	Not quite dry.
Holly	...	20,543	
Hornbeam	·86	26,411	Not quite dry.
Laburnum	...	18,000	Green, or fresh cut.
Lance wood	1·01	25,245	
Larch	·58	18,967	

---

[*] Phil. Trans. 1818.

[†] Phil. Trans. 1829, p. 128.

Kind of Wood.	Sp. grav.	Modulus in lbs.	Remarks.
Lime, or Linden	·675	18,309	
Maple	·735	23,947	Partly cross-grained.
Oak, English	...	20,000	
Oak, Hamburgh	·693	12,000	
Oak, Dantzic	·586	16,500	
Oak (from bog)	·67	14,500	
Ozier	...	18,700	
Pear	·72	18,115	
Pine, St. Petersburgh	...	10,500	Fresh.
Pine, St. Petersburgh	...	13,000	Four or five years old.
Pine, Memel	...	15,000	
Pine, American	...	14,750	
Plane	59	17,617	
Plum	79	23,700	
Poplar	333	9,473	
Satin wood	1·02	30,000	
Sallow	...	18,600	
Sycamore	...	22,900	
Teak	...	16,800	Old, partially decayed.
Teak, Africa	...	27,300	
Walnut	572	19,784	

A very interesting fact was noticed by Mr. Bevan in his researches, namely, that the resistance to torsion of timber varies nearly as its specific gravity, insomuch that if the specific gravity be known, the rule for finding the twist or torsion may be thus modified :—

$$\text{Torsion} = \frac{\lambda^2 l w}{30000\, s\, d^4}.$$

s being the specific gravity of the substance.

(74.) The following little table presents a comparative view of the torsion-resisting power of several metals :—

	lbs.
Cast steel	19·5
Shear steel	17
Blister steel	16·6
English iron	10·1
Swedish iron	9·5
Hard gun metal	5·0
Fine yellow brass	4·6
Copper	4·3
Tin	1·5
Lead	1·0

APPENDIX.

Experiments on long and short solid and hollow Pillars of Low Moor (Cast) Iron, No. 3 (from Mr. Hodgkinson's Paper in the Philosophical Transactions for 1840); and on Bars of Cast Iron, showing their transverse strength, by Messrs. Fairbairn and Hodgkinson.

Table I.	Solid uniform Cylinders of Cast Iron, with rounded ends.
Table II.	Ditto, with flat ends.
Table III.	Short solid Pillars of Cast Iron, flat at the ends.
Table IV.	Hollow Cylindrical Pillars, flat at the ends, of Low Moor Iron, No. 3.
Table V.	Ditto, rounded at the ends, cast in dry sand.
Table VI.	Hollow uniform Cylindrical Pillars.
Table VII.	Results of Experiments by Messrs. Fairbairn and Hodgkinson on Bars of Cast Iron from the principal Iron Works of the United Kingdom.

Table I.—Solid uniform Cylinders of Cast Iron.
(With rounded ends.)

Length.	Diameter.	Mean Diameter.	Deflexion of middle of Pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	inch.	lbs.	lbs.	lbs.	
60·5	·50	...	·07	58			
		·50	·49	113	136		
60·5	·50	...	·04	97		143	
			·23	136	150		
60·5	·77	·77	...	...	780	780	
60·5	·77		...	...	780		
60·5	1·29		·03	2,141			These pillars were cast in green sand.
		1·295	·17	4,549			
			·34	4,997	5,149		
60·5	1·30		·00 ?	2,141		5,465	
			bent	2,757			
			·07	5,445	5,781		Pillar weighed 19 lbs. 14 oz.
30·25	·50		bent	248			
			·15	472	526		
30·25	·50	·50	·02	304		539	
			·09	472	535		Broke near the middle.
30·25	·50		...	...	556		Broke ¾ inch from middle.
30·25	·77	...	·02	1,717			
		·77	·10	2,390	2,726	2,726	
30·25	·99		·04	2,745			
		·99	·13	4,985	6,105		T : C : : 76 : 23.
30·25	·99		·02	3,641		6,105	
			·07	4,985	6,105		Bent in different directions.
30·25	1·29		·01	12,287			Neutral line well defined.
		1·29	·07	16,115	17,515		T : C : : 96 : 34.
30·25	1·29		·08	12,287		17,235	
			·21	16,115	16,955		Small flaw in tensile part.
20·1666	1·00	1·01	...	...	15,737	15,737	63 : 67.
20·1666	1·02		...	...	15,737		65 : 37.
20·1666	·785	·767	...	...	7,255	6,602	
20·1666	·75		...	...	5,950		
15·125	·50	...	·08	1,353			39 : 11.
			·20	1,801	1,857	1,904	
15·125	·75	·76	...	...	7,786	9,223	
15·125	·99	·99	...	...	19,855	19,752	60 : 39.
10·083	·77	...	...	...	19,152		
7·5625	·51	...	...	...	6,188		
7·5625	·49	...	...	...	4,578	5,262	32 : 17.
7·5625	·77	·77	...	...	23,893		Split at both ends.
7·5625	·77		...	...	22,003	22,948	
3·7812	·50	...	...	...	15,233		
3·7812	·50	·50	...	...	14,981	15,107	Ends split.

Table II.—Solid uniform Cylinders of Cast Iron.

(With flat ends.)

Length.	Diameter.	Mean Diameter.	Deflexion of middle of Pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Remarks.
inches.	inch.	inch.	inch.	lbs.	lbs.	lbs.	
60·5	·51	·51	...	...	483	487	Had discs 2 in. diam. at each end.
60·5	·77	·77	·07	1,162		2,456	
60·5	1·01	·997	·05 ·14	4,123 5,467	5,971	6,238	
60·5	1·28	1·29	...	...	16,331	16,064	
60·5	...	1·56	...	...	...	28,962	A wedge broke out, showing neutral line.
30·25	·50	...	·05	1,090	1,662		
30·25	·78	·77	·08	4,357	...	8,811	61 : 15. Crack at neutral line.
30·25	...	1·01	·05 ·07 ·13	... 21,897	... ...	20,310 ...	Cracked at neutral line. 65 : 35.
20·1666	...	·51	...	...	...	3,830	
20·1666	1·03	1·022	...	...	32,007	31,804	56 : 46. Broke at two places near middle.
15·125	...	·51	...	...	...	6,764	32 : 19.
15·125	...	·775	...	...	...	21,509	
15·125	...	1·00	...	...	...	40,250	
12·1	...	·50	...	...	...	7,195	3 : 2.
12·1	...	·785	...	...	...	24,287	1 : 1.
10·0833	...	·50	...	...	...	8,931	13 : 12.
10·0833	...	·768	...	...	...	25,953	1 : 1. Broke in several pieces, but always at middle.
7·5625	...	·50	...	...	...	11,255	12 : 13.
7·5625	...	·777	...	...	...	32,007	4 : 5.
3·7812	...	·50	...	...	...	17,468	20 : 30. Broke in middle, but showed a ridge or wedge in centre.
2·0	...	·52	...	...	...	22,867	Bent and cracked half across in middle.
1·0	...	·52	...	...	...	24,616	Broke by wedge sliding off.

TABLE III.—Short Solid Pillars of Cast Iron, flat at the ends.

Diameter of Pillar.	Length.	Value of b.	Value of c.	Breaking weight.	Calculated breaking weight from formula* $y = \frac{bc}{b + \frac{3c}{4}}$.
in.	in.			lbs.	
·50	12·1	8327	21559	7,195	7328
·50	10·083	11353	21559	8,931	8872
·50	7·5625	18515	21559	11,255	11508
·50	3·7812	60155	21559	17,468	16992
·777	20·166	16713	52064	15,581	15604
·775	15·125	27005	51797	21,059	21241
·785	12·1	41300	53142	24,287	27043
·768	10·083	52096	50865	25,923	29363
·777	7·5625	88547	52064	32,007	36130
1·022	20·1666	44218	90074	31,804	35631
1·000	15·125	66746	86238	40,250	43797

* For the meaning of this formula see p. 248.

TABLE IV.—Hollow Cylindrical Pillars, flat at the ends, of Low Moor Iron, No. 3, Length of each pillar, 7 feet 6¾ inches, except otherwise specified.

No. of Exp.	Kind of Pillar.	Deflexion.	Weight producing the deflexion.	Weight with which the pillar sunk.	Value of x from formula* $x = \frac{W}{D^{3.55} - d^{3.55}}$	Ratio of the thickness of metal on opposite sides at the place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
1	Hollow uniform cylinder, same as in exp. 1 of preceding table. External diam. 1·78 in. Internal do. 1·21 Length of cylinder, 7 ft. 4¾ in. Weight, 31 lbs.	·02	2,813				The thinner was found to be the compressed side. – The weight of this and every subsequent pillar is the weight of the whole length of 7 ft. 6¾ in.
		·03	3,821				
		·00	Unloaded				
		·05	4,829				
		·20	12,001				
		·50	16,705				
		·66	17,489	17,840	2973·7	1 : 5	
2	Cylinder same as No. 2 of preceding table. External diam. 1·74 in. Internal do. 1·187 Length of cylinder, 7 ft. 4¾ in. Weight, 30¼ lbs.	·12	11,217				This pillar was reduced as No. 7 of last table, with similar negative results.
		·32	15,137				
		·48	15,921				
		direction changed					
		·54	16,313	16,705	3031·5	7 : 11	
3	External diam. 1·76 in. Internal do. 1·18	Bent.	2,141				Reduced as the last, or more, and gave way between middle and one end.
		·03	2,749				
		·09	6,677				
		·36	15,233				
		·54	16,241	16,745	2968·7	1 : 3	
4	External diam. 1·75 in. Internal do. 1·11 Weight, 32 lbs.	·01	2,237				A sound casting.
		·19	16,477				
		·55	20,509	20,957	3586.9	1 : 2	

* Where W = breaking weight; D, d, the external and internal diameters; and x the breaking weight of a pillar 7 ft. 6¾ inches long and 1 inch diameter.

TABLE IV.—*continued.*

No. of Exp.	Kind of Pillar.	Deflexion.	Weight producing the deflexion.	Weight with which the pillar sunk.	Value of s from formula $s = \frac{W}{D^{3.55} - d^{3.55}}$	Ratio of the thickness of metal on opposite sides at the place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
5	External diam. 2·04 in.	·04	3 589				
	Internal do. 1·46	·08	14,703				
	Length, 7 ft. 4¾ in.	·37	29,977				
	Weight, 35½ lbs.	·52	31,601	32,413	3573·8	1 : 1	
6	External diam. 2·01 in.	·03	3,589				Small bubble at the fracture.
	Internal do. 1·368	·08	18,667				
	Length, 7 ft.	·00	Unloaded				
	Weight, 37¾ lbs.	·38	28,353				
		·58	29,977	30,789	3290·3	7 : 10	
7	External diam. 2·01 in.	Bent	4,251				Cylinder same as that in Exp. 3 of last table. It was made straight, its ends firmly imbedded, and it was reduced as in Exp. 2, breaking in the middle and at three of the reduced places.
	Internal do. 1·415	·10	21,857				
	Length, 7 ft. 4¾ in.	·01	Unloaded				
	Weight, 36½ lbs.	·14	25,917				
		·25	27,541	28,353	3214·8	5 : 11	
8	External diam. 1·99 in.	Bent	1,456				Reduced as before. Broke at a small flaw near the middle.
	Internal do. 1·31	·20	15,605				
	Length, 7 ft. 5·8 in.	·55	24,205				
	Weight before reduction, 39 lbs.	·90	26,731	27,067	2988·3	6 : 7	
9	External diam. 2·23 in.						Same as that in No. 5, last table; it was now reduced as before, and broke at middle, and at one of the reduced places near the middle.
	Internal do. 1·54						
	Length, 7 ft. 4¾ in.						
	Weight, 47 lbs.			40,569	3099·0	4 : 9	
10	Uniform solid cylinder cast in green (moist) sand.	Bent	4,135				
	Diameter, 1·76 in.	Bent	10,855				
	Length, 7 ft. 6½ in.	·35	21,219				
	Weight, 56 lbs.	·65	22,787	23,179	3115·5		
11	Uniform solid cylinder cast in dry sand.	·20	16,115				
	Diameter, 1·72 in.	·28	18,355				
	Length, 6 ft. 6¾ in.	·44	20,595				
	Weight, 53½ lbs.	·65	21,715	21,995	3207·7		

TABLE V.—Hollow Cylindrical Pillars, rounded at the ends, of Low Moor Iron, No. 3, cast in dry sand; length of cylinder, 7 feet 6¾ inches.

No. of experiment.	Kind of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking pressure.	Value of x from formula* $x=\frac{W}{D^{3\cdot76}-d^{3\cdot76}}$	Remarks.
		inch.	lbs.	lbs.	lbs.	
1	External diam. 1·78 in.	·03	2,237			With 5585 lbs. the pillar sunk, but was not allowed to break. Its elasticity was very little injured, and afterwards showed no defect of strength.
	Internal do. 1·21	·32	4,829			
	Weight of cylinder, 31 lbs.	·49	5,333	5,585	834·37	
2	External diam. 1·74 in.	·02	2,141			Sunk with 5711 lbs.
	Internal do. 1·187	·13	4,325			
	Weight, 30¼ lbs.	·48	5,585	5,711	933·13	
3	External diam. 2·01 in.	·04	2,237			The thickness of the metal at the fracture was on opposite sides as 19 to 42.
	Internal do. 1·1415	·31	6,845			
	Weight, 36½ lbs.	·75	8,105	8,357	826·20	
4	External diam. 2·33 in.	·24	11,169			Thickness of metal on opposite sides nearly as 1 to 4; the thin side was compressed.
	Internal do. 1·70	·37	12,737			
	Weight, 46½ lbs.	·72	14,697	15,089	903·28	
5	External diam. 2·23 in.	·01	2,237			
	Internal do. 1·54	·22	8,357			
	Weight, 47 lbs.	·69	12,137	12,389	808·21	
6	External diam. 2·24 in.	·015	2,141			
	Internal do. 1·735	·34	12,445			
	Weight, 34¾ lbs.	·88	12,669	13,341	1041·7	
7	External diam. 2·24 in.	·02	4,325			The pillar was reduced to half its thickness at the ends, and 3-4ths at half-way between middle and ends, without failing in the reduced parts. Sunk by bending.
	Internal do. 1·58	·21	13,521	13,913	917·62	
8	External diam. 2·49 in.	·01?	4,123			Reduced as before, and bent.
	Internal do. 1·89	·40	18,623			
	Weight, 48¾ lbs.	·52	19,239	19,855	996·24	
9	External diam. 2·47 in.	·01?	3,211			At the place of fracture the opposites were in thickness as 7 to 9.
	Internal do. 1·98	·23	17,391			
	Weight, 41 lbs.	·62	18,667	19,003	1123·5	
10	External diam. 2·46 in.	Bent	2,141			Thickness of sides at fracture as 3 to 4.
	Internal do. 1·55	·04	9,103			
	Weight, 49 lbs.	·49	18,083			
		·65	18,615	19,147	989·95	

* Where W = the breaking weight; D, d, the external and internal diameters of the pillar; and x the breaking weight of a pillar 7 ft. 6¾ in. long and 1 inch in diameter.

TABLE V.—*continued.*

No. of Experiment.	Kind of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking pressure.	Value of s from formula $s=\frac{W}{D^{3.76}-d^{3.76}}$	Remarks.
		inch.	lbs.	lbs.	lbs.	
11	External diam. 2·73 in. Internal do. 2·17 Weight, 48 lbs.	Bent ·03 ·70	3,603 12,105 22,787	23,963	949·48	
12	External diam. 2·74 in. Internal do. 2·155 Weight, 51¼ lbs.	·02 ·14 1·10	3,603 21,219 27,491	27,883	1059·5	Thickness of metal at fracture as 2 to 3 nearly.
13	External diam. 3·01 in. Internal do. 2·48 Weight, 50½ lbs.	·07 ·23 ·75	16,115 21,219 25,923	26,707	819·46	Thickness of metal at fracture as 9 to 15.
14	External diam. 3·36 in. Internal do. 2·823 Weight, 59½ lbs.	·09 ·32 1·10	16,115 30,627 40,335	40,973	895·02	Thickness of metal at fracture as 19 to 34.
15	External diam. 3·36 in. Internal do. 2·63 Weight, 77¾ lbs.	Bent ·09 ·30 ·90 1·07	3,355 16,115 33,424 40,511 49,494	50,477	880·11	Thickness of metal at fracture as 5 to 7.
16	Solid uniform pillar, rounded at the ends. Diameter, 2·24 in. Weight, 93 lbs.	Bent ·02 ·25	2,141 7,541 20,273	21,281	2026·6	
	SHORTER PILLARS.					
17	External diam. 1·78 in. Internal do. 1·21 Length, 4 ft. 9 in.			13,693	927·86	
18	External diam. 2·31 in. Internal do. 1·67 Length, 4 ft. 9 in.			36,382	1005·3	Thickness of metal at fracture as 10 to 21.
19	External diam. 1·85 in. Internal do. 1·36 Length, 2 ft. 7 in. Weight of 2 ft. 5 in. = 8 lbs. 15½ oz.	·35	32,587	33,763	784·9	Thickness of metal at fracture as 11 to 15.

TABLE VI.—Hollow uniform Cylindrical Pillars.

No. of Experiment.	Length of pillar.	External diameter.	Internal diameter.	Weight of pillar.	Breaking weight.	Value of *b*.	Value of *c*.	Calculated breaking weight from formula $S = \frac{bc}{b + \frac{3c}{4}}$	Remarks.
	feet.	inch.	inch.	lbs. oz.	lbs.	lbs.	lbs.	lbs.	
1	2·5208	1·26	·767	6 2	33,679	38,807·6	86,178·5	32,331	Not perfectly sound.
2	2·5208	1·26	·781	6 1	32,867	38,274	84,310	31,790	Core not quite in middle. Thickness of metal on opposite sides, 3 : 4.
3	2·1666	1·25	·768	5 2	35,303	48,461·7	83,882	36,501	Air bubbles in casting.
4	2·1666	1·17	·752	4 7	31,195	36,887	69,283·2	28,764	Core in centre, T : C : : 43 : 4.
5	1·9166	1·16	·7705	3 9	30,383	42,633	64,844·7	30,291	Do., T : C : : 11 : 18.
6	1·6805	1·21	·77	3 9	41,751	64,599·2	75,130·6	40,128	Core in centre.
7	1·6805	1·14	·805	2 11	27,135	46,408	56,193	29,449	
8	1·4166	1·15	·91	1 11	25,511	50,927	42,636	26,191	
9	1·3333	1·15	·92	1 9½	25,105	54,730	41,053	26,273	
10	1·2604	1·16	·932	1 8	26,729	61,304·1	41,133·8	27,364	Core in centre, T : C : : 52 : 64.
11	1·2604	1·08	·77	1 12	27,135	61,570·2	49,457	30,863	
12	1·1667	1·15	·792	2 0	37,285	91,909	59,953	40,257	
13	·7333	1·13	·91	0 13½	34,037	133,000	38,704	31,750	

TABLE VII.—Results of Experiments by Messrs. Fairbairn and Hodgkinson on Bars of Cast Iron from the principal Iron Works of the United Kingdom.

No. of Iron in the scale of strength.	Names of Iron.	Specific gravity.	Modulus of elasticity in lbs. per square inch.	Mean breaking weight (S) of bars 4½ feet long, in lbs.	Ultimate deflexion of the 4½-feet bars.	Power of the 4½-feet bars to resist impact.	Colour.	Quality.
1	Ponkey, No. 3, cold blast	7·122	17211000	578	1·747	992	Whitish gray	Hard
2	Devon, No. 3, hot blast	7·251	22473650	537	1·09	589	White	Hard
3	Oldberry, No. 3, do.	7·300	22733400	530	1·005	549	White	Hard
4	Carron, No. 3, do.	7·056	17873100	527	1·365	711	Whitish gray	Hard
5	Coed-Talon, No. 3, do.	6·970	14707900	518	1·577	782	Dullish gray	Hard
6	Beaufort, No. 3, do.	7·069	16802000	517	1·599	807	Dullish gray	Hard
7	Butterley	7·033	15379500	502	1·815	889	Dark gray	Soft
8	Bute, No. 1, cold blast	7·066	15163000	491	1·764	872	Bluish gray	Soft
9	Windmill End, No. 2, do.	7·071	16490000	489	1·581	765	Dark gray	Hard
10	Old Park, No. 2, do.	7·049	14607000	485	1·621	718	Gray	Soft
11	Carron, No. 2, do.	7·066	17270500	476	1·313	630	Dull gray	Rather hard
12	Beaufort, No. 2, hot blast	7·108	16301000	474	1·512	729	Dull gray	Hard
13	Low Moor, No. 2, cold blast	7·055	14509500	472	1·852	855	Dark gray	Soft
14	Low Moor, No. 3, do.	7·052	13918740	467	1·944	908	Dark gray	Rather harder
15	Buffery, No. 1, do.	7·079	15381200	463	1·550	721	Gray	Rather hard
16	Carron, No. 2, hot blast	7·046	16085000	463	1·337	619	Grayish blue	Rather hard
17	Brimbo, No. 2, cold blast	7·017	14911666	459	1·748	815	Light gray	Rather hard
18	Apedale, No. 2, hot blast	7·017	14852000	456	1·730	791	Light gray	Stiff
19	Oldberry, No. 2, cold blast	7·059	14307500	455	1·811	812	Dark gray	Rather soft
20	Pentwyn, No. 2	7·038	15193000	455	1·484	650	Bluish gray	Hard
21	Maesteg, No. 2	7·088	13959500	454	1·957	886	Dark gray	Rather soft
22	Muirkirk, No. 1, cold blast	7·113	14003550	454	1·734	770	Bright gray	Fluid
23	Adelphi, No. 2, do.	7·080	13815500	449	1·759	777	Light gray	Soft

24	Blaina, No. 3, do.	7·159	14281466	448	1·726	747	Bright gray	Hard
25	Devon, No. 3, do.	7·295	22907700	448	1·790	354	Light gray	Hard
26	Gartsherrie, No. 3, hot blast	7·017	13894000	447	1·557	998	Light gray	Soft
27	Frood, No. 2, cold blast	7·031	14112666	447	1·825	841	Light gray	Open
28	Lane End, No. 2	7·028	15787666	444	1·414	629	Dark gray	Soft
29	Carron, No. 3, cold blast	7·094	16246966	446	1·336	594	Gray	Soft
30	Dundyvan, No. 3. do.	7·087	16534000	443	1·469	674	Dull gray	Rather soft
31	Maesteg (marked red)	7·038	13971500	442	1·887	830	Bluish gray	Fluid
32	Corbyn's Hall, No. 2	7·007	13845866	442	1·687	727	Gray	Soft
33	Pontypool, No. 2	7·080	13136500	440	1·857	816	Dull blue	Rather soft
34	Wallbrook, No. 3	6·979	15394766	440	1·443	625	Light gray	Rather hard
35	Milton, No. 3, hot blast	7·051	15852500	438	1·368	585	Gray	Rather hard
36	Buffery, No. 1, do.	6·998	13730500	436	1·640	721	Dull gray	Soft
37	Level, No. 1, do.	7·080	15452500	432	1·516	699	Light gray	Soft
38	Pant, No. 2	6·975	15280900	431	1·251	511	Light gray	Rather hard
39	Level, No. 2, hot blast	7·031	15241000	429	1·358	570	Dull gray	Soft
40	W. S. S., No. 2	7·041	14953333	429	1·339	554	Light gray	Soft
41	Eagle Foundry, No. 2, hot blast	7·038	14211000	427	1·512	618	Bluish gray	Soft
42	Elsicar, No. 2, cold blast	6·928	12586500	427	2·224	992	Gray	Soft
43	Varteg, No. 2, hot blast	7·007	15012000	426	1·450	621	Gray	Hard
44	Colthau, No. 1, do.	7·128	15510066	424	1·532	716	Whitish gray	Rather soft
45	Carrol, No. 2, cold blast	7·069	17036000	419	1·231	530	Gray	Hard
46	Muirkirk, No. 1, hot blast	6·953	13294400	419	1·570	656	Bluish gray	Soft
47	Bierley, No. 2	7·185	16156133	418	1·222	494	Dark gray	Soft
48	Coed-Talon, No. 2, hot blast	6·969	14322500	416	1·882	772	Bright gray	Soft
49	Coed-Talon, No. 2, cold blast	6·955	14304000	413	1·470	600	Gray	Rather soft
50	Monkland, No. 2, hot blast	6·916	12259500	403	1·762	709	Bluish gray	Soft
51	Ley's Works, No. 1, do.	6·957	11539333	392	1·890	742	Bluish gray	Soft
52	Milton, No. 1, do.	6·976	11974500	369	1·525	538	Gray	Soft and fluid
53	Plaskynaston, No. 2, do.	6·916	13341633	357	1·366	517	Light gray	Rather soft
	ANTHRACITE IRONS.							
54	Yniscedwyn Anthracite, No. 1, hot blast	7·078	13741400	458	1·730	785	Grayish blue	Soft

TABLE VII.—*continued.*

No. of Iron in the scale of strength.	Names of Iron.	Specific gravity.	Modulus of elasticity in lbs. per square inch.	Mean breaking weight (S) of bars 4½ feet long, in lbs.	mean	Ultimate deflexion of the 4½-feet bars.	Power of the 4½-feet bars to resist impact.	Colour.	Quality.
	Yniscedwyn Anthracite,								
55	No. 2, hot blast	7·095	15334000	508		1·529	709	Grayish blue	Harder
56	No. 3, do.	7·168	16194327	520		1·525	785	Whitish gray	Rather harder
	Ystalyfera Anthracite,								
57	First sample, No. 1, hot blast	6·992	11555635	429	411	2·252	771	Bluish gray	
	Second do. do. do.	7·098	14044420	392·3		1·445			
58	First do. No. 2, do.	7·053	13973270	454	467	1·788	769	Dark gray	Rather soft
	Second do. do. do.	7·258	15686750	480·7		1·505			
59	First do. do. do.	7·133	13436806	466	484	1·825	751	Whitish gray	
	Second do. do. do.	7·352	18391425	502		1·324			

The values of S in the above table are for bars 4 feet 6 inches long; therefore, the breaking weight of any beam of iron will be found by the formula—

$$W = \frac{4{\cdot}5\, b\, d^2\, S}{2}.$$

Also, to find the deflexion (δ) of a beam, taking from a weight (w) the modulus of elasticity (ε) from the table, we have

$$\delta = \frac{432\, w\, l^3}{b\, d^3\, \varepsilon}.$$

INDEX.

PAGE
Abutments of bridges, foundations of 156
——— irregular thickness of 187
——— of old bridges 157
Accelerated motion, analytical determination of 41
——— manner of calculating 13
Accelerating forces, nature of 12
Acceleration of a wheel and axle, acted on by a weight 31
Addendum of teeth 53
Air, resistance of, to moving bodies 25
America, rope bridges of 208
Angers suspension bridge, fall of 212
Angle of resistance, the limiting 107
Angles of a frame, effect of pressure on 120
Angular velocity of a rotating body 27
Annular wheels 46
Applegath and Cowper's printing machine 88
Applegath's patent printing machine 111
Arc, circular, as a figure for arches 152
Arch, the angular 144
—— calculations of rupture in the 147
—— the loaded catenarian 146
—— with circular intrados 145
—— effect of loading the crown of an 149
——— very heavy haunches 149
—— flat (plate bande) 144
—— laying out of, for building 152
—— elliptical 146
—— with level roadway 145
—— line of pressure in 150
—— parabolic 146
—— Perronet's observation of its settlement 150
—— point of rupture in the 151
—— its practical character 151
—— Robison's observation of rupture in the 151
—— sinking of the, on removal of the centre 138
—— turning of about the upper edges at the crown 153
Arch-stones, conditions required to slide on each other 147
——— of turning on each other 148
——— laying of on the centres 158
——— thickness of in different bridges 159
Arcole, bridge of, noticed 226
Argentino, (theatre), roof of 128
Asia, central, rope bridges of 201
Axes of rotation, spontaneous 34
——— three principal 35
Axis of rotation, rotation of a body round the shortest when free 36
Axis, pressure on, in a rotating loaded wheel 32
—— rotation about fixed 26

PAGE
Axle and wheel, proportional diameter of to raise a weight in the shortest time 33
Axles, friction of 107, 110

Ball, motions of, when struck out of the direction of the centre of gravity . 35
Bamberg, timber bridge at 175
Band wheels 63
Barlow's experiments on the tensile strength of timber 236
Barreling (lathe-work) 75
Beams, cast-iron, strongest section of 180
——— deflection of 254
——— hidden knots and faults in, to discover 130
——— rules for calculating the strength of 251
——— supported at one and both ends, rules and best forms for . . . 253
Bell-crank lever 59
Bevan's experiments on torsion 269
Bevel-wheels 46
————— application of the epicycloid for the teeth of . . . 53
Bhootan, chain bridges in 208
Birmingham theatre, roof-truss of 129
Bishopwearmouth, iron bridge 205
Blackfriars bridge, arches of 145, 152
——————— centre of 137
——————— thickness of the archstones of 159
Bolts of suspension bridge chains, strength of 214
Bones, tubular structure of 260
Boston, iron bridge at 207
Bourbon, suspension bridge in Island of 226
Bow-and-string rafter roof 124
Bow-drill 59
Bracket, strength of 121
Bréant's timber-preserving solution 168
Brick, as a material for bridge building 151
Bridge, over the Adda, Milan 158
——— Blackfriars 137, 138, 145, 152, 159
——— Bow 138
——— Chester (new) 138, 159, 162
——— Essex (Dublin) 138
——— Gloucester Over 153, 159
——— Hutcheson (Glasgow) 159
——— Hexham (faulty foundations) 154
——— Lary (near Plymouth) 154
——— over the Lea Cut, Eastern Counties Railway 159
——— London 138, 155, 156, 157, 159, 164
——— at Maidenhead (Railway) 157, 159
——— at Mantes 138
——— St. Maxence 157
——— Neuilly 137, 138, 156, 159
——— over the Taaf (Pont ŷ pryd) 149, 159, 167
————— Ribble, abutment of 156
——— Tongueland 159
——— Toulouse 159
——— of the Santissima Trinita (Florence) 167
——— Turin (over the Dora) 138, 159, 164
——— Victoria (Durham Railway) 159
——— Waterloo 138, 159, 167
——— Wellesley (Limerick) spandrels of 158
——— Westminster 239, 155, 156, 158
——— (timber) at Bamberg 175
————— over the Cismone (Italy) 172
——————————— Clyde (Glasgow) 171
——————————— James River Falls (lattice) 175
————— at Kandel (Berne) 171
————— near Kandy (Ceylon) 174

PAGE
Bridge (timber), Ladykirk and Norham 176
———— Migneron's proposed (at Paris) 130
———— over the Onondaga Creek Valley, U. S. 171
—————— Ottawa (Canada) 176
———— near Portsmouth, U. S. 174
———— over the Rhine (Schaffhausen) 172
—————— Schuylkill (Philadelphia, U. S.) 175
———— Trajan's, revived by Mr. Price 174
———— at Trenton, over the Delaware, U. S. 177
———— over the Whitadder (girder) 170
—— (iron) Bishopwearmouth 205
——— Boston 207
——— Britannia (Menai Strait) tubular 184
——— Buildwas, over the Severn 205
——— Busse's wrought-iron girder 182
——— Colebrook-dale, over the Severn 204
——— Conway (tubular) 193, 204
——— over the Dee (Chester), fall of 179
——— Dublin and Drogheda Railway (lattice) 182
——— Gainsborough (tubular girder) 183
——— Galton 207
——— Southwark 206
——— Tewkesbury 207
—— (suspension) Arcole (Paris) 226
————— Belu's 216
————— Broughton, fall of 211
————— Brown's (Capt.) 214
————— in the Isle of Bourbon 214, 226
————— Freiburg 217
————— Hammersmith 214, 215, 225
————— Hungerford Market 214, 215, 224
————— Menai Strait 213, 214, 220
————— over the Niagara 225
——————— Schuylkill cataract 216
————— Union, over the Tweed 214, 215, 226
————— over the Tees (Yorkshire) 208
Bridges, centerings, for arches of 135
—— of curved timber arches 173
—— cutwaters for 155
—— forms of arches for 152
—— Dredge's form of suspension 227
—— foundations of piers and abutments 153
—— general character of 151
—— girder 169, 179
—— lattice girder 169, 182
—— laying the arch-courses of 158
—— materials used in building 151
—— of stone 151
——— timber 168
——— iron 177
—— Remington's method of constructing 170
—— tables of the principal 230
Brighton suspension pier, broken by a storm 213
Britannia tubular bridge 184
Buildwas iron bridge 207
Buff wheels (friction) for varying wheel 66
Burnett's process for preserving timber 168
Busse's wrought-iron bridge 182
Brown's (Capt.) chain links 214
Buffon's remarks on transverse strength of wood 261

Caissons, making foundations by 155
Cam wheels 61, 65
Cannon ball, resistance of air to the flight of 26

PAGE
Catenarian arch, the loaded . . . 146
Catenary curve, form and principles of . . . 209
Centres, principles of . . . 135
——— removal of . . . 138
Centering of Blackfriars' bridge . . . 137
——— Chester new bridge . . . 162
——— London bridge . . . 138
——— Neuilly bridge . . . 137
——— Turin bridge . . . 162
Centrifugal force, calculation of . . . 37
——— at earth's equator . . . 38
——— on railways neutralised . . . 40
Chains for suspension bridges . . . 214
——— thrust of, on the towers of a bridge . . . 214
Chester new bridge, description of . . . 162
Chester railway bridge, fall of . . . 180
China bridges in Asia and America . . . 207
——— chain bridges in . . . 208
Circle, centre of gyration in the . . . 30
——— three principal axes of rotation in . . . 35
——— moment of inertia of, to find . . . 42
Circular arcs, arches of . . . 152
Cismone, bridge over the . . . 172
Claus's proposed wooden arch . . . 173
Cofferdams for laying foundations in water . . . 155
Cohesion as affecting the strength of materials . . . 233
Colebrook-dale iron bridge . . . 203
Colladon's dynamometer . . . 97
Collier's wheel-cutting machine . . . 86
Collision of bodies . . . 8
Compensating pendulum, the . . . 21
Compression of iron tubes . . . 243
——— pillars of iron, wood, and stone . . . 245, 271
——— laws of rupture by . . . 240
——— tensile strength of cast iron compared . . . 243
——— of wrought iron . . . 243
Cones, friction . . . 65
Connecticut river, bridge over . . . 171
Convertible pendulum, the . . . 29
Conway tubular bridge . . . 193, 203
——— suspension bridge . . . 223
Corrosive sublimate for preserving timber . . . 168
Corrugation, strengthening plate by . . . 187
Cost of bridges . . . 160, 164, 166
Coulomb's investigation of the arch . . . 146
Coupling, Higson's friction . . . 70
Couplings for driving shafts . . . 69
Crank and connecting-rod . . . 59
Crown of an arch, effect of loading . . . 149
——— wheels . . . 46, 65
Crushing of cast iron . . . 242
——— physical circumstances of . . . 241
——— of wood . . . 241
——— stone . . . 243
Curb roof . . . 126
Curve, motion in a . . . 16
Curved timber for bridge arches . . . 173
——— roofs . . . 130
Cutwaters, forms of . . . 155
Cycloid, principle of . . . 17
Cycloidal pendulum . . . 17
——— teeth for rack and trundle . . . 51
Cylindrical tubular girder, formula for strength of . . . 188

PAGE
Definition of dynamics 1
Deflection of loaded beams 254
——— injury to elasticity by 255
——— of railway bars from the passage of carriages 256
Delaware, timber bridge over the (U. S.) 117
Delorme's curved timber roof-truss 130
——— domical roofs 134
Density, definition of 1
Detrusion 267
Dolcoath mine, time of oscillation of a pendulum in 18
Dome, nature of 134
—— of the church of the Mad. della Saluta 134
—— over the Edinburgh Register Office 135
——— ——— Halle aux Blès 134
—— of the Pantheon 135
Drill, common bow 59
Drilling machine 84
——— Whitworth's self-acting 85
Drury-lane theatre, roof-truss of 129
Dry-rot, prevention of 168
Dufour's experiment on iron wire 217
Duhamel's experiments on cut-beams 251
Dynamics, historical notice of 43
Dynamometer, Colladon's lever 97
——— Morin's self-registering 102, 103
——— Prony's friction 95
——— Regnier's 94

Earth, centrifugal force at equator of 38
——— time of revolution of a body round it at the surface 38
Eccentric wheels, varieties of 60
Edinburgh Register Office, dome over 135
Edwards, builder of the Pont ŷ pryd 159, 167
Elasticity, determination of 8
——— practical nature and moduli of 233
——— for different substances 234
——— of beams, injury of by deflection 255
——— ——— cast iron 264
——— ——— wrought iron 267
Elevation above the sea-level, effect of on the pendulum 18
Ellipse as a figure for arches 152
——— three principal axes of rotation in 36
Elliptical arch (flat) drawing of 152
——— ——— with the archivolt a circular arc 153, 165
——— ——— thrust of on abutments 156
——— ——— the 146
——— tubular girders, formula for 188
——— wheels, for variable velocity 66
Encaissement, laying foundation by 154
Epicycloid, application of to bevelled wheels 59
——— generation of for teeth of wheels 49
Equilibrium of three forces 117
——— (See *Frames, Roofs, Domes, Centres, Walls,* and *Bridges.*)

Face turning 75
Fairbairn's experiments on effect of temperature on cast iron beams 266
——— ——— transverse strength of cast iron 262, 278
——— ——— impact 256
——— ——— the tensile strength of rolled iron plates 238
——— ——— hot and cold blast iron 237, 256
——— ——— for the tubular bridges 185
——— patent tubular girders 182
Falling bodies, laws of 13
Fastening of chains in suspension bridges 215

PAGE
Finlay's suspension bridge . . . 216
Flat arch, laying out of . . . 152
Force, accelerating . . . 12
—— nature of . . . 1
Forces, effect of two . . . 3
——— equilibrium of three . . . 117
——— moment of . . . 118
——— parallelogram of . . . 4, 118
Forge-hammer, centre of percussion in . . . 30, 61
———— Nasmyth's patent steam . . . 79
Foundations of bridges . . . 153
———— London bridge . . . 157, 161
Framework, principles of . . . 120
Frazer's account of Indian rope bridge . . . 208
Friction . . . 99
——— gear . . . 47
——— cones, for varying velocity . . . 65
——— clutch, or coupling . . . 70
——— Morin's apparatus and experiments . . . 101
——— of axles . . . 107, 110
——— tables of, for plane surfaces . . . 108
Freiburg suspension bridge . . . 217
Fungi, effect of on wood . . . 168

Gainsborough, tubular girder bridge at . . . 183
Galileo's theory of projectiles . . . 24
———— transverse strain . . . 250
Galton, iron bridge at . . . 207
Gauthey, observation of the falling arch . . . 151
——— table by, of rupture in arches . . . 151
Gear, friction . . . 47
Gearing . . . 57
Girders, nature of . . . 169
——— cast-iron, forms of . . . 179
———— effect of long-continued pressure on . . . 268
———— of strongest form of section . . . 179, 258, 263
——— trussed (wood) . . . 170, 258
———— (cast-iron) . . . 179
——— kinds of . . . 258
——— (hollow) of rectangular, elliptical, and circular section, formulæ for strength of . . . 188
Glasgow, wooden bridge at . . . 171
Gloucester Over bridge . . . 153
Gothic building, buttresses of . . . 140
Graham's compensating pendulum . . . 21
Granite used in London bridge . . . 162
——— strength of . . . 244
Gravity, Galileo's experiments on . . . 12
——— law of . . . 13
——— stability of centre of . . . 119
——— calculation of rules for estimating time, space, and velocity . . . 41
Grubenmann's bridges . . . 172
Gudgeons, friction of . . . 107, 110
——— form and strength of . . . 57
Gyration, determination of centre of . . . 30, 43

Hall, Capt. Basil, mention of an American rope-bridge . . . 208
Halle aux Blès, roof of . . . 134
Hammering, influence of, on tenacity of metals . . . 239
Hammersmith suspension bridge . . . 226
Harrison's compensating pendulum . . . 21
Haslar Lake (Portsmouth), proposed suspension bridge over . . . 226
Haunches of an arch, effect of much material in . . . 149
Headslocks for turning lathes . . . 73

PAGE
Higson's friction coupling . . . 70
Historical sketch of theoretical dynamics . . . 43
Hodgkinson on the experiments relative to the tubular bridges . . . 208
Hodgkinson's experiments on the strength of materials (See *Strength of Materials*)
Hoe's fast printing machine . . . 94
Hooke's wheel . . . 47
Hollow spandrels (Telford's) . . . 158
Humboldt, mention of South American bridge by . . . 208
Hungerford Market suspension bridge . . . 224
Hypocycloidal teeth for wheels . . . 51

Impact, laws of . . . 9
——— and motion on beams . . . 255
——— power of hot and cold blast iron to resist . . . 256
——— of railway carriages on the rails, experiments on . . . 256
Inclination of roadway in bridges . . . 152
Inclined plane, motion on . . . 14
Inertia, moment of . . . 27, 42
————— in beams . . . 254
——— of matter . . . 3
Insects, tubular skeleton of . . . 259
Intermittent motion in wheelwork . . . 68
Intrados and extrados of an arch . . . 145
Iron as a bridge-building material . . . 152, 177
—— arches, expansion of . . . 178
—— bridges (See *Bridges*.)
———— table of . . . 231
———— Busse's girder . . . 182
—— expansion of . . . 152, 178
—— oxydation of . . . 178
—— sulphate of, to preserve timber . . . 168
—— wire, tensile strength of . . . 216
—— plate, tensile strength of . . . 238
—— corrugated strength of . . . 187
—— roof trusses . . . 133
—— cast, character of . . . 237, 263
——— compressive strength of . . . 242
——— detrusive strength of . . . 267
——— effect of temperature on . . . 266
——— elasticity of . . . 264
——— hot and cold blast, chemical analysis of . . . 263
——— pillars, strength of . . . 245
——— tensile strength of . . . 237
——— transverse strength of . . . 261
——— torsion of . . . 269
—— wrought, compression of . . . 243
———— detrusive strength of . . . 267
———— elasticity of . . . 267
———— tensile strength of . . . 238
———— transverse strength of . . . 266
Involute teeth for rack and pinion . . . 51
——— of a circle, generation of . . . 54

Jamaica, time of oscillation of the pendulum at . . . 18
James River falls, lattice bridge over . . . 170
Jhoola bridge . . . 207
Joints of arch stones, bad filling in of . . . 159
—— when affected by a tensile force . . . 239

Kandel, bridge over the . . . 171
Kater's compensating pendulum . . . 22
Kerfed timber . . . 131
Key-stone of arches . . . 159

PAGE
King-post, nature of . . . 125
Knots in timber beams, to detect . . . 130
Kyan's timber-preserving process . . . 168

Ladykirk and Norham bridge . . . 176
Larnier, roof truss by . . . 129
Lary bridge, foundations of . . . 154
Lathes, their construction . . . 71
Latitude, effects of on the pendulum . . . 19
Latitudes, centrifugal force at different . . . 39
Lattice bridges, iron . . . 182
———— timber . . . 170
Lavé's girder . . . 257
Lever, the bell-crank . . . 59
Links (chain bridges), sizes of . . . 214
Little's printing machine . . . 91
Loads on suspension bridges, effect of . . . 211
London bridge, centre of . . . 138
———— description of . . . 160
Long's bridges . . . 171
Lubricants, influence of on friction . . . 107

Machinery, elements of . . . 58
Machine, drilling (Nasmyth's and Whitworth's) . . . 84
——— forge hammer, steam (Nasmyth's) . . . 79
——— lathe (Scott and Sinclair's) . . . 71
——— planing (Whitworth's) . . . 76
——— printing . . . 88
———— (Little's) . . . 91
———— Hoe's fast . . . 94
———— Applegath's patent (*Times*) . . . 111
——— punching and plate cutting (Nasmyth's) . . . 82
——— wheel-cutting (Collier's) . . . 86
Machines, centre of percussion in . . . 30
Mahavillaganga (Ceylon), bridge over the . . . 174
Maidenhead (Railway) bridge . . . 157
Mansarde roof . . . 126
Martin's compensating pendulums . . . 22
Mass, definition of . . . 1
Materials for bridge building . . . 151
Mathematical illustrations of dynamical problems . . . 40
Mâtura (Magazin de), roof over the . . . 130
Maxence, St., bridge, abutments of . . . 157
Measure of forces, parallelogram for . . . 118
Medium, effect of, on projectiles . . . 25
Menai suspension bridge, effect of temperature on . . . 213
———— description of . . . 220
—— tubular bridge . . . 184
Mercury, bichloride of, to preserve wood . . . 168
Mersey, proposed suspension bridge over . . . 216
Metal, increased strength of mixtures of . . . 239
Miers, mention of Chilian rope bridge . . . 208
Migneron's proposed Pont de la Cité . . . 130
Merulius lachrymans, or dry rot . . . 168
Modulus of elasticity . . . 234
Moment of a force . . . 118
——— inertia of geometrical figures . . . 42, 254
——— a wall supported by a buttress . . . 140
———— assisted by a shore . . . 140
———— supporting earth . . . 142
———— pressed by a sloping roof . . . 140
Momentum, calculation of . . . 7
——— methods of estimating . . . 11
Montrose suspension bridge . . . 226
Morin's friction measuring apparatus . . . 101

PAGE
Morin's tachometer . . . 102
Moscow riding house, roof truss of . . . 130
Motion, Newton's laws of . . . 3
——— parallel . . . 63
——— pendulous . . . 22
——— progressive and rotatory of a ball struck out of the line through its centre . . . 34
——— rolling . . . 36
Movements, mechanical . . . 58
Mont Blanc, time of oscillation of a pendulum on . . . 18
Muschenbroek's experiments on wood . . . 236

Nasmyth's punching and plate-cutting machine . . . 83
——— steam forge-hammer . . . 79
Neill and Crawford's malleable iron roofs . . . 134
Neuilly bridge . . . 152, 153
——— centre of . . . 137
Neutral axis, position of, in a girder . . . 169
——— nature of . . . 250
Newton, rule for resistance of air to moving bodies . . . 251
Newton's laws of motion . . . 3
Niagara suspension bridge . . . 225
Nicholson's iron front roof truss . . . 328
Nogent bridge, Perronet's observation of its settlement . . . 150
Norman roof . . . 133

Odontograph, Mr. Willis's . . . 55
Onondaga Creek Valley, bridge over . . . 171
Oscillation, centre of calculated . . . 27, 48
——— in a pendulum . . . 29
Oscillations in suspension bridge from loads and winds . . . 211
Ottawa, bridge over . . . 176
Oxydation of iron . . . 178

Pantheon, dome of the . . . 135
Palladio's bridge over the Cismone . . . 172
Parabolic ribs for roof trusses . . . 131
——— path of a projectile . . . 24
——— arch . . . 146
Parallel motion . . . 63
Parallelogram of forces . . . 4, 40
——— the mechanical . . . 118
Parallelopiped, three principal axes of rotation of . . . 36
Paris, fall of a suspension bridge at . . . 215
Pasley's experiments on revetments . . . 141
Patapsco river railway bridge . . . 173
Paul, St. (Rome), roof truss of the church of . . . 127
Pendulous motion in bodies . . . 22
Pendulum, the convertible . . . 29
——— compensating . . . 21
——— cycloidal . . . 17
——— determination of the length of a seconds' . . . 17
——— effect of distance from the earth's centre on . . . 18
——— a change of latitude on . . . 19
——— rules to calculate and correct . . . 18
——— principle of the . . . 16
Percussion, centre of . . . 29, 61
Permanent axes of rotation . . . 35
Perronet's observation of settlement in an arch . . . 150
——— centre for Neuilly bridge . . . 137
——— method of removing centres . . . 138
Pesth, suspension bridge at . . . 225
Piers, foundations of . . . 153
——— various thickness of in bridges . . . 155

PAGE
Piles, Wiebeking's method of preserving . . . 168
Pillars, strength of, under compression . . . 245, 271
——— of different materials, strength of . . . 249
Pin-wheels . . . 46
Piscataqua, timber bridge over the . . . 174
Pitch-circle of a wheel . . . 47
Planing machine (Whitworth's self-acting) . . . 76
Plate-bande or flat arch . . . 144
Plate-cutting machine . . . 83
Polarized light as a detector of the neutral axis . . . 250
Polygon, the mechanical . . . 129
——— timber arches, fault of . . . 172
Practices of engineers very various in bridge-work . . . 157, 159
Preserving timber, agents for . . . 168
Pressure of arch-stones on the centre . . . 136
——— earth against revetments . . . 141
——— long continued, effect of on pillars . . . 249
Printing machine, construction of the . . . 88
——— the *Times* . . . 111
Progression with rotatory motion . . . 34
Projectiles, Galileo's theory of . . . 23
——— resistance of a medium to . . . 24
——— rules to calculate the flight of . . . 24
Pryd, Pont ŷ, description of . . . 167
——— failure of . . . 149
Prussian method of bending timber . . . 131
Pulleys, fast and loose, speeds by . . . 68
Punching machine . . . 82

Quito, time of oscillation of a pendulum at . . . 19

Rafter, angle of inclination of, to give the least pressure on a tie-beam . 125
——— bow-and-string . . . 124
——— calculation of thrust of, on a roof . . . 124
Railway-bars, deflection of, by passing trains . . . 256
——— carriages, pendulous motion of . . . 23
——— curves, compensation of centrifugal force in . . . 40
——— iron roof truss . . . 133
——— viaduct, fall of through bad foundations . . . 153
Raised tie-beams . . . 132
Range of projectiles . . . 24
Rectangular tubes, great strength of . . . 188
Rectilinear converted into circular motion . . . 59
Regnitz, bridge over the . . . 175
Remington's timber bridges . . . 170
Rennie's experiments on friction . . . 106
——— observation of expansion in the Southwark bridge arches . . 178
——— values for tensile strength of cast-iron . . . 237
Resistance of air to projectiles . . . 25
——— limiting angle of . . . 107
Revetments, nature of . . . 141
Reversing movements . . . 62, 77
Ribs (for timber arches), curvature of . . . 175
—— with tie-beams for bridges . . . 177
Ritter's bridge over the Kandel . . . 171
Riveting adopted in the Menai tubes . . . 197
Robison's observation of the falling arch . . . 151
Rod, the place of its centre of oscillation . . . 28
——— gyration . . . 30
—— to find its moment of inertia . . . 42
Roof, bow-and-string rafter . . . 124
—— curb or mansarde . . . 126
—— curved-beam (Delorme's) . . . 130
—— by M. Larnier . . . 129

PAGE
Roof, iron, at Birmingham (railway) . . . 133
—— of Birmingham theatre . . . 129
———— Drury Lane theatre . . . 129
———— the Magazin de la Mâture . . . 130
———— Moscow riding house . . . 130
—— Norman . . . 133
—— of St. Paul (Rome) . . . 127
———— the Teatro d'Argentino . . . 128
—— simple rafter . . . 123
—— truss, general . . . 127
—— —— Nicholson's . . . 128
Rope bridges of central Asia and America . . . 207
Rotation of arch stones on their edges . . . 148
———— cause of . . . 26
———— about a fixed axis, investigation of . . . 26, 42
———— of a body round the earth . . . 38

Savart's laws of torsion . . . 268
Schaffhausen timber bridge . . . 172
Schuylkill, timber bridge over . . . 175
Scott and Sinclair's lathe . . . 76
Screw-cutting machine . . . 75
Sea level, effect of vertical distance from on the pendulum . . . 18
Second's pendulum, determination of . . . 17
Section of chain, variation of (in suspension bridges) . . . 210
———— strongest form of, for tensile strain . . . 236
Self-registering dynamometer . . . 102
—————— tachometer . . . 103
Semi-circular arch . . . 145
———————— calculation of rupture in . . . 147
———————— for bridges . . . 152
Settlement of timber arches . . . 174
Shafts, coupling, for driving machinery . . . 69
———— of wheelwork . . . 57
Shores, calculation of their effect . . . 140
Shortest axis, tendency of bodies to rotate round the . . . 36
Sinking of arches on removing their centres . . . 138, 166
Slide rest, in turning lathes . . . 73, 75
Smeaton's cutwaters . . . 155
————— error in the foundations of Hexham bridge . . . 154
————— method of lightening arch spandrels . . . 158
South America, rope bridges in . . . 208
Southwark bridge . . . 178, 206
Span for suspension bridges . . . 213
Spandrels, space (falling bodies) . . . 41
————— filling in of, injudicious . . . 158
————— of Chester bridge . . . 163
—————— London bridge . . . 161
—————— the Pont ŷ pryd . . . 149, 168
—————— Turin bridge . . . 166
—————— Waterloo bridge . . . 158
—————— Wellesley (Limerick) bridge . . . 158
—————— Westminster bridge . . . 188
Speed-pulleys . . . 65
Speeds, wheels for giving three . . . 67
Sphere, its three principles, axes of rotation . . . 35
————— moment of inertia . . . 42
Spindles of wheelwork . . . 57
Spontaneous rotation, centre of . . . 34
Springing course in the arch . . . 158, 165
Spur wheels . . . 45
Stability of centre of gravity . . . 119
Statics in reference to structures . . . 117
Steam vessels, dynamometer for . . . 97

PAGE
Stems of plants, tubular form of 260
Stirling's cast iron 262
Stone, as a material for bridge building 152
—— bridges, table of 230
—— compression, strength of 243
—— physical character of 244
—— equilibrium of structures in 139
—— pillars, strength of 248
—— transverse strength of 267
Storms, effect of, on suspension and other bridges 194, 202, 213, 222
Strength of a beam, to estimate 257
——— general comparison of 267
——— of materials 233
Struts in a roof, function of 125
Suspension bridges of Asia and America 207
———————— principles of 208
———————— table of 232
———————— See *Bridges.*

Tachometer, Morin's 102
Tarabita (rope suspension bridge) 208
Tees, early chain bridge over 208
Teeth, addendum of 53
—— forms of, for wheels 49, 55
———————— rack and pinion 51
———————————— trundle 51
—— least number for wheel and pinion 51
—— relative number of in a pair for different arcs of action 52
—— strength of 57
Telford's cutwaters 155
——— paved foundations for piers 155
——— light spandrels 158
——— experiments on strength of iron bars 216
—————————— iron wire 219
Temperature, effect of on suspension bridges 213
—————————— the Conway tube 194
———————————— pendulum 20
Tenacity of metals increased by hammering 239
Tensile strength of cast iron 237
———————— wrought iron 238
———————— iron wire 239
—————————— plate 238
———————— mixed metals 239
———————— riveted plates 239
———————— timber 236
Tension, laws of 235
——— in a chain bridge 209
—————————— variation of 211
Tewkesbury iron bridge 207
Tie-beam 123
Tilt hammer 61, 79
Timber bridges, table of 231
——————— See *Bridges.*
——— curved for roof-houses 130
——————— bridge arches 173
——— preserving processes for 168
——— Prussian mode of bending 131
——— tensile strength of 236
——— pillars, strength of 248
Time, a specific feature in dynamics 1
—— of flight of projectiles 24
——— descent of a body by gravity 18, 41
Times printing machine 111
Tongueland bridge, hollow spandrels of 158

PAGE
Torsion, principles of 267
Town's lattice bridges 170
Transverse strain 250
Traverse movement 61
Traversing machine 163
Tredgold's experiments on cast iron 237, 262, 263
Trestle bridges 171, 181
Trezzo (Milan) ancient bridge at 158
Triangular framing, strength of 122
Trinita (Santissima) bridge at Florence 167
Trundle 45
Trussed girders (See *Girders.*)
Trusses for bridge spans 171
Tubes, ability of, to resist strains 202
—— compression, strength of 243
Tubular girders, Fairbairn's patent 183
———— formulæ to calculate strength of 188
———— comparison with solid 259
——— bridges over the Conway river and Menai straits 184
——— structures in nature 259
Turin, bridge at 164
Turning-lathe 73

Unguents, effect of in reducing friction 107
Uniform motion 2
———— of two bodies, their local relation after a lapse of time 6
Union suspension bridge over the Tweed 226
Utica and Syracuse railway bridge 171

Velocity of falling bodies, to find 13, 41
——— combination in wheel-work 64
——— nature of 2
——— of projectiles, overcome by the resistance of the air 25
——— a rolling body 36
——— angular, of rotating bodies 17
——— of wheels, to produce variable 66
Vibration, effect of, on wrought iron 179
——— of suspension bridges 211
Vicat's experiments on iron wire 36

Walls, church 140
—— curved, on London and Birmingham Railway 143
—— equilibrium of 139
———— under any pressure 144
—— supported by buttresses 140
———— shores 141
Water, elliptico-circular arch to facilitate passage of 153
Waterloo bridge 156, 167
Wellesley bridge spandrels 158
Wernwag's bridge over the Schuylkill (U. S.) 175
Westminster bridge 152, 156, 158
Wheel and axle, acceleration of by motion of through weights 31
———— pressure of suspended weight on the spindle of when revolving 32
———— proportions of, to raise a weight in the shortest time 33
Wheels, annular 46
—— bevelled 46, 53
—— band 63
—— buff (friction) 47, 66
—— centre of gyration in 30
—— crown 46, 65
—— eccentric 60
—— with epicycloidal teeth 49
—— with involute teeth 54

PAGE
Wheels, Hooke's 47
——— with internal pinion 63
——— instruments to describe the teeth of 56
——— pin 46
——— pitch circle of 47
——— and racks 51
——— pair of (ordinary construction) 53
——— (toothed) to set out 48
——— for transmitting two speeds 67
——— White's 63
Wheelwork 45
Whitadder, bridge over the 170
Whitworth's planing machine 76
———— drilling and boring machine 85
Wiebeking's bridges 174
———— method of preserving piles in water 168
Willis's odontograph 56
Windlass, stability of 120
Winds, effect of on tubular and suspension bridges 202, 213, 222
Wire rope for suspension bridges 214
Wood, as a bridge-building material 152, 168
——— preservation of, against fungi 168
——— transverse strength of 268
——— tensile strength of 236
——— compression, strength of 241
——— bridge of (See *Bridges*).

Zinc, bichloride of, for preserving timber 168

G. Woodfall and Son, Printers, Angel Court, Skinner Street, London.

2

R XII Cost.
LI. XR Prec.

OCT 28 1892
JAN 12 1893

Dynamics, construction of machinery
Cabot Science 006870969
3 2044 092 014 950